CS 6840 – Algorithmic Game Theory (3 pages) Spring 2012

Lecture 30 Scribe Notes

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1 Lecture 30 – Monday 02 April 2012 - A Common Framework

1.1 Overview of next two lectures

- checking how far we've gotten to greedy and item auction goals
- put two auction types into common framework, so we can make general statements

1.2 Recap of 2 Auctions

Both Greedy and Item

- are combinatorial auctions
- $S =$ set of items
- $v_i(A) \geq 0$ value for set A, user i
- assume free disposal: $v_i(A) \geq v_i(B)$ if $A \supseteq B$

where the latter two assumptions are the only assumptions we have on value so far.

1.2.1 Greedy (Wednesday) - from Lucier-Borodin paper

- bid: sets, and bids $b_i(A)$ (i.e., not all sets need bids)
- select max $\frac{b_i(A)}{\sqrt{|A|}}$ among sets A still available
- critical value pricing (i.e. smallest bid that would have won item; natural analog to second price)

1.2.2 Item-Auction (Friday)

- bid: bid per item $= b_i(a) \geq 0$
- for each item, run first or second price to determine winner and price

1.3 (λ, μ) Framework

1.3.1 Greedy

From original analysis, we have

1. If $\Theta_i(A)$ is critical bid for set A, then

$$v_i(O) - \Theta_i(O) \leq v_i(A_i)$$

   if A_i is allocated by algorithm for all sets O (in a Nash). In other words, player i could just bid for set O.

2. If algorithm is a c-approximation, then

$$\sum_i \Theta_i(O_i) \leq c \sum_i b_i(A_i)$$

   if A_i is algorithm's allocation and O_i is optimal.

3. Assumes bidders are conservative: $b_i(A) \leq v_i(A)$.

Claim: This is a (λ, μ)-smoothness proof. Recap of smoothness:

Smoothness had cost and utility versions. This is a *utility* version. We have strategy s^* that leads to solution maximizing $\sum_i U_i(s^*)$, where $s^* = (s_1^*, ..., s_n^*)$ and $U_i(s^*)$ is utility of player i in resulting outcome.

If, for all s,

$$\sum_i U_i(s_i^*, s_{-i}) \geq \lambda \sum_i U_i(s^*) - \mu \sum_i U_i(s)$$

then game is (λ, μ)-smooth. This implies

1. if s is Nash, and s^* is opt, then

$$\sum_i U_i(s) \geq \frac{\lambda}{1+\mu} \sum_i U_i(s^*)$$

   (i.e., price of anarchy bound), and

2. if all players have no regret $s^1, ..., s^T$, then

$$\frac{1}{T} \sum_{t=1}^{T} \sum_i U_i(s^*) \geq \frac{\lambda}{1+\mu} \sum_i U_i(s^*)$$

So can we convert greedy auction bound to (λ, μ)-smoothness bound form?

- idea: one optimal strategy s^* for player i is to bid for O_i only; $b_i(O_i) = v_i(O_i)$
  - critical value $= 0$ since sets disjoint
  - this is as good as possible
- so take $\lambda = 1$, $\mu = c$

Issue: How do we evaluate $\sum_i U_i(s_i^*, s_{-i})$?

$$\begin{aligned}\sum_i U_i(s_i^*, s_{-i}) &= \sum_i v_i(O_i) - \sum_i \Theta_i(O_i) \\ &\geq \sum_i v_i(O_i) - c\sum_i v_i(A_i)\end{aligned}$$

due to c-approximation and conservation assumption.

1.3.2 Item Auctions

Interjections:

- Fire Drill!!. 15 minute loss.
- Note: we never even proved Nash exists, and/or if we can find it. But we do know learning exists and has nice properties.

Note that first-price doesn't fit into (λ, μ) framework. Second price *almost* does:

Suppose $O_1, ..., O_n$ is an optimal allocation. Then

- for each item a, $p(a) = max_i\{b_i(a)\}$
- if $v_i(O_i) \geq \sum_{a\in O_i} p(a)$, then bids $b_i = p(a) + \epsilon$ for all $a \in O_i$.

At Nash,

$$v_i(A_i) \geq v_i(O_i) - \sum_{a\in O_i} p(a)$$

(otherwise they would have bid for O_i). But then

$$\begin{aligned}\sum_i v_i(A_i) &\geq \sum_i [v_i(O_i) - \sum_{a\in O_i} p(a)] \\ &= \sum_i v_i(O_i) - \sum_a p(a) \\ &= \sum_i v_i(O_i) - \sum_i \sum_{a\in A_i} p(a) \\ &\geq \sum_i v_i(O_i) - \sum_i v_i(A_i)\end{aligned}$$

So this is *almost* smoothness with $\lambda = 1$, $\mu = 1$ (the right side is good). But there is a **problem:** No clear bidding strategy for O_i!. Remember we found bids by adding ϵ to price in current solution; in other words, solution s^* is not independent of s. More next time.