Finish Parallel Determinant & Perf Matching
Start flow

Recall: "clow" = "directed closed walk" **in bidirected K_n with self loops**

lowest numbered vertex (head) must
be at start + end of walk, and
not in the interior

"clow sequence" = sequence of clows whose
heads (lowest numbered vertices)
are in $\wedge$ strictly increasing order

$length(clow) = \#$ of hops

$length(clow\ sequence) = \sum lengths$

$weight(clow) = \prod (labels\ of\ edges)$

$weight(clow\ seq) = (-1)^{length-(\#\ clows)} \cdot \prod_{clow\ C} weight(C),$

For every perm of $[n]$ there is a corresp.
clow sequence.

Each cycle of the perm is a clow starting/ending
at its head, clows organized with heads
increasing.

$weight(clow\ seq\ corresp\ to\ \pi)$

$= $ contrib of π to determinant polynomial.

To sum up weights of all clow sequences,
use a DAG with vertices u_{hij}

$1 \leq h \leq i \leq n$, $1 \leq j \leq n$

and sink vertex t.
(u_{111} is the start vertex)

Edge $\quad u_{hij} \longrightarrow u_{h'i'j'} \qquad$ if ...

$\qquad$ (a) $h = h'$, $\quad j' = j+1$, $\quad i' > h$. $\quad$ label $\ -x_{ii'}$

$\qquad$ (b) $h < h'$, $\quad j' = j+1$, $\quad i' = h'$ $\quad$ label $\ + x_{ih}$

Also $\quad u_{hin} \to t \qquad$ for all h, i. $\qquad$ label $\ + x_{ih}$

A clow sequence of length n corresponds
to a perm. __if__ __and__ __only__ __if__ every
vertex occurs at most once.
(Ignoring 2nd occurrence of head in each clow.)

If clow seq $C_1, C_2, \ldots, C_k$ contains
a vertex at least twice, let
C_i be the highest numbered
clow containing a vertex j that
appears at least once more in
$C_j, C_{i+1} \ldots, C_n$.

C_1 C_2 7 C_3 (with labels 10, 9, j, 5, 4, 11, i, b) C_4 C_5

This 1:1 Corresp.

C_1 C_2 C_3 C_4 5
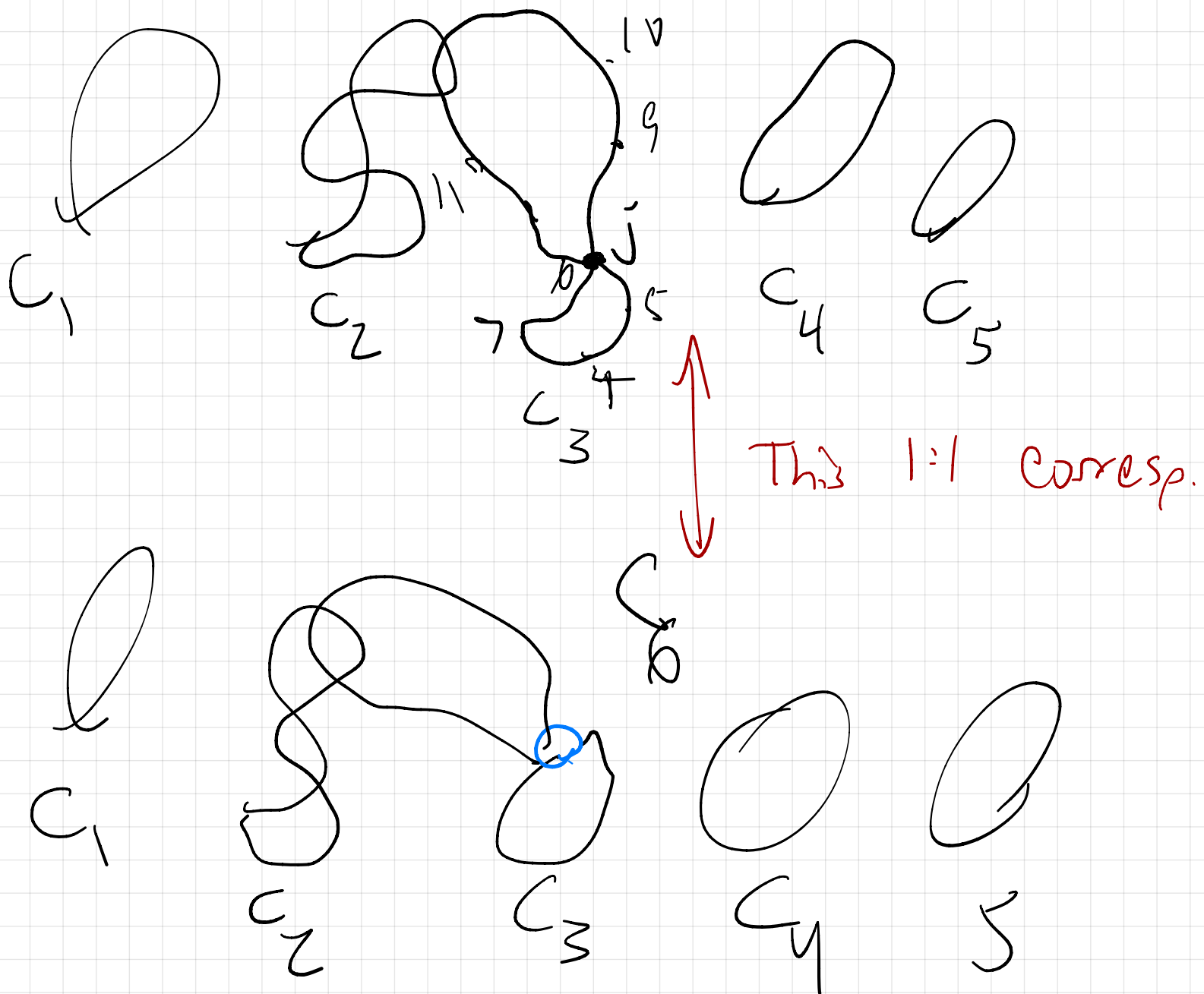
Mulmuley: Use random edge weights $w(e)$, sampled independently from $[100m]$.

① whp, min-weight matching is unique.

② when it is unique, it can be found by evaluating $\det(A)$ and various submatrices, where

$$A_{ij} = \begin{cases} 2^{w(e)} & \text{if} \quad e = (i,j) \in E \\ 0 & \text{o.w.} \end{cases}$$