Low Precision Arithmetic

CS6787 Lecture 10 — Fall 2021

Memory as a Bottleneck

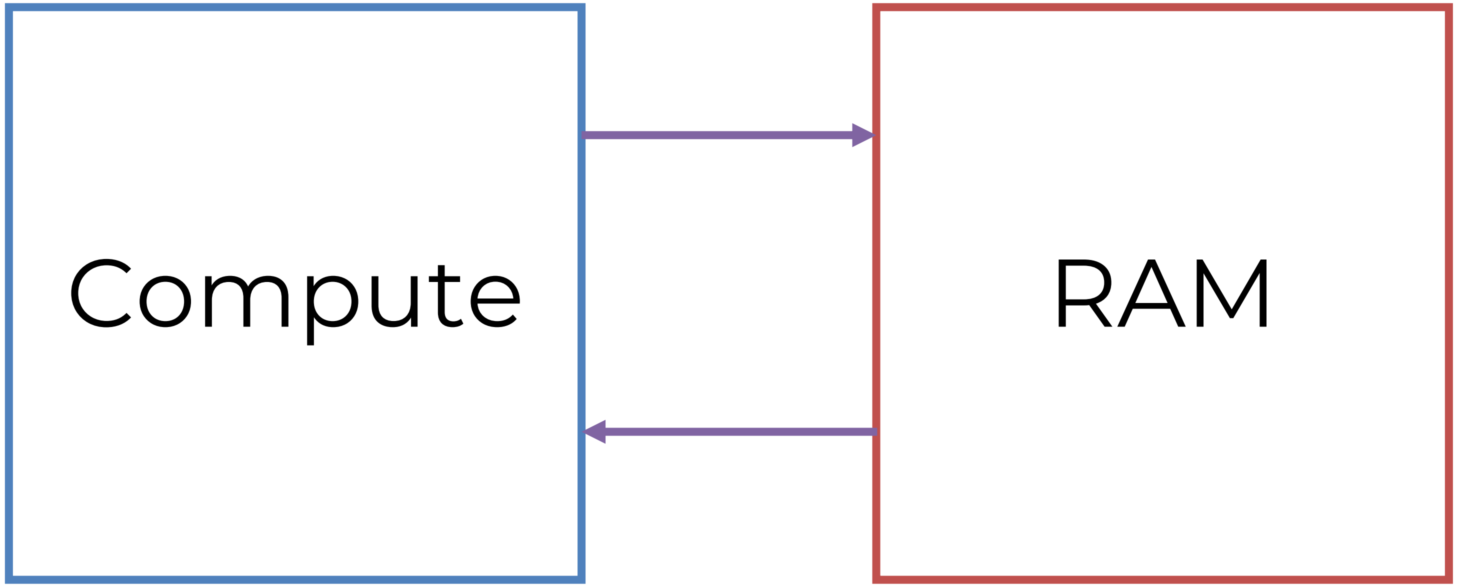
- So far, we've just been talking about **compute**
  - e.g. techniques to decrease the amount of compute by decreasing iterations

- But machine learning systems need to process **huge amounts of data**

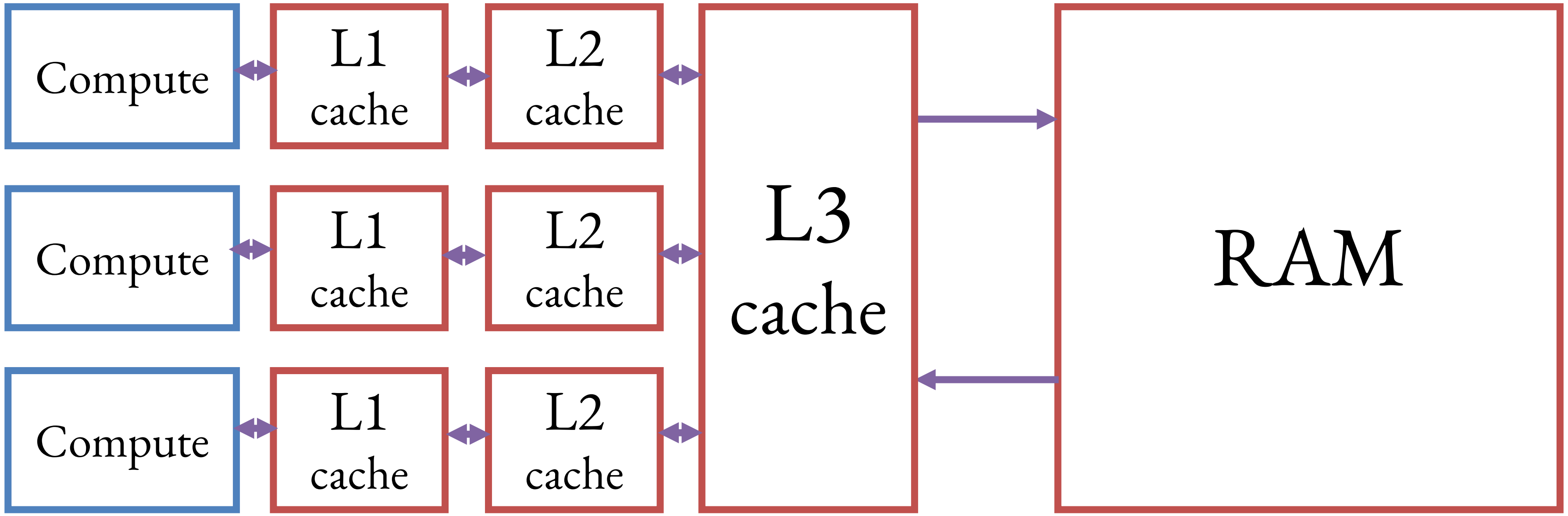
- Need to **store, update, and transmit** this data

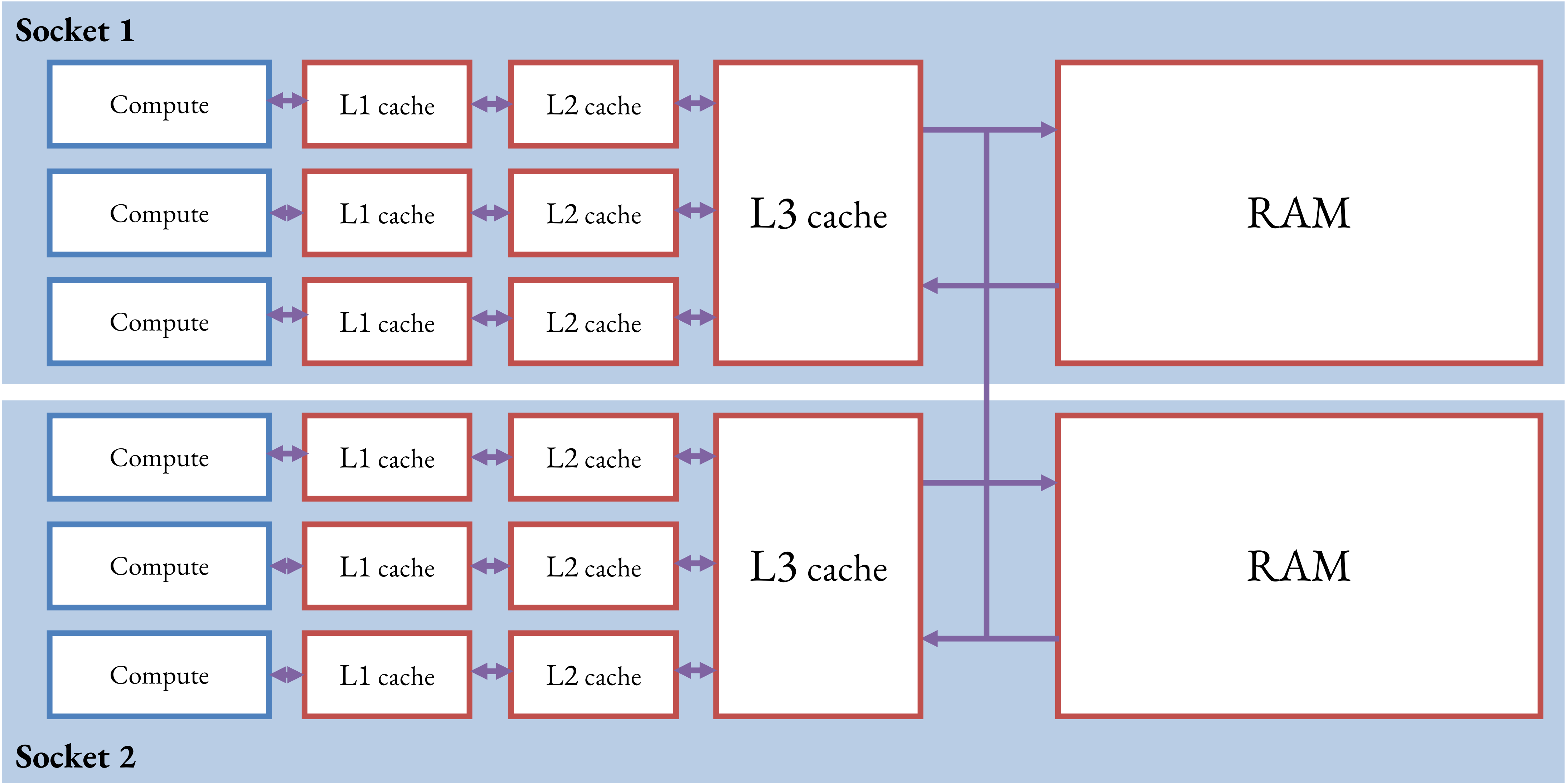
- As a result: **memory** is of critical importance
  - Many applications are memory-bound

Memory: The Simplified Picture



Memory: The Multicore Picture




Socket 1
Compute
L1 cache
L2 cache
Compute
L1 cache
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Compute
L1 cache
L2 cache
L3 cache
RAM
Compute
L1 cache
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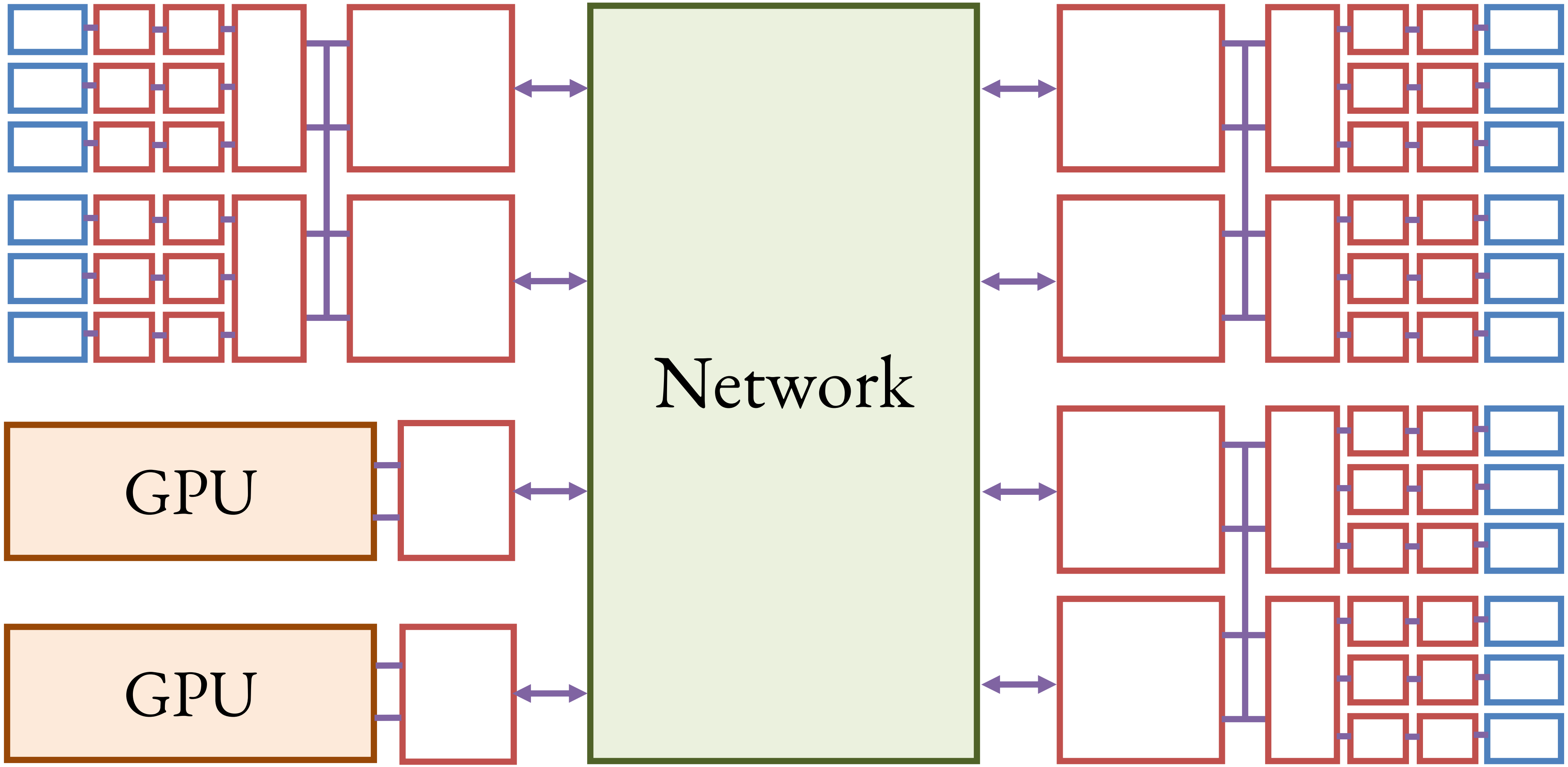
Compute
L1 cache
L2 cache
Compute
L1 cache
L2 cache
L3 cache
RAM
Socket 2

Memory: The Distributed Picture



What can we learn from these pictures?

- Many more **memory** boxes than **compute** boxes
  - And even more as we zoom out
- Memory has a **hierarchical structure**
- **Locality matters**
  - Some memory is closer and easier to access than others
  - Also have standard concerns for CPU cache locality

What limits us?

- **Memory capacity**
  - How much data can we store locally in RAM and/or in cache?
- **Memory bandwidth**
  - How much data can we load from some source in a fixed amount of time?
- **Memory locality**
  - Roughly, how often is the data that we need stored nearby?
- **Power**
  - How much energy is required to operate all of this memory?

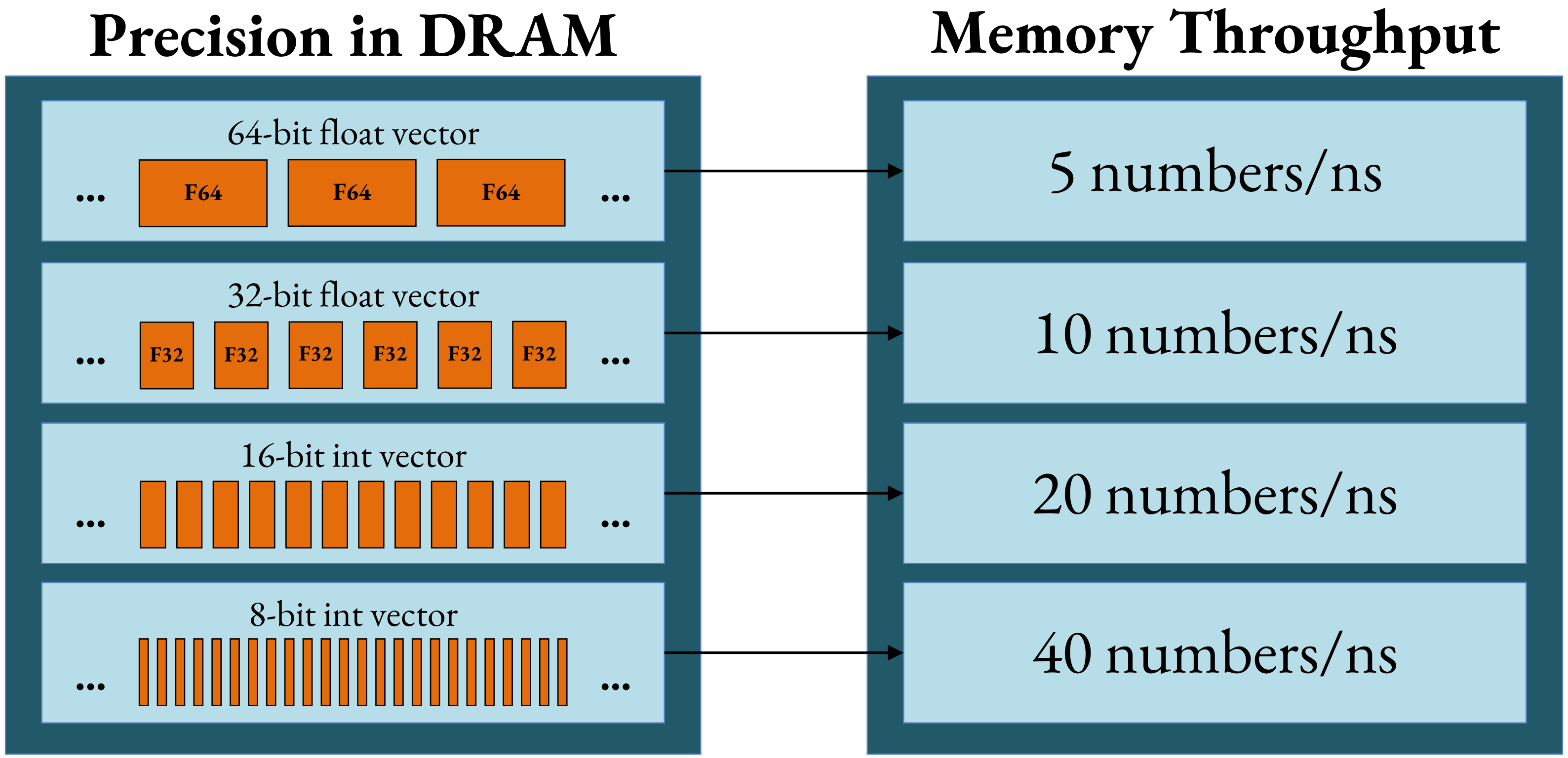
One way to help:
Low-Precision Arithmetic

Low-Precision Arithmetic

- Traditional ML systems use 32-bit or 64-bit **floating point numbers**
- **But do we actually need this much precision?**
  - Especially when we have inputs that come from noisy measurements
- Idea: instead use **8-bit or 16-bit numbers** to compute
  - Can be either floating point or fixed point
  - On an FPGA or ASIC can use arbitrary bit-widths

Low Precision and Memory

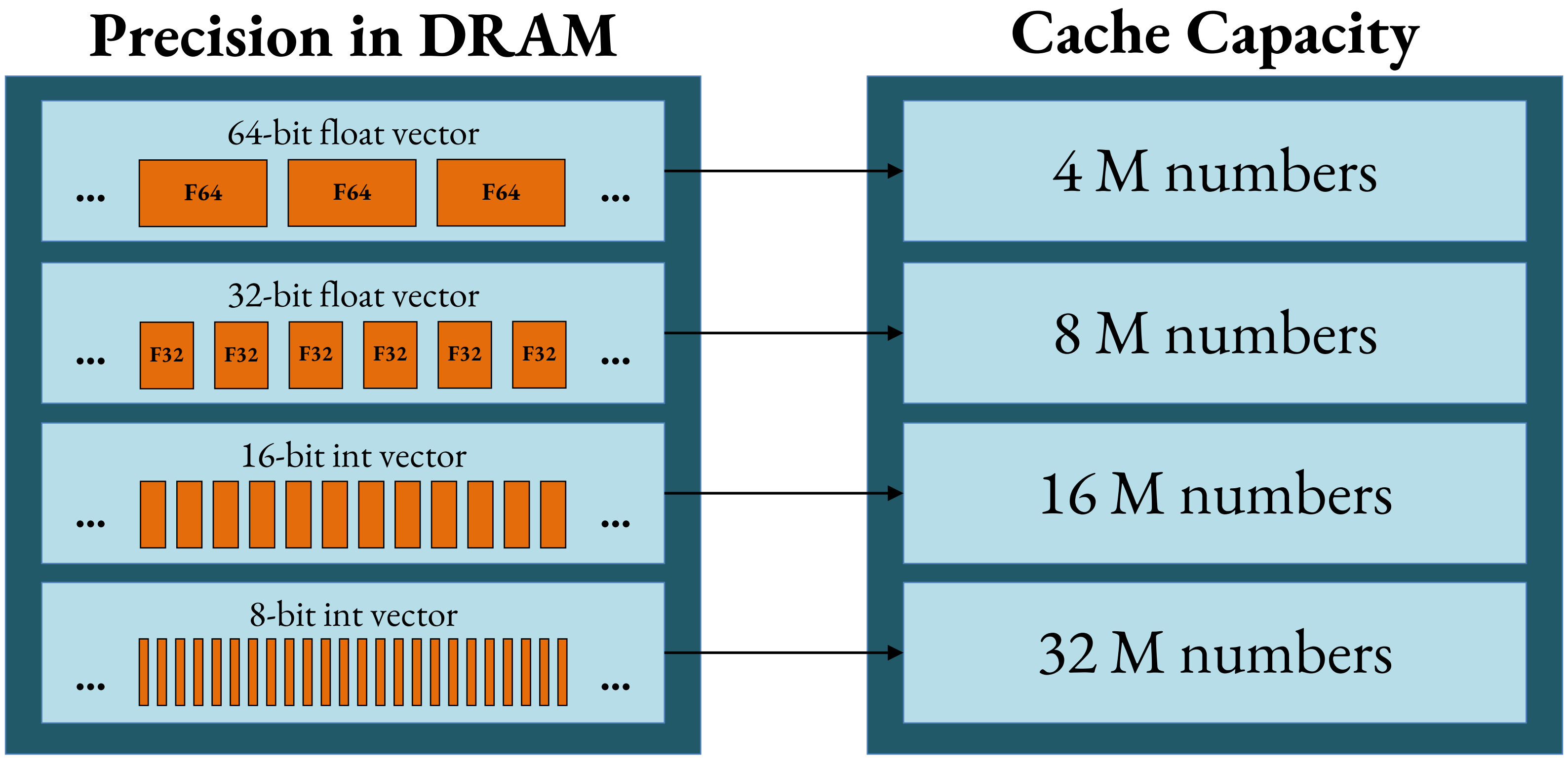
- Major benefit of low-precision: **uses less memory bandwidth**



(assuming ~40 GB/sec memory bandwidth)

Low Precision and Memory

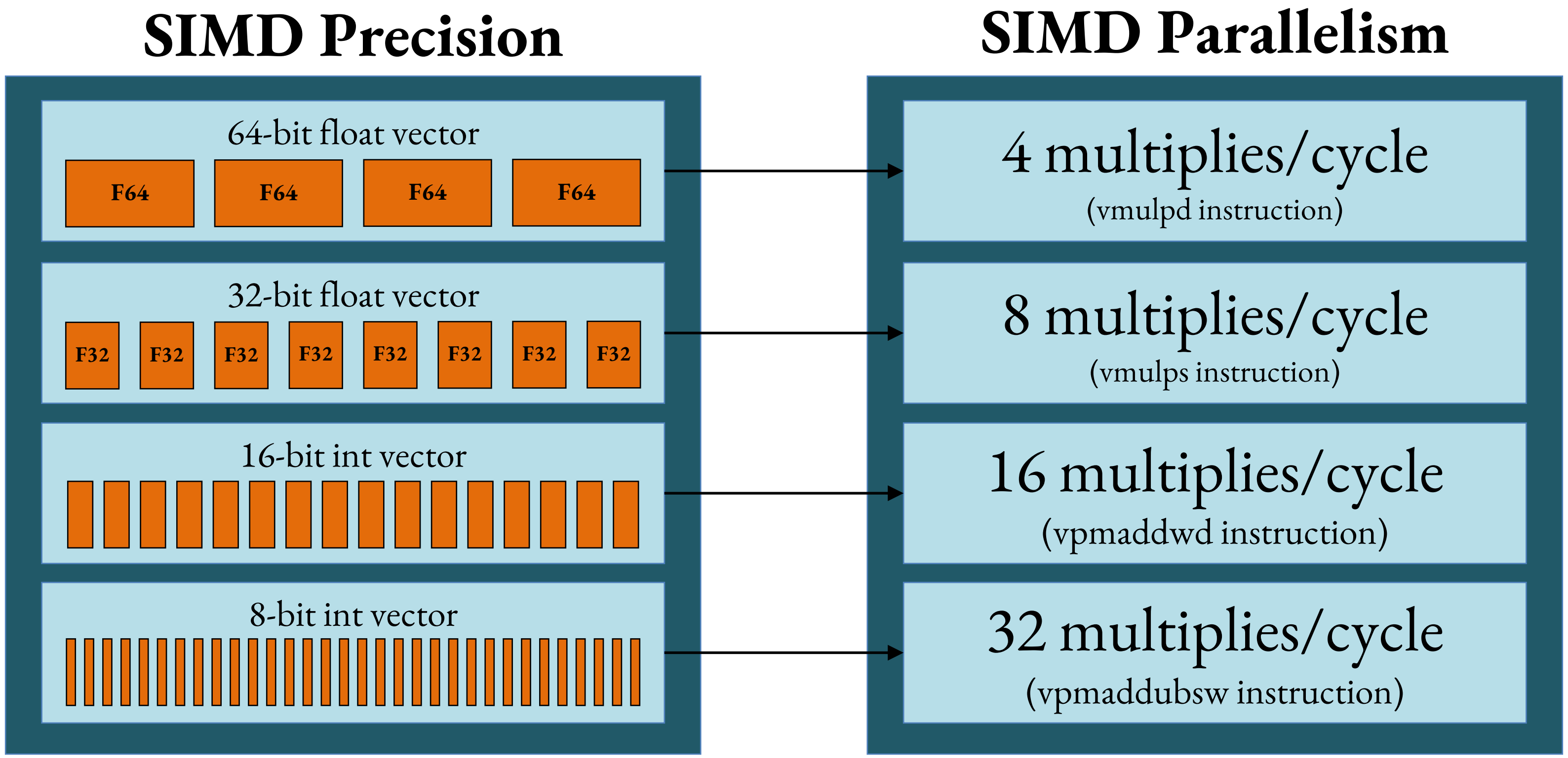
- Major benefit of low-precision: **takes up less space**



(assuming ~32 MB cache)

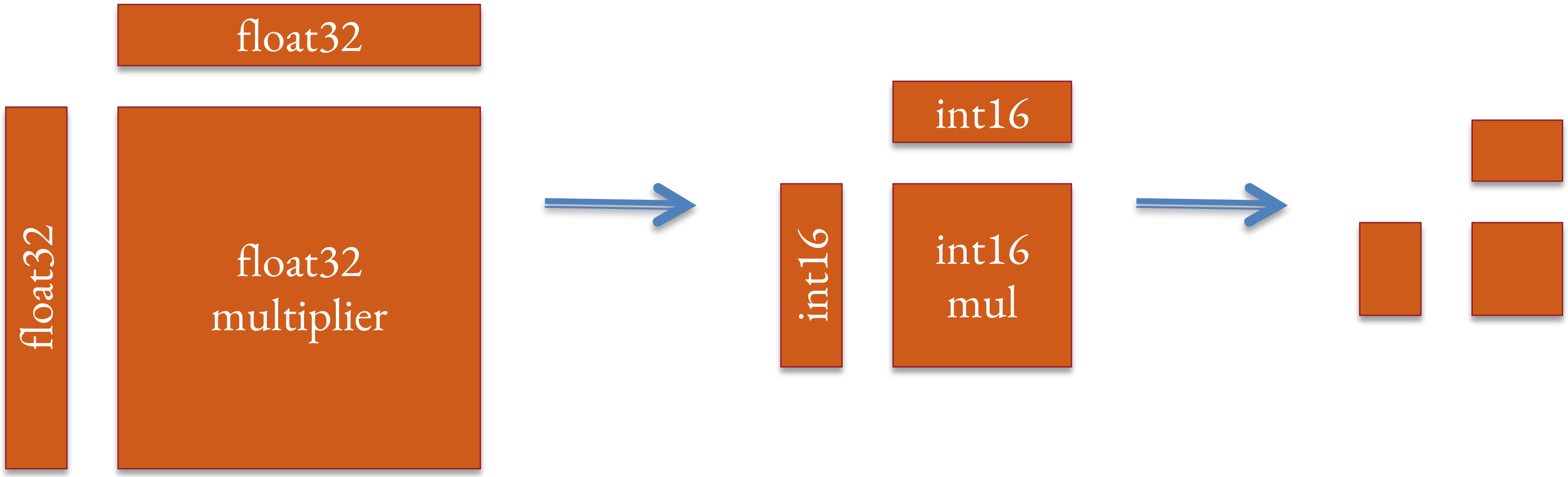
Low Precision and Parallelism

- Another benefit of low-precision: use **SIMD instructions** to get more parallelism on CPU

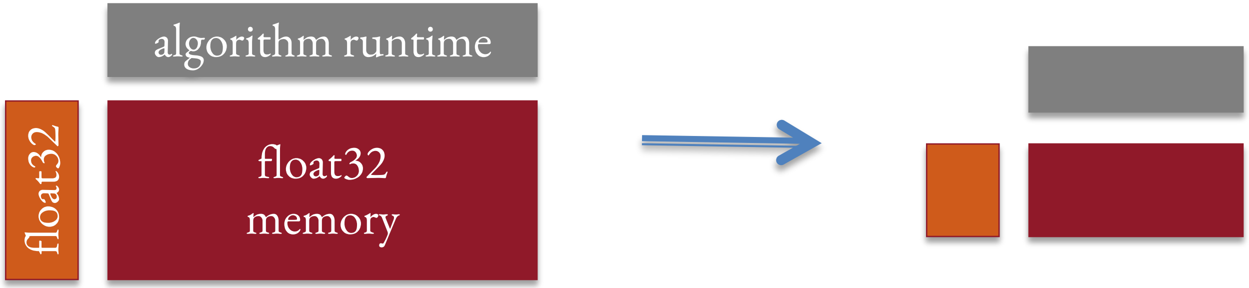


Low Precision and Power

- Low-precision computation can even have a super-linear effect on energy



- Memory energy can also have quadratic dependence on precision



Effects of Low-Precision Computation

- **Pros**
  - Fit more numbers (and therefore more training examples) in memory
  - Store more numbers (and therefore larger models) in the cache
  - Transmit more numbers per second
  - Compute faster by extracting more parallelism
  - Use less energy

- **Cons**
  - Limits the numbers we can represent
  - Introduces **quantization error** when we store a full-precision number in a low-precision representation

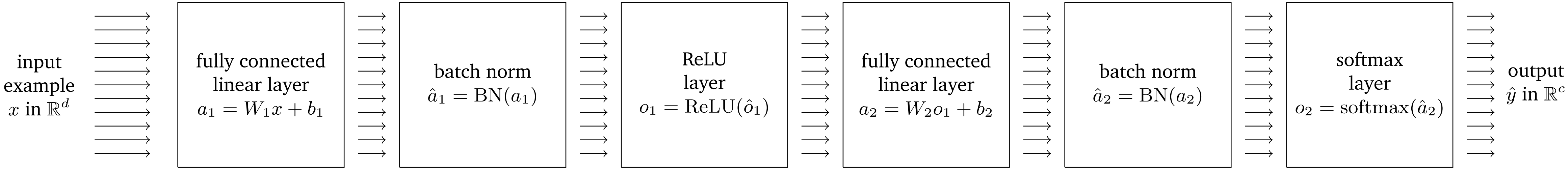
Numeric Formats in Machine Learning

How do we represent numbers as bit patterns on a computer?

A representative setup: DNN training

Many of the large-scale learning tasks we want to accelerate are deep learning tasks.

A **deep neural network (DNN)** looks like this:



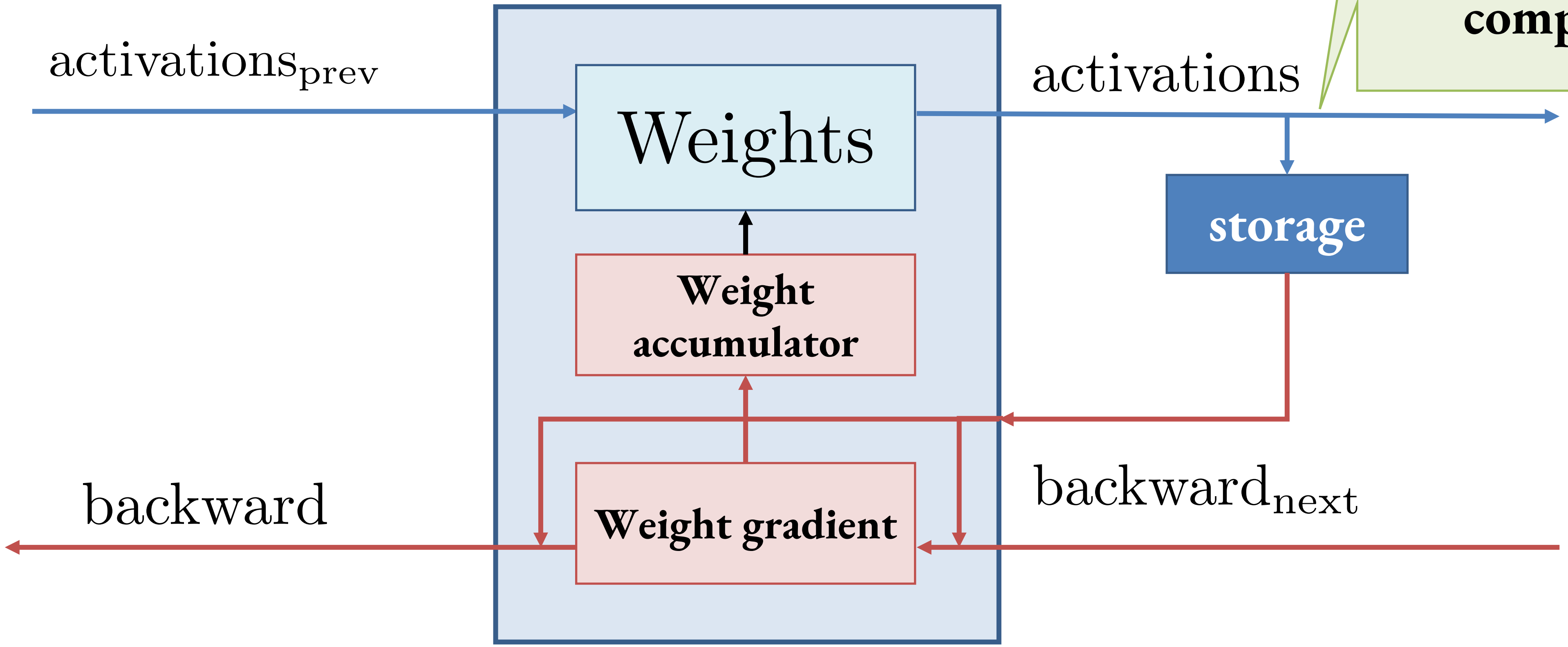
Many layers connected to each other in series.

To train, we compute the loss gradient and run stochastic gradient descent:

$$w_{t+1} = w_t - \alpha_t \nabla f(w_t; x_t)$$

A representative setup: DNN training

- Standard method of computing gradient for SGD uses backp
  - Computationally, it looks like this on the level of a single layer



All of the signals here are vectors of real numbers.

But how are they stored on a computer?

The standard approach
Single-precision floating point (FP32)

- 32-bit floating point numbers

31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
sign	8-bit exponent								23-bit mantissa																						

- Usually, the represented value is

$$\text{represented number} = (-1)^{\text{sign}} \cdot 2^{\text{exponent}-127} \cdot 1.b_{22}b_{21}b_{20}\ldots b_0$$

- Has a machine epsilon (measures relative error) of $\varepsilon_{\text{machine}} \approx 6.0 \times 10^{-8}$

An example

- Let's convert the number **6.5** to floating point.

$$
\begin{aligned}
6.5 &= 13 \times 2^{-1} = (8+4+1) \times 2^{-1} \\
&= 1101_b \times 2^{-1} = 1.101_b \times 2^2 \\
&= 1.101_b \times 2^{(129-127)} \\
&= 1.101_b \times 2^{(10000001_b-127)}
\end{aligned}
$$

```
1 10000001 10100000000000000000000
```

What is the machine epsilon?

Or, confusingly, twice this.

- Represents the relative error of the floating-point format
  - One half the distance between 1 and the next-largest floating point number
  - If there are **m** mantissa bits, $\varepsilon_{\text{machine}} \approx 2^{-m-1}$
  - Because the smallest representable number > **1** is $1 + 2^{-m}$

Relative error bound. If $x \in \mathbb{R}$ is any number in range of the format, and $\hat{x}$ is the nearest number representable in the format, then

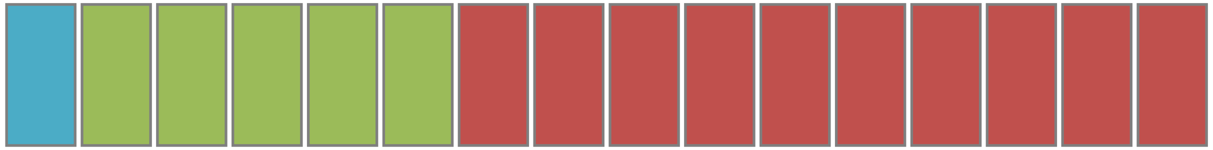
$$|\hat{x} - x| \leq \varepsilon_{\text{machine}} \cdot |x|.$$

Similarly, if $x, y \in \mathbb{R}$ are two floating-point numbers, $\star$ is any primitive numerical operation (e.g. $+$, $\times$, etc.), and $\circledast$ is the floating-point "version" of that op, then

$$|(x \circledast y) - (x \star y)| \leq \varepsilon_{\text{machine}} \cdot |x \star y|.$$

A low-precision alternative
FP16/Half-precision floating point

- 16-bit floating point numbers



1-bit sign | 5-bit exponent | 10-bit significand

- Usually, the represented value is

$$x = (-1)^{\text{sign bit}} \cdot 2^{\text{exponent}-15} \cdot 1.\text{significand}_2$$

Numeric properties of 16-bit floats

- A larger machine epsilon (**larger rounding errors**) of $\varepsilon_{\text{machine}} = 4.9 \times 10^{-4}$
  - Compare 32-bit floats which had $\varepsilon_{\text{machine}} \approx 6.0 \times 10^{-8}$
- A smaller overflow threshold (**easier to overflow**) at about 6.5×10^{4}
  - Compare 32-bit floats where it's 3.4×10^{38}
- A larger underflow threshold (**easier to underflow**) at about 6.0×10^{-8}.
  - Compare 32-bit floats where it's 1.4×10^{-45}

With all these drawbacks, does anyone use this?

Half-precision floating point support

- Supported on most **modern machine-learning-targeted GPUs**
  - E.g. efficient implementation as far back as NVIDIA Pascal GPUs

Pascal Hardware Numerical Throughput

GPU	DFMA (FP64 TFLOP/s)	FFMA (FP32 TFLOP/s)	HFMA2 (FP16 TFLOP/s)	DP4A (INT8 TIOP/s)	DP2A (INT16/8 TIOP/s)
GP100 (Tesla P100 NVLink)	5.3	10.6	21.2	NA	NA
GP102 (Tesla P40)	0.37	11.8	0.19	43.9	23.5
GP104 (Tesla P4)	0.17	8.9	0.09	21.8	10.9

Table 1: Pascal-based Tesla GPU peak arithmetic throughput for half-, single-, and double-precision fused multiply-add instructions, and for 8- and 16-bit vector dot product instructions. (Boost clock rates are used in calculating peak throughputs. TFLOP/s: Tera Floating-point Operations per Second. TIOP/s: Tera Integer Operations per Second. https://devblogs.nvidia.com/parallelforall/mixed-precision-programming-cuda-8/

- Good empirical results for **deep learning**

Micikevicius et al. "Mixed Precision Training." on arxiv, 2017.

Another common option
Bfloat16 — "brain floating point"

- Another 16-bit floating point number

1-bit sign

8-bit exponent

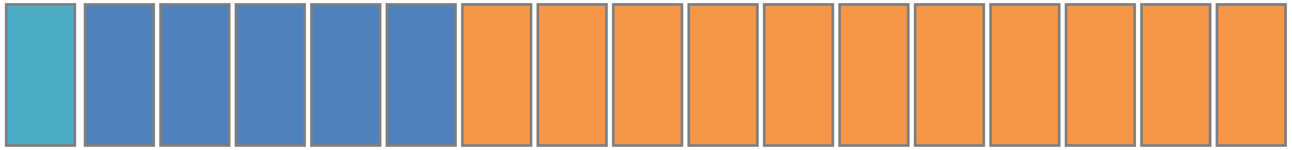
7-bit significand

$$\varepsilon_{\text{machine}} = 3.9 \times 10^{-3}$$

- Main benefit: numeric range is **now the same as single-precision float**
  - Since it looks like a truncated 32-bit float
  - This is useful because ML applications are **more tolerant to quantization error** than they are to overflow

An alternative to low-precision floating point
Fixed point numbers

- **p + q + 1** –bit fixed point number



1-bit sign

p-bit integer part

q-bit fractional part

- The represented number is

$$
\begin{aligned}
x &= (-1)^{\text{sign bit}} \left(\text{integer part} + 2^{-q} \cdot \text{fractional part} \right) \\
&= 2^{-q} \cdot \text{whole thing as signed integer}
\end{aligned}
$$

Arithmetic on fixed point numbers

- **Simple and efficient**
  - Can just use preexisting integer processing units
  - **Lower power** than floating point operations with the same number of bits
- **Mostly exact**
  - Can always convert to a higher-precision representation to avoid overflow
- Can represent a **much narrower range of numbers than float**
- **Has an absolute error bound, not relative error bound**

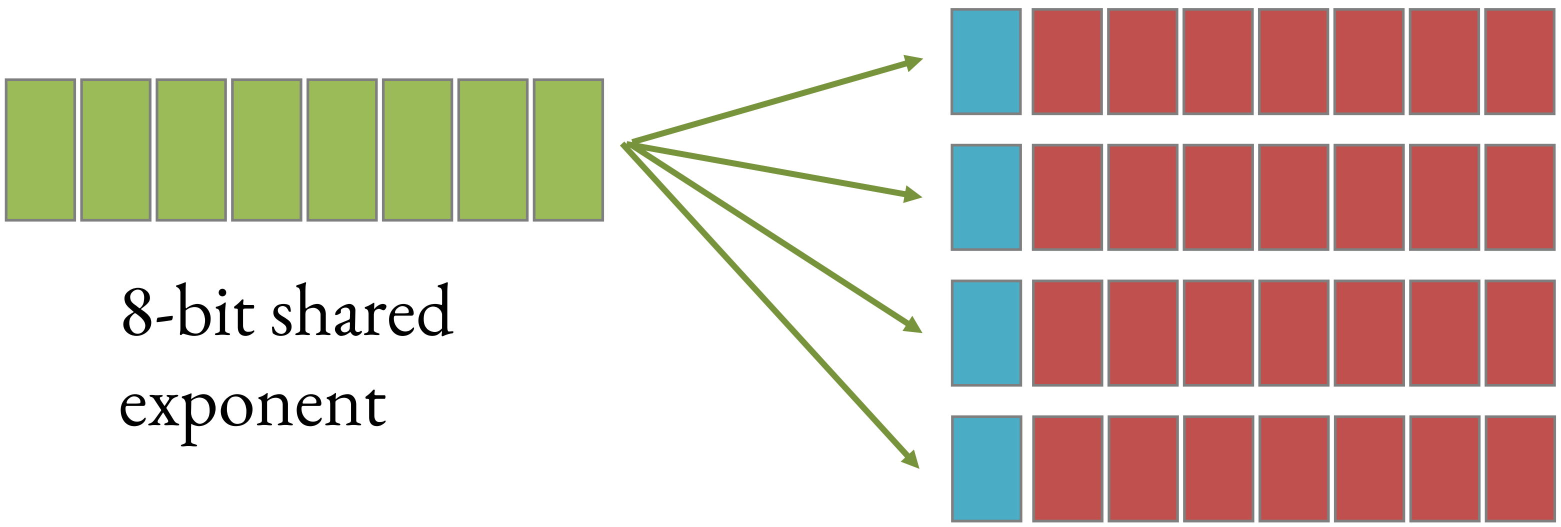
Support for fixed-point arithmetic

- **Anywhere integer arithmetic is supported**
  - CPUs, GPUs
  - Although not all GPUs support 8-bit integer arithmetic
  - And AVX2 does not have all the 8-bit arithmetic instructions we'd like
- Particularly effective on **FPGAs and ASICs**
  - Where floating point units are costly
- Sadly, very **little support for other precisions**
  - **4-bit operations** would be particularly useful

A powerful hybrid approach
Block Floating Point

- Motivation: when storing a vector of numbers, often these numbers all lie in the same range.
  - So they will have the same or similar exponent, if stored as floating point.

- **Block floating point** shares a single exponent among multiple numbers.



A more specialized approach
Custom Quantization Points

- Even more generally, we can just have a list of $\mathbf{2^b}$ numbers and say that these are the numbers a particular low-precision string represents
  - We can think of the bit string as indexing a number in a dictionary

- Gives us total freedom as to range and scaling
  - But **computation can be tricky**

- Some **research into using this with hardware support**
  - *"The ZipML Framework for Training Models with End-to-End Low Precision: The Cans, the Cannots, and a Little Bit of Deep Learning"* (Zhang et al 2017)

Low-precision formats in general

- These are some of the **most common formats used in ML**
  - …but we're not limited to using only these formats!
- There are **many other things we could try**
  - For example, floating point numbers with different exponent/mantissa sizes
  - Fixed point numbers with nonstandard widths
- Problem: there's **no hardware support** for these other things yet, so it's hard to get a sense of how they would perform.
  - Need to **simulate**

Other Numerical Formats Used Rarely

- **BigFloats**
  - Higher-precision floating-point numbers that are implemented in software
  - Are sometimes necessary when you need very high precision, such as for very poorly conditioned problems

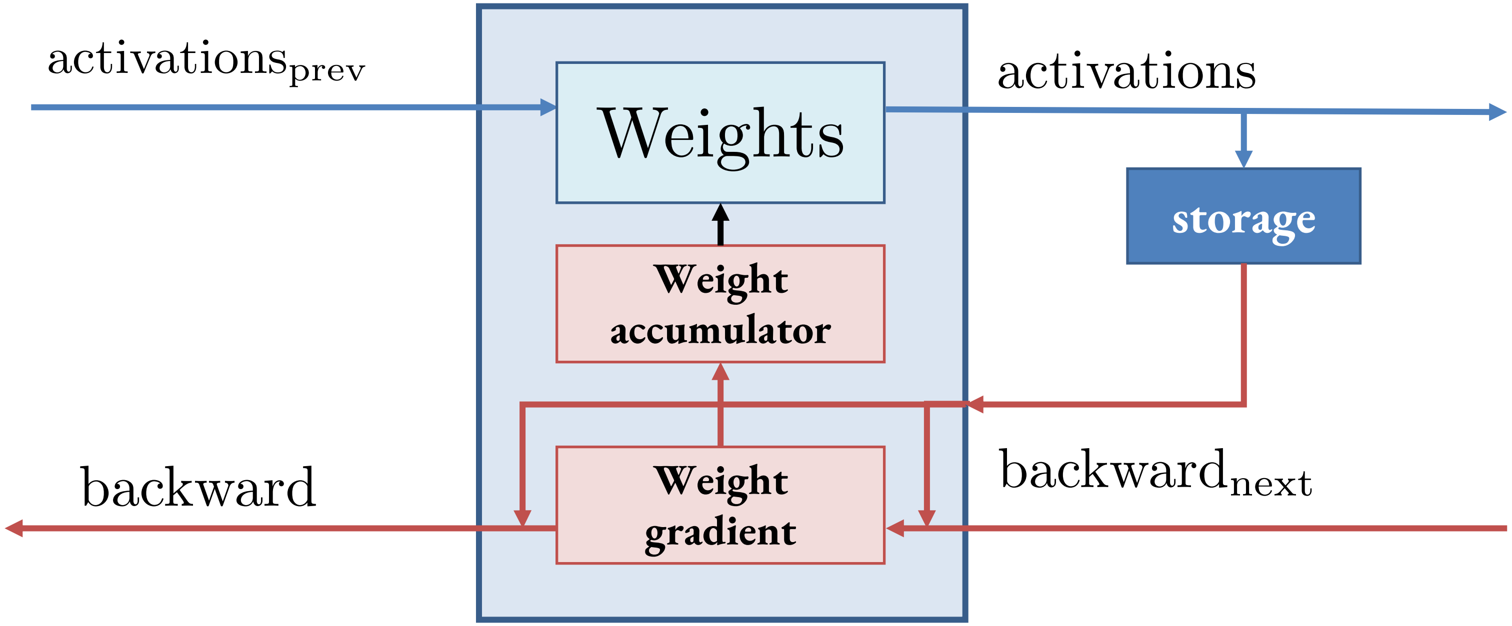
- Exact arithmetic with **rational numbers**
  - Lets you do arithmetic with no error
  - Numbers have variable length, because they require arbitrarily large integers
  - Can also support countable **field extensions of the rational numbers**
  - But these are very rarely used because of performance implications

Low-Precision SGD

Using low-precision arithmetic for training

How is precision used for training

- Recall our training diagram
  - Each of these signals forms a class of numbers
  - Generally, we assign a precision to each of the classes, and different classes can have different precisions



Number classes extended from "Understanding and Optimizing Asynchronous Low-Precision Stochastic Gradient Descent," ISCA 2017:

- **D**ataset numbers
- **M**odel/weight numbers
- **G**radient numbers
- **C**ommunication numbers
- Activation numbers
- Backward pass numbers
- Weight accumulator
- Linear layer accumulator

Quantize classes independently

- Using low-precision for different number classes has **different effects on throughput**.
  - Quantizing the **dataset numbers** improves memory capacity and overall training example throughput
  - Quantizing the **model numbers** improves cache capacity and saves on compute
  - Quantizing the **gradient numbers** saves compute
  - Quantizing the **communication numbers** saves on expensive inter-worker memory bandwidth

Quantize classes independently

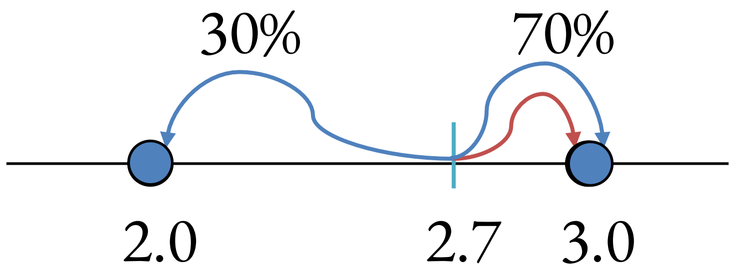
- Using low-precision for different number classes has **different effects on statistical efficiency and accuracy**.
  - Quantizing the **dataset numbers** means you're solving a different problem
  - Quantizing the **model numbers** adds noise to each gradient step, and often means you can't exactly represent the solution
  - Quantizing the **gradient numbers** can add errors to each gradient step
  - Quantizing the **communication numbers** can add errors which cause workers' local models to diverge, which slows down convergence

Theoretical Guarantees for Low Precision

- Reducing precision adds noise in the form

Using this, we can prove **guarantees** that SGD converges with a low precision model.

- Two approaches to rounding:
  - **biased rounding** – round to nearest number
  - **unbiased rounding** – round randomly: $\boldsymbol{E}[Q(x)] = x$



Why unbiased rounding?

- Imagine running SGD with a low-precision model with update rule

$$w_{t+1} = \tilde{Q}\left(w_t - \alpha_t \nabla f(w_t; x_t, y_t)\right)$$

  - Here, **Q** is an unbiased quantization function

- In expectation, this is **just gradient descent**

$$\begin{aligned}
\mathbf{E}[w_{t+1} | w_t] &= \mathbf{E}\left[\tilde{Q}\left(w_t - \alpha_t \nabla f(w_t; x_t, y_t)\right) \middle| w_t\right] \\
&= \mathbf{E}\left[w_t - \alpha_t \nabla f(w_t; x_t, y_t) | w_t\right] \\
&= w_t - \alpha_t \nabla f(w_t)
\end{aligned}$$

Implementing unbiased rounding

- To implement an unbiased to-integer quantizer:

$$\text{sample } u \sim \mathrm{Unif}[0,1], \quad \text{then set } Q(x) = \lfloor x + u \rfloor$$

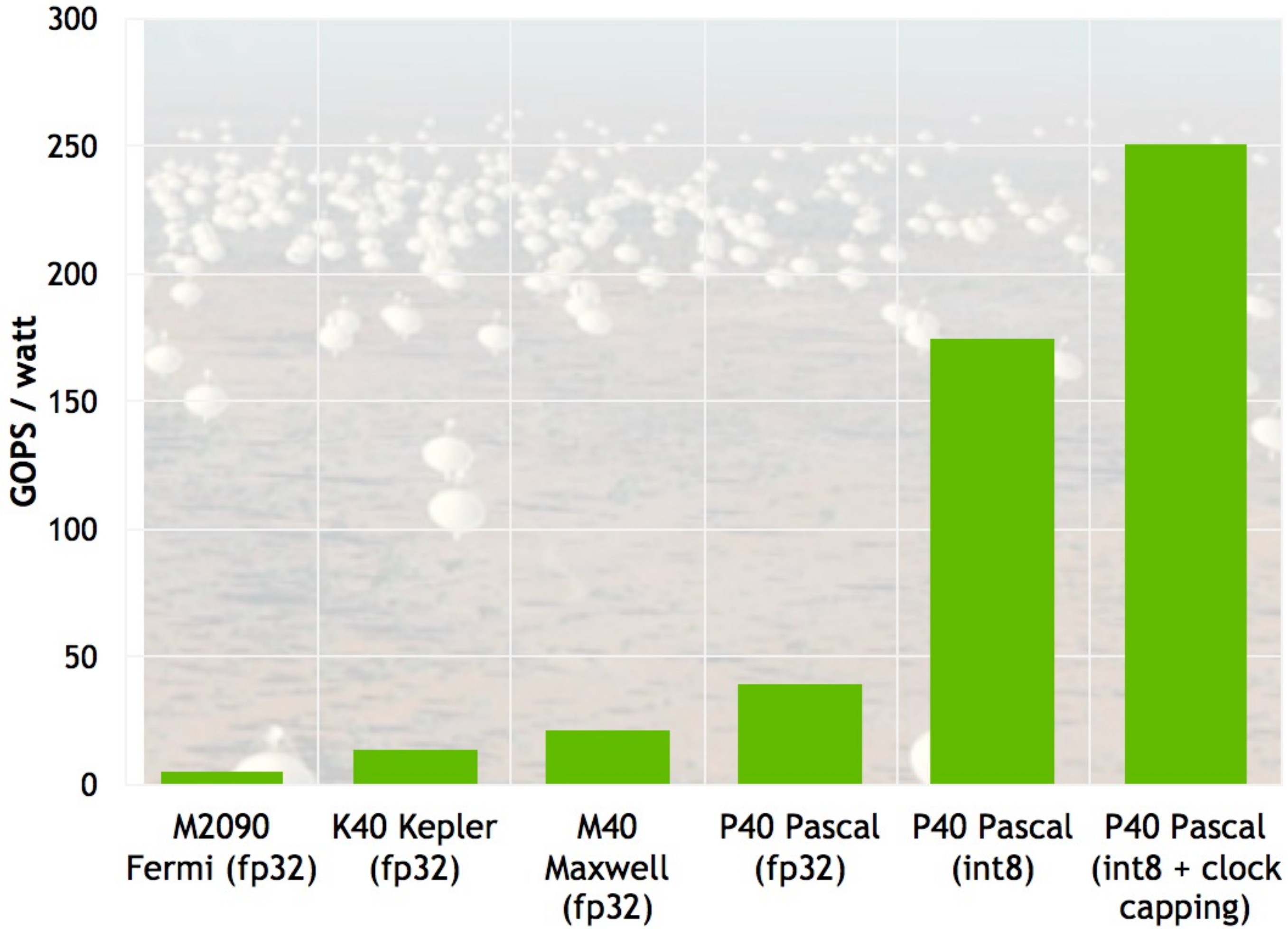
- Why is this unbiased?

$$\begin{aligned}
\mathbf{E}[Q(x)] &= \lfloor x \rfloor \cdot \mathbf{P}(Q(x) = \lfloor x \rfloor) + (\lfloor x \rfloor + 1) \cdot \mathbf{P}(Q(x) = \lfloor x \rfloor + 1) \\
&= \lfloor x \rfloor + \mathbf{P}(Q(x) = \lfloor x \rfloor + 1) = \lfloor x \rfloor + \mathbf{P}(\lfloor x + u \rfloor = \lfloor x \rfloor + 1) \\
&= \lfloor x \rfloor + \mathbf{P}(x + u \geq \lfloor x \rfloor + 1) = \lfloor x \rfloor + \mathbf{P}(u \geq \lfloor x \rfloor + 1 - x) \\
&= \lfloor x \rfloor + 1 + (\lfloor x \rfloor + 1 - x) = x.
\end{aligned}$$

Doing unbiased rounding efficiently

- We still need an efficient way to do unbiased rounding
- **Pseudorandom number generation can be expensive**
  - E.G. doing C++ rand or using Mersenne twister takes many clock cycles
- Empirically, we can use **very cheap** pseudorandom number generators
  - And still get good statistical results
  - For example, we can use XORSHIFT which is just a cyclic permutation

Benefits of Low-Precision Computation



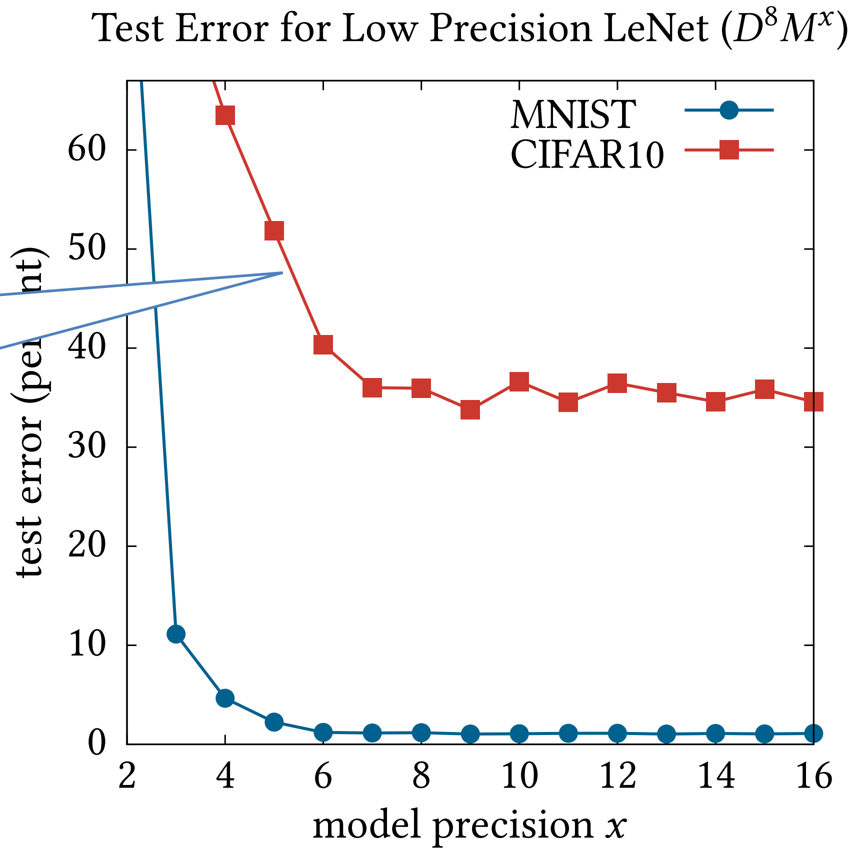
From https://devblogs.nvidia.com/parallelforall/mixed-precision-programming-cuda-8/

Drawbacks of low-precision

- The draw back of low-precision arithmetic is the **low precision**!
- Low-precision computation means we accumulate **more rounding error** in our computations
- These rounding errors can add up throughout the learning process, resulting in **less accurate learned systems**
- The trade-off of low-precision: **throughput/memory vs. accuracy**

Example: Low-Precision Neural Net

Performance eventually breaks down as precision is lowered



(b) Test accuracies of low-precision SGD on LeNet neural network after 5000 passes, for various datasets.

Demo

Memory Locality and Scan Order

Memory Locality: Two Kinds

- Memory locality is needed for **good cache performance**
- **Temporal locality**
  - Frequency of reuse of the same data within a short time window
- **Spatial locality**
  - Frequency of use of data nearby data that has recently been used
- **Where is there locality in stochastic gradient descent?**

Problem: no dataset locality across iterations

- The training example at each iteration is chosen randomly
  - Called a **random scan order**
  - Impossible for the cache to predict what data will be needed

$$w_{t+1} = w_t - \alpha_t \nabla f(w_t; x_t, y_t)$$

- Idea: process examples in the order in which they are stored in memory
  - Called a **systematic scan order** or **sequential scan order**
  - **Does this improve the memory locality?**

Random scan order vs. sequential scan order

- **Much easier to prove theoretical results** for random scan
- But **sequential scan has better systems performance**
- In practice, **almost everyone uses sequential scan**
  - There's no empirical evidence that it's statistically worse in most cases
  - Even though we can construct cases where using sequential scan does harm the convergence rate

Other scan orders

- **Shuffle-once**, then sequential scan
  - Shuffle the data once, then systematically scan for the rest of execution
  - Statistically very similar to random scan at the state
- **Random reshuffling**
  - Randomly shuffle on every pass through the data
  - Gets better upper bounds for SGD
    - e.g. for convex, gets O(1/t) versus O(1/sqrt(t))
  - Very commonly used with deep learning

Demo

Questions?

- Upcoming things
  - Project feedback coming soon!