Memory Bandwidth and Low Precision Computation

CS6787 Lecture 10 — Fall 2018

Memory as a Bottleneck

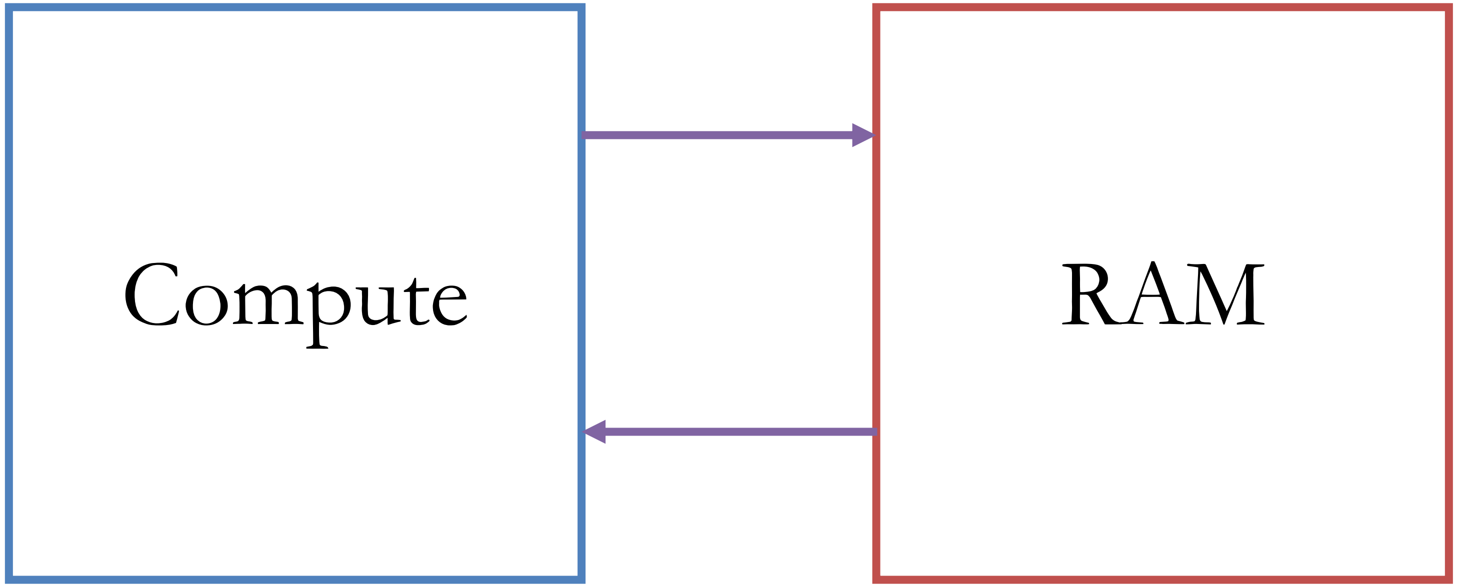
- So far, we've just been talking about **compute**
  - e.g. techniques to decrease the amount of compute by decreasing iterations

- But machine learning systems need to process **huge amounts of data**

- Need to **store, update, and transmit** this data

- As a result: **memory** is of critical importance
  - Many applications are memory-bound

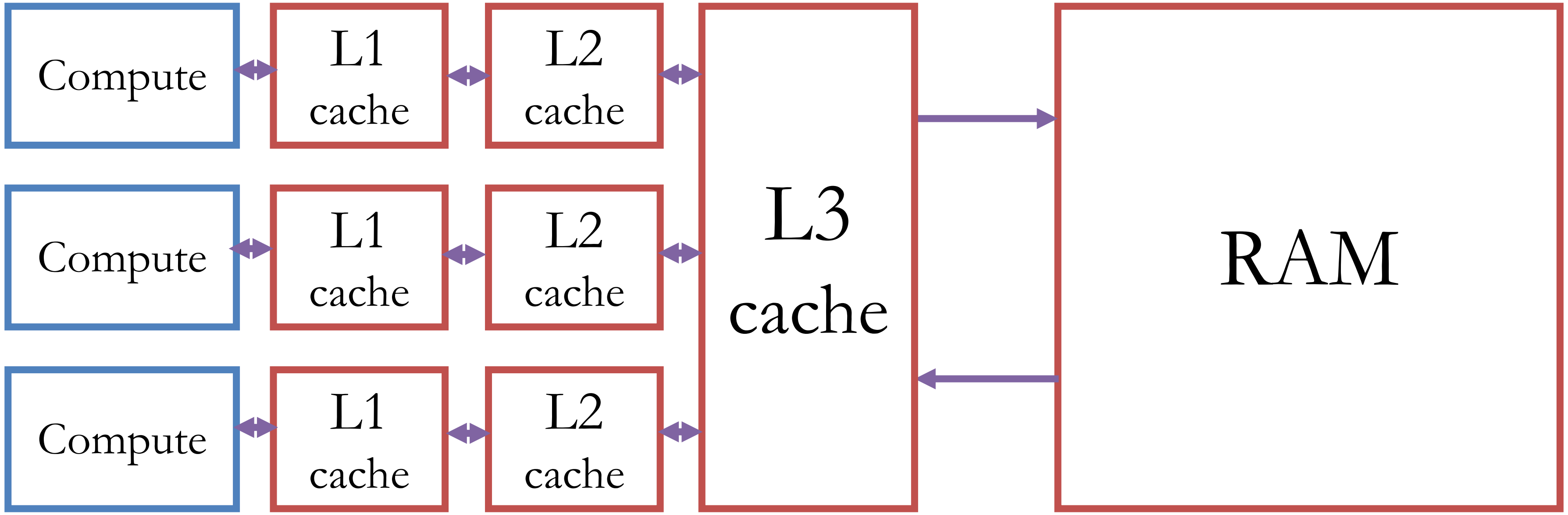
Memory: The Simplified Picture



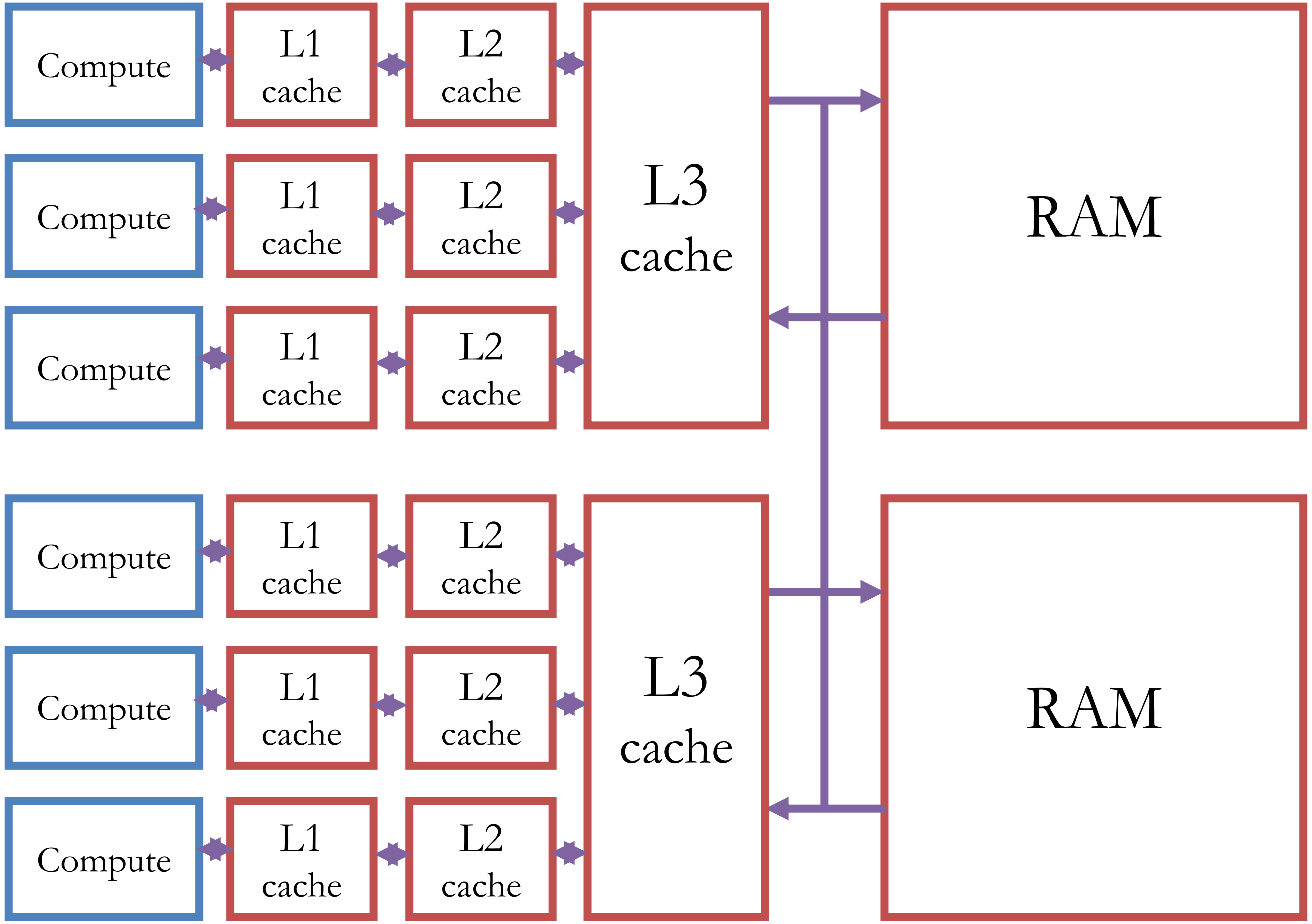
Compute

RAM

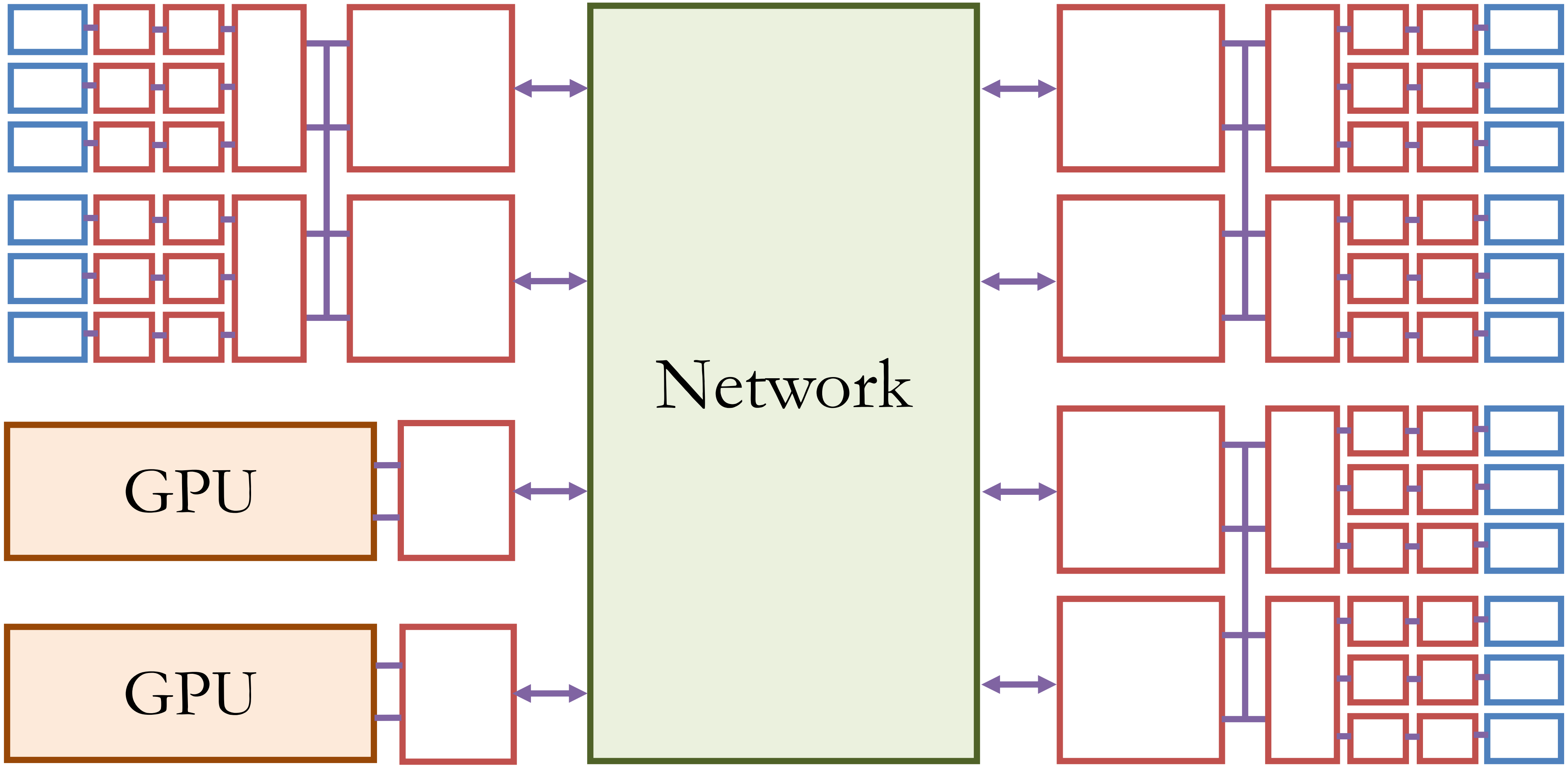
Memory: The Multicore Picture



Memory: The Multisocket Picture



Memory: The Distributed Picture



What can we learn from these pictures?

- Many more **memory** boxes than **compute** boxes
    - And even more as we zoom out
- Memory has a **hierarchical structure**
- **Locality matters**
    - Some memory is closer and easier to access than others
    - Also have standard concerns for CPU cache locality

What limits us?

- **Memory capacity**
  - How much data can we store locally in RAM and/or in cache?
- **Memory bandwidth**
  - How much data can we load from some source in a fixed amount of time?
- **Memory locality**
  - Roughly, how often is the data that we need stored nearby?
- **Power**
  - How much energy is required to operate all of this memory?

One way to help:
Low-Precision Computation

Low-Precision Computation

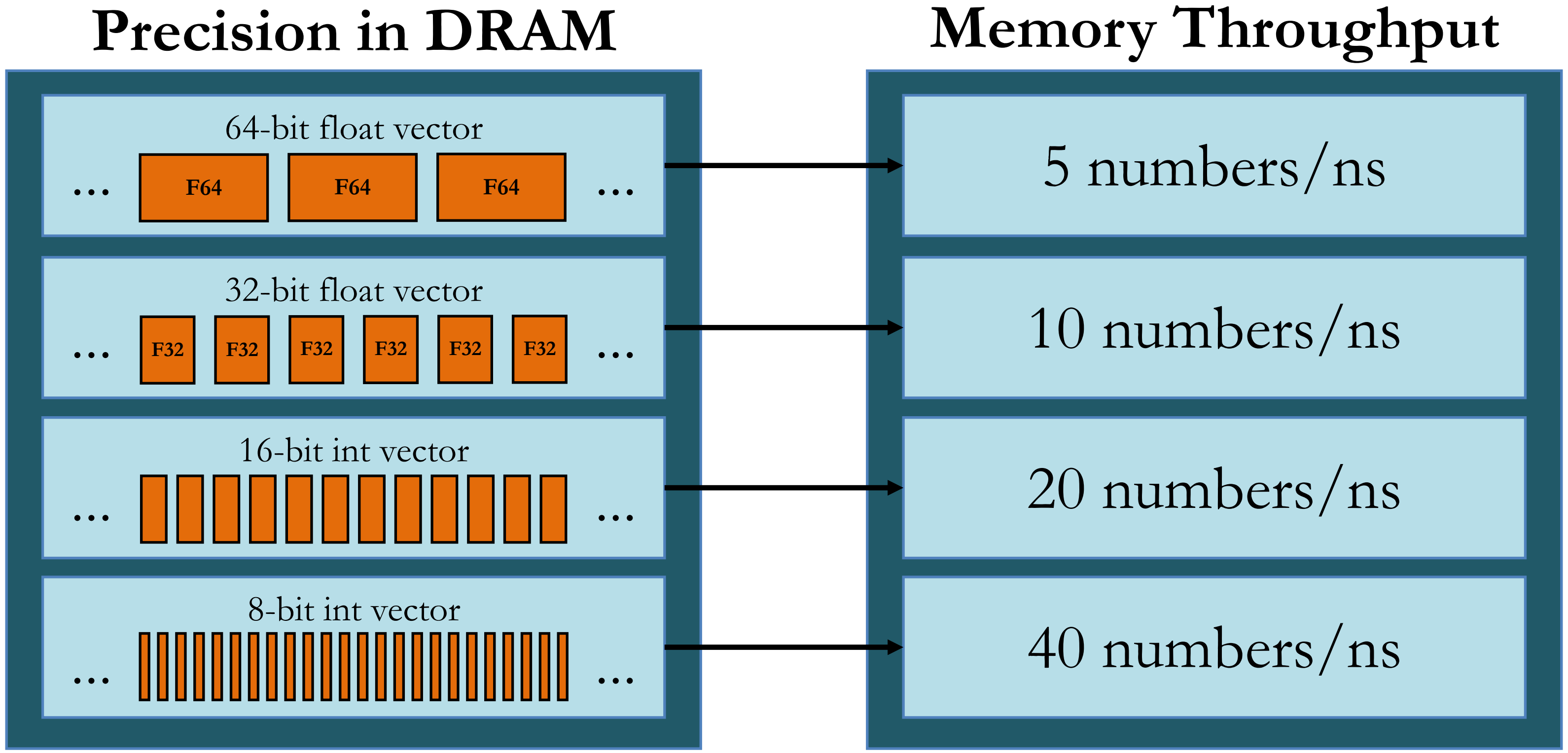
- Traditional ML systems use 32-bit or 64-bit **floating point numbers**

- **But do we actually need this much precision?**
  - Especially when we have inputs that come from noisy measurements

- Idea: instead use **8-bit or 16-bit numbers** to compute
  - Can be either floating point or fixed point
  - On an FPGA or ASIC can use arbitrary bit-widths

Low Precision and Memory

- Major benefit of low-precision: **uses less memory bandwidth**



Precision in DRAM

64-bit float vector

32-bit float vector

16-bit int vector

8-bit int vector

Memory Throughput

5 numbers/ns

10 numbers/ns

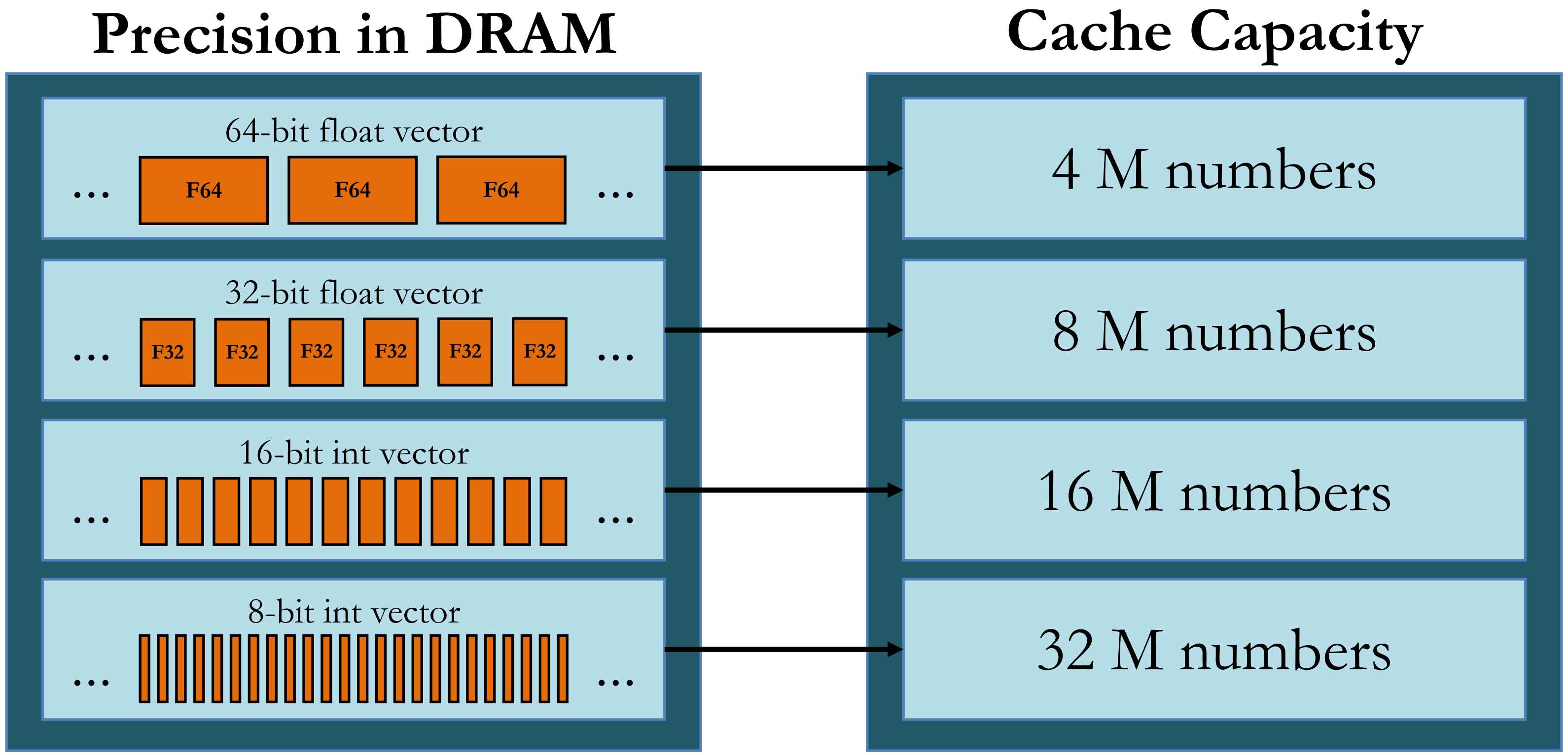
20 numbers/ns

40 numbers/ns

(assuming ~40 GB/sec memory bandwidth)

Low Precision and Memory

- Major benefit of low-precision: **takes up less space**



Precision in DRAM

64-bit float vector
… F64 F64 F64 …

32-bit float vector
… F32 F32 F32 F32 F32 F32 …

16-bit int vector
… …

8-bit int vector
… …

Cache Capacity

4 M numbers

8 M numbers

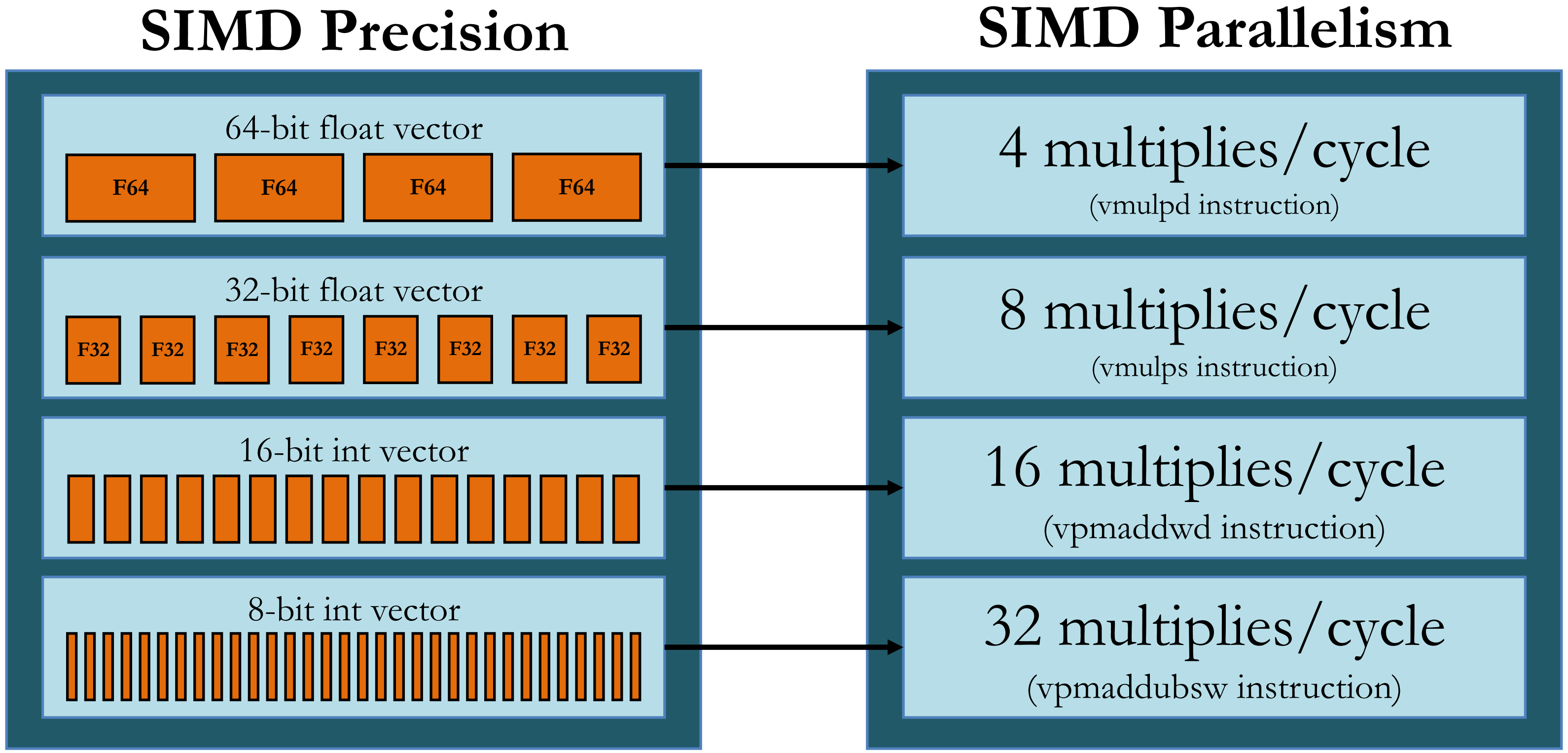
16 M numbers

32 M numbers

(assuming ~32 MB cache)

Low Precision and Parallelism

- Another benefit of low-precision: use **SIMD instructions** to get more parallelism on CPU



SIMD Precision

64-bit float vector

32-bit float vector

16-bit int vector

8-bit int vector

SIMD Parallelism

4 multiplies/cycle
(vmulpd instruction)

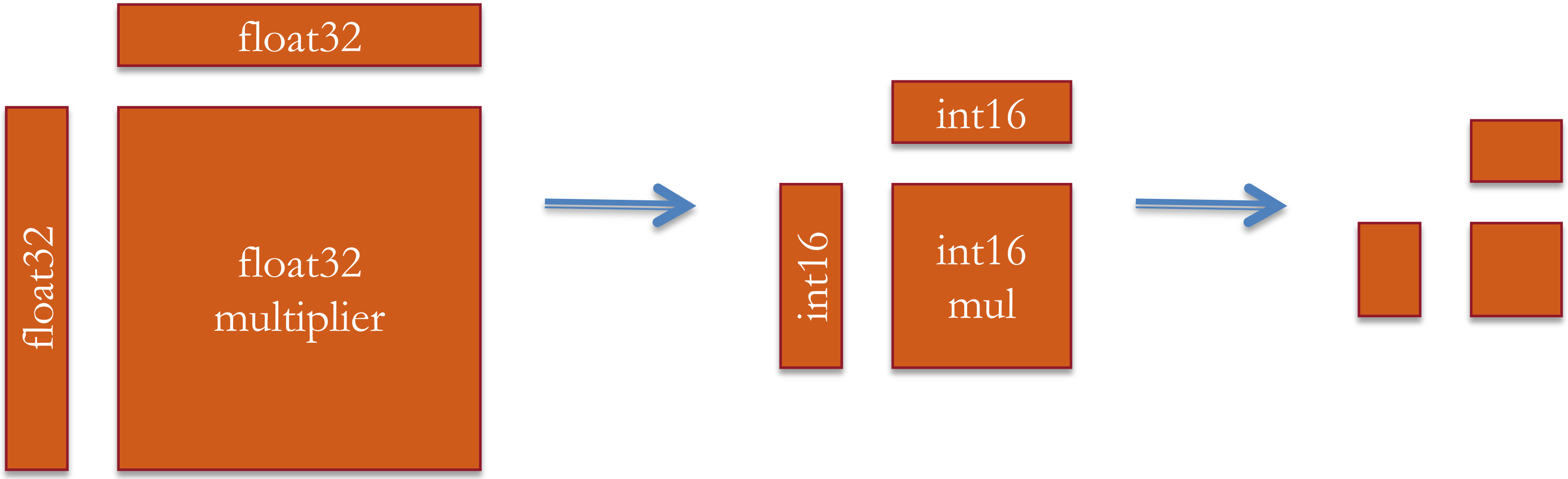
8 multiplies/cycle
(vmulps instruction)

16 multiplies/cycle
(vpmaddwd instruction)

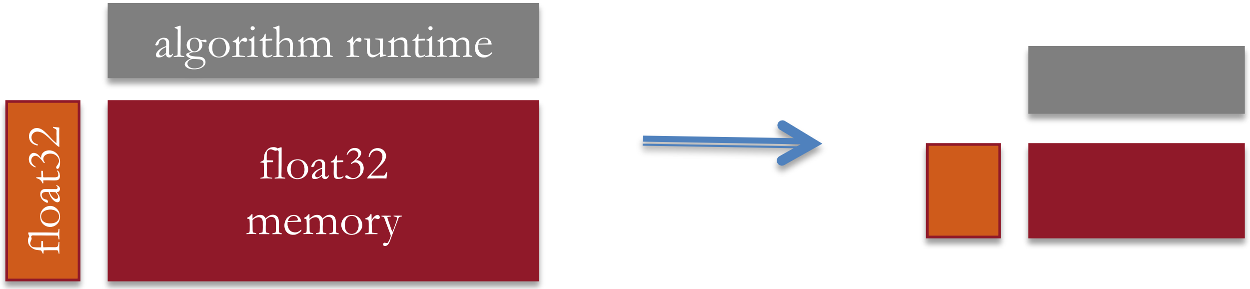
32 multiplies/cycle
(vpmaddubsw instruction)

Low Precision and Power

- Low-precision computation can even have a super-linear effect on energy



- Memory energy can also have quadratic dependence on precision



Effects of Low-Precision Computation

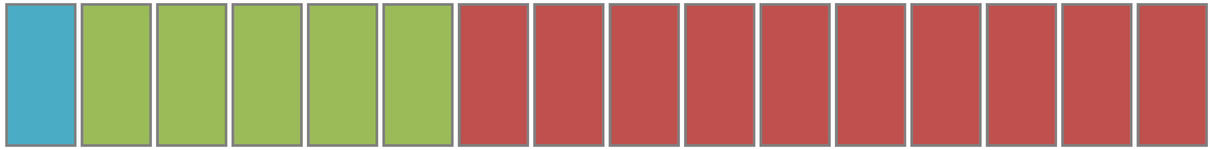
- **Pros**
  - Fit more numbers (and therefore more training examples) in memory
  - Store more numbers (and therefore larger models) in the cache
  - Transmit more numbers per second
  - Compute faster by extracting more parallelism
  - Use less energy

- **Cons**
  - Limits the numbers we can represent
  - Introduces **quantization error** when we store a full-precision number in a low-precision representation

Ways to represent low-precision numbers

FP16/Half-precision floating point

- 16-bit floating point numbers



1-bit
sign

5-bit
exponent

10-bit
significand

- Usually, the represented value is

$$x = (-1)^{\text{sign bit}} \cdot 2^{\text{exponent}-15} \cdot 1.\text{significand}_2$$

Arithmetic on half-precision floats

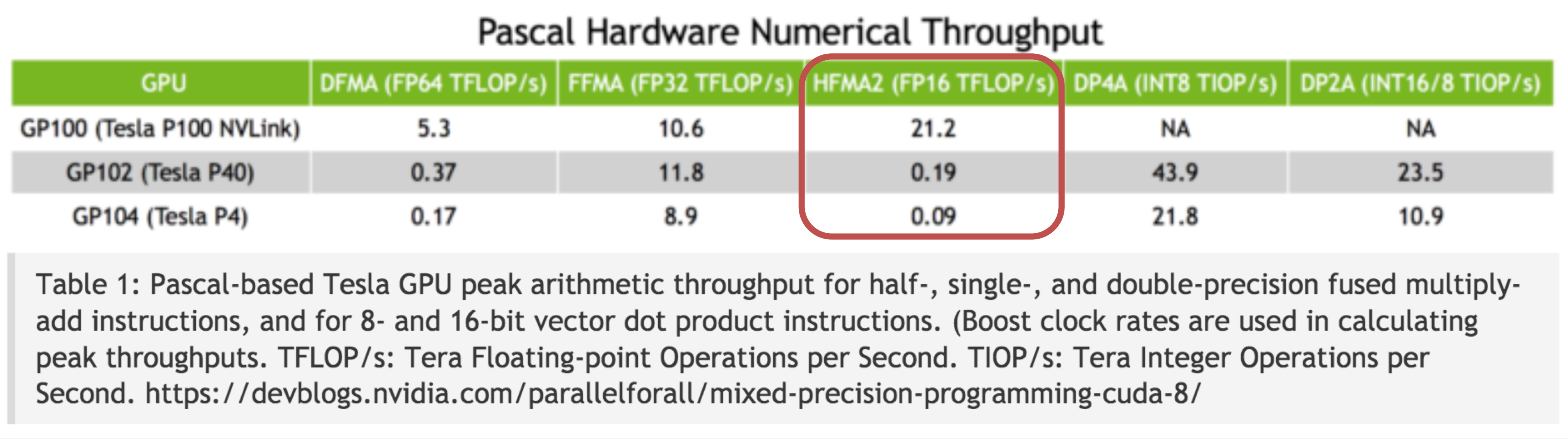
- **Complicated**
  - Has to handle adding numbers with different exponents and signs
  - To be efficient, needs to be **supported in hardware**

- **Inexact**
  - Operations can experience overflow/underflow just like with more common floating point numbers, but it happens more often

- Can represent a **wide range of numbers**
  - Because of the exponential scaling

Half-precision floating point support

- Supported on some **modern GPUs**
  - Including new efficient implementation on NVIDIA Pascal GPUs



Pascal Hardware Numerical Throughput

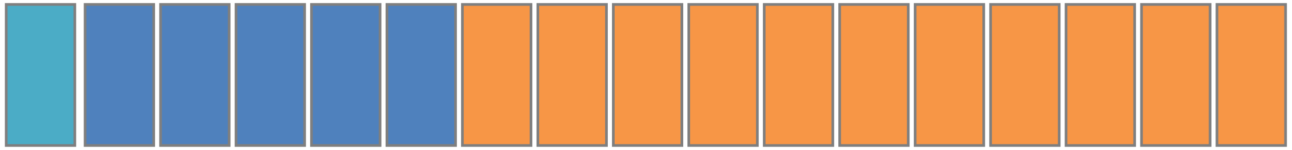
GPU	DFMA (FP64 TFLOP/s)	FFMA (FP32 TFLOP/s)	HFMA2 (FP16 TFLOP/s)	DP4A (INT8 TIOP/s)	DP2A (INT16/8 TIOP/s)
GP100 (Tesla P100 NVLink)	5.3	10.6	21.2	NA	NA
GP102 (Tesla P40)	0.37	11.8	0.19	43.9	23.5
GP104 (Tesla P4)	0.17	8.9	0.09	21.8	10.9

Table 1: Pascal-based Tesla GPU peak arithmetic throughput for half-, single-, and double-precision fused multiply-add instructions, and for 8- and 16-bit vector dot product instructions. (Boost clock rates are used in calculating peak throughputs. TFLOP/s: Tera Floating-point Operations per Second. TIOP/s: Tera Integer Operations per Second. https://devblogs.nvidia.com/parallelforall/mixed-precision-programming-cuda-8/

- Good empirical results for **deep learning**

Fixed point numbers

- **p + q + 1** –bit fixed point number



1-bit
sign

p-bit
integer part

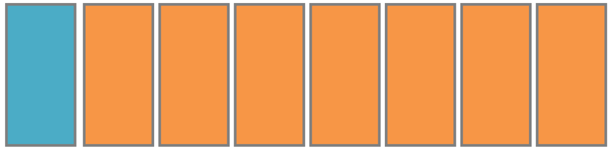
q-bit
fractional part

- The represented number is

$$
\begin{aligned}
x &= (-1)^{\text{sign bit}} \left(\text{integer part} + 2^{-q} \cdot \text{fractional part}\right) \\
&= 2^{-q} \cdot \text{whole thing as signed integer}
\end{aligned}
$$

Example: 8-bit fixed point number

- It's common to want to represent numbers between **-1** and **1**
  - To do this, we can use a fixed point number with **all fractional bits**



1-bit
sign

7-bit
fractional part

- If the number as an integer is **k**, then the represented number is

$$x = 2^{-7} \cdot k \in \left\{ -1, -\frac{127}{128}, \ldots, -\frac{1}{128}, 0, \frac{1}{128}, \ldots, \frac{126}{128}, \frac{127}{128} \right\}$$

More generally: scaled fixed point numbers

- Sometimes we don't want the decimal point to lie between two bits that we are actually storing
  - We might want more tight control over what our bits mean

- Idea: pick a real-number scale factor **s**, then let integer **k** represent

$$x = s \cdot k$$

- This is a **generalization of traditional fixed point**, where

$$s = 2^{-\#\text{ of fractional bits}}$$

Arithmetic on fixed point numbers

- **Simple**
  - Can just use preexisting integer processing units

- **Mostly exact**
  - Underflow impossible
  - Overflow can happen, but is easy to understand
  - Can always convert to a higher-precision representation to avoid overflow

- Can represent a **much narrower range of numbers than float**

Example: Exact Fixed Point Multiply

- When we multiply two integers, if we want the result to be exact, we need to convert to a representation with more bits

- For example, if we take the product of two 8-bit numbers, the result should be a 16-bit number to be exact.
  - **Why? 100 x 100 = 10000** which can’t be stored as an 8-bit number

- To have exact fixed point multiply, we can do the same thing
  - Since **fixed-point operations are just integer operations behind the scenes**

Support for fixed-point arithmetic

- **Anywhere integer arithmetic is supported**
  - CPUs, GPUs
  - Although not all GPUs support 8-bit integer arithmetic
  - And AVX2 does not have all the 8-bit arithmetic instructions we'd like

- Particularly effective on **FPGAs and ASICs**
  - Where floating point units are costly

- Sadly, very **little support for other precisions**
  - **4-bit operations** would be particularly useful

Custom Quantization Points

- Even more generally, we can just have a list of $\mathbf{2^b}$ numbers and say that these are the numbers a particular low-precision string represents
  - We can think of the bit string as indexing a number in a dictionary

- Gives us total freedom as to range and scaling
  - But **computation can be tricky**

- Some **recent research into using this with hardware support**
  - "The ZipML Framework for Training Models with End-to-End Low Precision: The Cans, the Cannots, and a Little Bit of Deep Learning" (Zhang et al 2017)

Recap of low-precision representations

- **Half-precision floating-point**
  - Complicated arithmetic, but good with hardware support
  - Difficult to reason about overflow and underflow
  - Better range
  - No 8-bit support as of yet

- **Fixed-point**
  - Simple arithmetic, supported wherever integers are
  - Easy to reason about overflow, but has worse range
  - Supports 8-bit and 16-bit arithmetic, but limited 4-bit support

Low-Precision SGD

Recall: SGD update rule

$$w_{t+1} = w_t - \alpha_t \nabla f(w_t; x_t, y_t)$$

- There are a lot of numbers we can make low-precision here
  - We can quantize the input dataset **x, y**
  - We can quantize the model **w**
  - We can try to quantize within the gradient computation itself
  - We can try to quantize the communication among the parallel workers

Several Broad Classes of Numbers

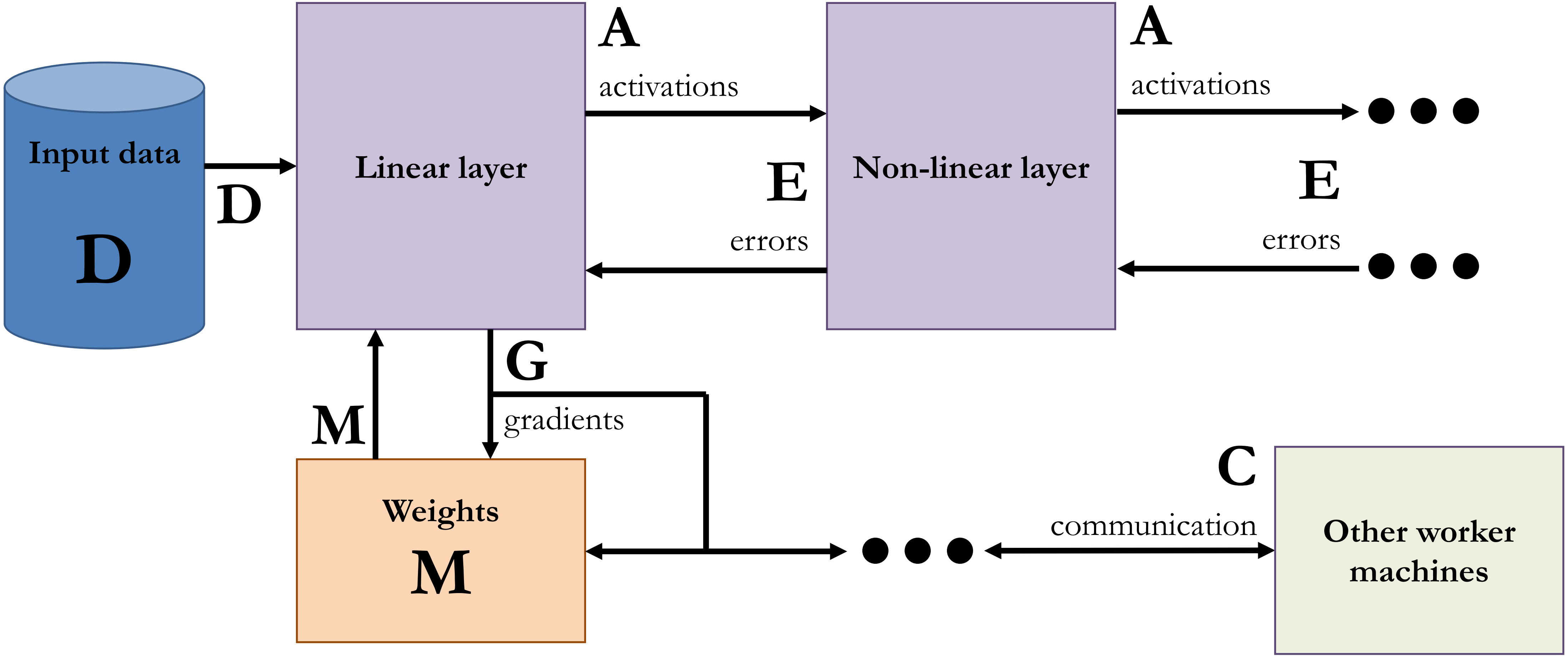
For SGD on deep-learning-like objectives

- **D**ataset numbers
  - used to store the immutable input data
- **M**odel/**W**eight numbers
  - used to represent the vector we are updating
- Numbers used to store gradients as we are computing them
  - **A**ctivation numbers: used to compute/store the forward pass/loss of a neural network
  - **E**rror numbers: used to compute the gradients in the backwards pass
  - **G**radient numbers: used to store the gradients as they are computed
- **C**ommunication numbers
  - used to communicate among parallel workers

De Sa et al 2017, Wu et al 2018

Several Broad Classes of Numbers

For SGD on deep-learning-like objectives



Quantize classes independently

- Using low-precision for different number classes has **different effects on throughput**.
    - Quantizing the **dataset numbers** improves memory capacity and overall training example throughput
    - Quantizing the **model/error numbers** improves cache capacity and saves on compute
    - Quantizing the **gradient/activation numbers** saves compute
    - Quantizing the **communication numbers** saves on expensive inter-worker memory bandwidth

Quantize classes independently

- Using low-precision for different number classes has **different effects on statistical efficiency and accuracy**.
  - Quantizing the **dataset numbers** means you're solving a different problem
  - Quantizing the **model numbers** adds noise to each gradient step, and often means you can't exactly represent the solution
  - Quantizing the **gradient/activation/error numbers** can add quantization errors to each gradient step
  - Quantizing the **communication numbers** can add errors which cause workers' local models to diverge, which can slow down convergence

Theoretical Guarantees for Low Precision

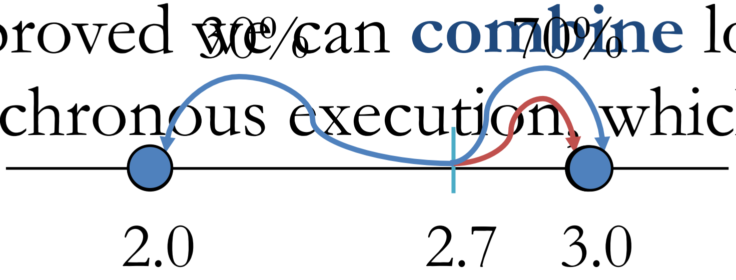
- Reducing precision adds noise in the for

- Two approaches to rounding:
  - **biased rounding** – round to nearest number
  - **unbiased rounding** – round randomly: $\boldsymbol{E}[Q(x)] = x$

Using this, we can prove **guarantees** that SGD works with a low precision model.

Taming the Wild [NIPS 2015]

- In some of my work, proved we can **combine** low-precision computation with asynchronous execution, which we call BUCKWILD!



Why unbiased rounding?

- Imagine running SGD with a low-precision **m**odel with update rule

$$w_{t+1} = \tilde{Q}\left(w_t - \alpha_t \nabla f(w_t; x_t, y_t)\right)$$

  - Here, **Q** is an unbiased quantization function

- In expectation, this is **just gradient descent**

$$\begin{aligned}
\mathbf{E}[w_{t+1} | w_t] &= \mathbf{E}\left[\tilde{Q}\left(w_t - \alpha_t \nabla f(w_t; x_t, y_t)\right) \middle| w_t\right] \\
&= \mathbf{E}\left[w_t - \alpha_t \nabla f(w_t; x_t, y_t) | w_t\right] \\
&= w_t - \alpha_t \nabla f(w_t)
\end{aligned}$$

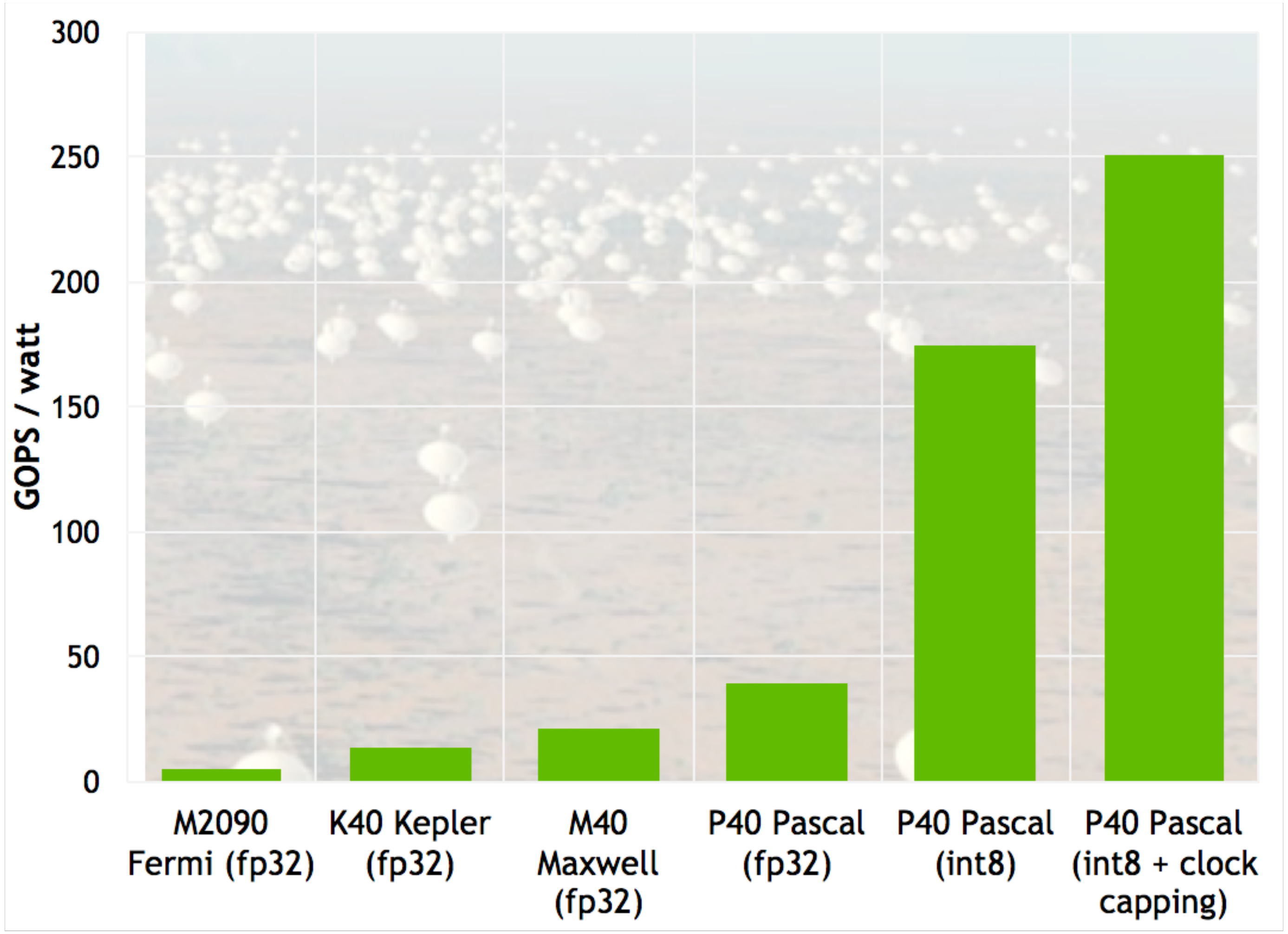
Doing unbiased rounding efficiently

- We still need an efficient way to do unbiased rounding

- **Pseudorandom number generation can be expensive**
    - E.G. doing C++ rand or using Mersenne twister takes many clock cycles

- Empirically, we can use **very cheap** pseudorandom number generators
    - And still get good statistical results
    - For example, we can use XORSHIFT which is just a cyclic permutation

Benefits of Low-Precision Computation



From https://devblogs.nvidia.com/parallelforall/mixed-precision-programming-cuda-8/

Memory Locality and Scan Order

Memory Locality: Two Kinds

- Memory locality is needed for **good cache performance**

- **Temporal locality**
  - Frequency of reuse of the same data within a short time window

- **Spatial locality**
  - Frequency of use of data nearby data that has recently been used

- **Where is there locality in stochastic gradient descent?**

Problem: no dataset locality across iterations

- The training example at each iteration is chosen randomly
  - Called a **random scan order**
  - Impossible for the cache to predict what data will be needed

$$w_{t+1} = w_t - \alpha_t \nabla f(w_t; x_t, y_t)$$

- Idea: process examples in the order in which they are stored in memory
  - Called a **systematic scan order** or **sequential scan order**
  - **Does this improve the memory locality?**

Random scan order vs. sequential scan order

- **Much easier to prove theoretical results** for random scan
- But **sequential scan has better systems performance**
- In practice, **almost everyone uses sequential scan**
  - There's no empirical evidence that it's statistically worse in most cases
  - Even though we can construct cases where using sequential scan does harm the convergence rate

Other scan orders

- **Shuffle-once**, then sequential scan
  - Shuffle the data once, then systematically scan for the rest of execution
  - Statistically very similar to random scan at the state

- **Random reshuffling**
  - Randomly shuffle on every pass through the data
  - Believed to be always at least as good as both random scan and sequential scan
  - But no proof that it is better

Demo

Questions?

- Upcoming things
  - Paper Review #7a or #7b — **due today**
  - Paper Presentation #8a and #8b **on Wednesday**