Structured Output Prediction: Generative Models

CS6780 – Advanced Machine Learning
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Reading:
Murphy 17.3 , 17.4, 17.5.1

Structured Output Prediction

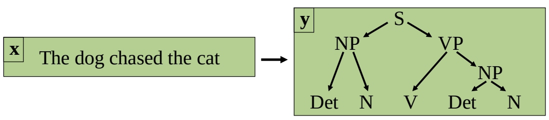
- Supervised Learning from Examples
  - Find function from input space X to output space Y

$$h: X \rightarrow Y$$

  such that the prediction error is low.
- Typical
  - Output space is just a single number
    - Classification: -1,+1
    - Regression: some real number
- General
  - Predict outputs that are complex objects

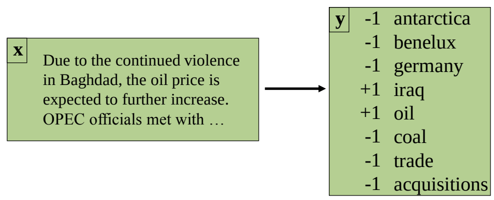
Examples of Complex Output Spaces

- Natural Language Parsing
  - Given a sequence of words *x*, predict the parse tree *y*.
  - Dependencies from structural constraints, since *y* has to be a tree.



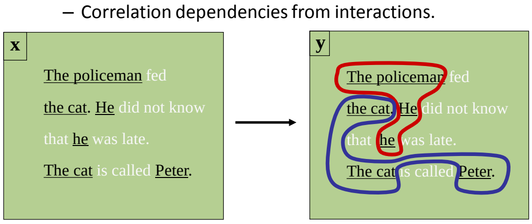
Examples of Complex Output Spaces

- Multi-Label Classification
  - Given a (bag-of-words) document x, predict a set of labels *y*.
  - Dependencies between labels from correlations between labels (“iraq” and “oil” in newswire corpus)



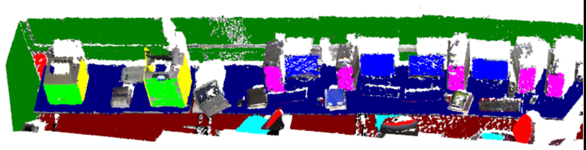
Examples of Complex Output Spaces

- Noun-Phrase Co-reference
  - Given a set of noun phrases *x*, predict a clustering *y*.
  - Structural dependencies, since prediction has to be an equivalence relation.
  - Correlation dependencies from interactions.



Examples of Complex Output Spaces

- Scene Recognition
  - Given a 3D point cloud with RGB from Kinect camera
  - Segment into volumes
  - Geometric dependencies between segments (e.g. monitor usually close to keyboard)



Part-of-Speech Tagging Task

- Assign the correct part of speech (word class) to each word in a document
  - "The/DT planet/NN Jupiter/NNP and/CC its/PRP moons/NNS are/VBP in/IN effect/NN a/DT mini-solar/JJ system/NN ,/, and/CC Jupiter/NNP itself/PRP is/VBZ often/RB called/VBN a/DT star/NN that/IN never/RB caught/VBN fire/NN ./."
- Needed as an initial processing step for a number of language technology applications
  - Information extraction
  - Answer extraction in QA
  - Base step in identifying syntactic phrases for IR systems
  - Critical for word-sense disambiguation (WordNet apps)
  - …

Why is POS Tagging Hard?

- Ambiguity
  - He will race/VB the car.
  - When will the race/NN end?
  - I bank/VB at CFCU.
  - Go to the bank/NN!
- Average of ~2 parts of speech for each word
  - The number of tags used by different systems varies a lot. Some systems use < 20 tags, while others use > 400.

The POS Learning Problem

- Example

sentence	POS
$\bar{x}_1 = (I, bank, at, CFCU)$	$\bar{y}_1 = (PRP, V, PREP, N)$
$\bar{x}_2 = (Go, to, the, bank)$	$\bar{y}_2 = (V, PREP, DET, N)$

Markov Model

- Definition
  - Set of States: $s_1, \ldots, s_k$
  - Start probabilities: $P(S_1=s)$
  - Transition probabilities: $P(S_i=s \mid S_{i-1}=s')$
- Random walk on graph
  - Start in state s with probability $P(S_1=s)$
  - Move to next state with probability $P(S_i=s \mid S_{i-1}=s')$
- Assumptions
  - Limited dependence: Next state depends only on previous state, but no other state (i.e. first order Markov model)
  - Stationary: $P(S_i=s \mid S_{i-1}=s')$ is the same for all i

Hidden Markov Model for POS Tagging

- States
  - Think about as nodes of a graph
  - One for each POS tag
  - special start state (and maybe end state)
- Transitions
  - Think about as directed edges in a graph
  - Edges have transition probabilities
- Output
  - Each state also produces a word of the sequence
  - Sentence is generated by a walk through the graph

Hidden Markov Model

- States: $y \in \{s_1, \ldots, s_k\}$
- Outputs symbols: $x \in \{o_1, \ldots, o_m\}$
- Starting probability $P(Y_1 = y_1)$
  - Specifies where the sequence starts
- Transition probability $P(Y_i = y_i \mid Y_{i-1} = y_{i-1})$
  - Probability that one states succeeds another
- Output/Emission probability $P(X_i = x_i \mid Y_i = y_i)$
  - Probability that word is generated in this state

=> Every output+state sequence has a probability

$$P(x, y) = P(x_1, \ldots, x_l, y_1, \ldots, y_l)$$
$$= P(y_1)P(x_1|y_1)\prod_{i=2}^{l} P(x_i|y_i)P(y_i|y_{i-1})$$

Estimating the Probabilities

- Fully observed data:
  - input/output sequence pairs

$$P(Y_i = a \mid Y_{i-1} = b) = \frac{\#\text{ of times state } a \text{ follows state } b}{\#\text{ of times state } b \text{ occurs}}$$

$$P(X_i = a \mid Y_i = b) = \frac{\#\text{ of times output } a \text{ is observed in state } b}{\#\text{ of times state } b \text{ occurs}}$$

  - Smoothing the estimates:
    - See Naïve Bayes for text classification
- Partially observed data (Y_i unknown):
  - Expectation-Maximization (EM)

HMM Prediction (Decoding)

Question: What is the most likely state sequence given an output sequence?

$$y^* = \underset{y \in \{y_1,\ldots,y_l\}}{\operatorname{argmax}} P(x_1, \ldots, x_l, y_1, \ldots, y_l)$$

$$= \underset{y \in \{y_1,\ldots,y_l\}}{\operatorname{argmax}} \left\{ P(y_1) P(x_1 \mid y_1) \prod_{i=2}^{l} P(x_i \mid y_i) P(y_i \mid y_{i-1}) \right\}$$

Going on a trip

- Deal: trip to any 3 cities in
  Germany -> Italy -> Spain
  for one low low price

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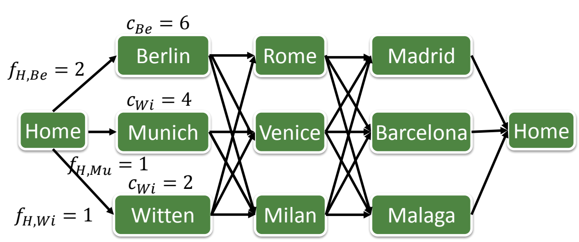
Country	City options
Germany	Berlin/Munich/Witten
Italy	Rome/Venice/Milan
Spain	Madrid/Barcelona/Malaga

Going on a trip

- Deal:
  - Each city i has an attractiveness score $c_i \in [0, 10]$
  - Each flight has an comfort score $f_{i,j} \in [0, 10]$
- Find the best trip!

Going on a trip



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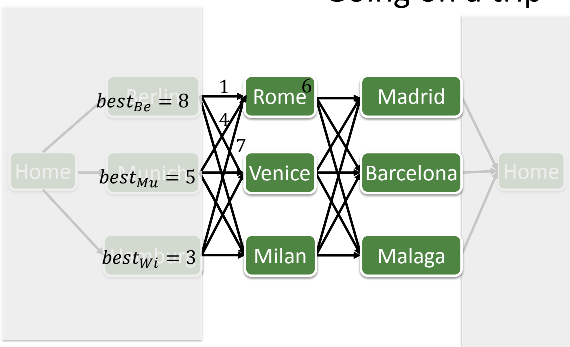

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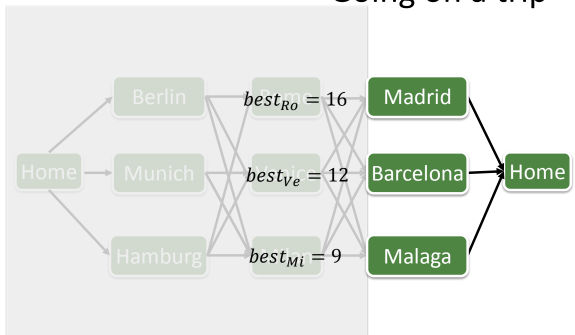

Viterbi Algorithm for Decoding

- Efficiently compute most likely sequence

$$\hat{y} = \underset{y \in \{y_1, \ldots, y_l\}}{\operatorname{argmax}} \left\{ P(y_1)P(x_1|y_1) \prod_{i=2}^{l} P(x_i|y_i)P(y_i|y_{i-1}) \right\}$$

- Viterbi Algorithm:

$$\delta_y(1) = P(Y_1 = y)P(X_1 = x_1|Y_1 = y)$$

$$\delta_y(i+1) = \max_{v \in \{s_1, \ldots, s_k\}} \delta_v(i) P(Y_{i+1} = y|Y_i = v) P(X_{i+1} = x_{i+1}|Y_{i+1} = y)$$

Viterbi Example

$P(X_i\|Y_i)$	I	bank	at	CFCU	go	to	the
DET	0.01	0.01	0.01	0.01	0.01	0.01	0.94
PRP	0.94	0.01	0.01	0.01	0.01	0.01	0.01
N	0.01	0.4	0.01	0.4	0.16	0.01	0.01
PREP	0.01	0.01	0.48	0.01	0.01	0.47	0.01
V	0.01	0.4	0.01	0.01	0.55	0.01	0.01

$P(Y_1)$	
DET	0.3
PRP	0.3
N	0.1
PREP	0.1
V	0.2

$P(Y_i\|Y_{i-1})$	DET	PRP	N	PREP	V
DET	0.01	0.01	0.96	0.01	0.01
PRP	0.01	0.01	0.01	0.2	0.77
N	0.01	0.2	0.3	0.3	0.19
PREP	0.3	0.2	0.3	0.19	0.01
V	0.2	0.19	0.3	0.3	0.01