Hard Constraints

"Shouldn't start driving if Red"

Don't collide $x \neq x_{collision}$

Preserve the ability to stop to avoid collisions.

Car dynamics $[x_{t+1} = f(x_t, u_t)]$

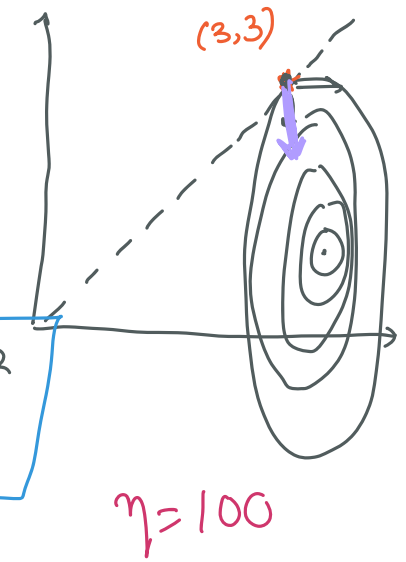
Steering angle

"Nice to have" Costs

Get out of intersection as fast as possible (especially if car)

$$2(x_1 - 4)^2 + (x_2 - 1)^2$$

$$x_1 - x_2 = 0$$



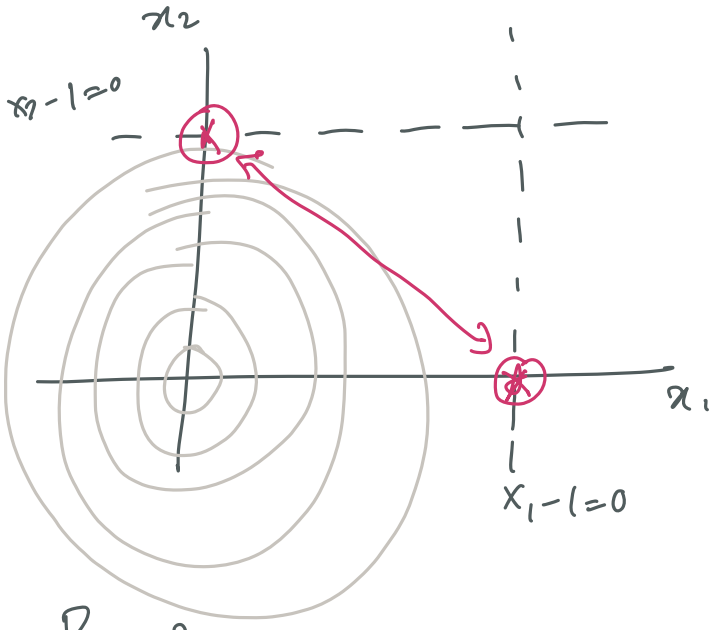
Penalty Method

$$\min_{x_1, x_2} \; \boxed{2(x_1 - 4)^2 + (x_2 - 1)^2} + \boxed{\eta (x_1 - x_2)^2}$$

$\eta = 100$

$$\nabla_{x_1} \quad 4(x_1 - 4) + 2\eta(x_1 - x_2) = 0$$

$$\nabla_{x_2} \quad 2(x_2 - 1) + 2\eta(x_1 - x_2) = 0$$



$$\max_{\lambda} \min_{x} \; f(x) - \lambda^T g(x)$$

$$\max_{\lambda_1, \lambda_2} \min_{x_1, x_2} \; \frac{1}{2}(x_1^2 + x_2^2) - \lambda_1(x_1 - 1) - \lambda_2(x_2 - 1)$$

Round 1

$\lambda_1 = 1, \quad x_2 = 0$

$$\min_x \; \frac{1}{2}(x_1^2 + x_2^2) - 1 \cdot (x_1 - 1)$$

$$\nabla x_1 = x_1 - 1 = 0$$

$$\nabla x_2 = x_2 = 0$$

Round 2

$x_2 = 1, \quad \lambda_1 = 0$

Primal Aug La (Augmented Lagrangian)

FTRL on λ (gradient descent)

Best respon of x

$$\min_x \max_\lambda \; f(x) - \lambda^T g(x)$$

$$\lambda_{t+1} = \lambda_t - \eta g(x)$$

gradient ascent

What should x play?

$$\min_x \; f(x) - \lambda(x) g(x)$$

$$\min_x \; f(x) - (\lambda_t - \eta g(x)) g(x)$$

$$x_{t+1} = \min_x \; f(x) - \lambda_t g(x) + \eta g^2(x)$$