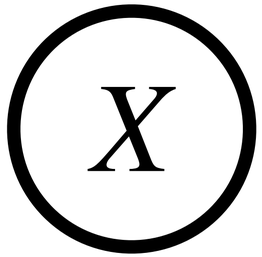
Causal Confounds in Sequential Decision Making

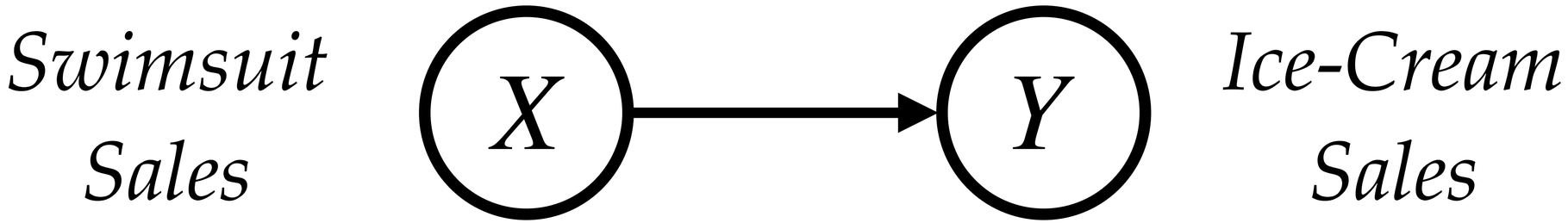
Gokul Swamy



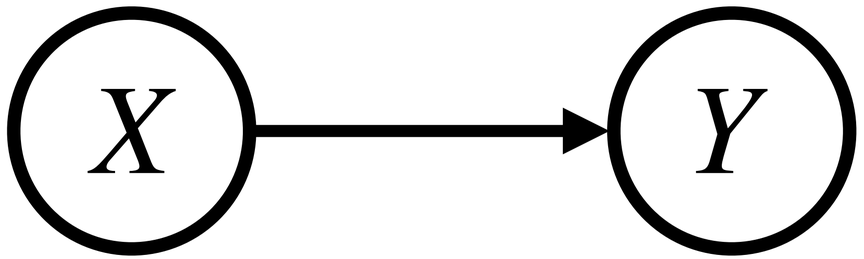

(joint work with Sanjiban Choudhury, Drew Bagnell, Steven Wu)


X

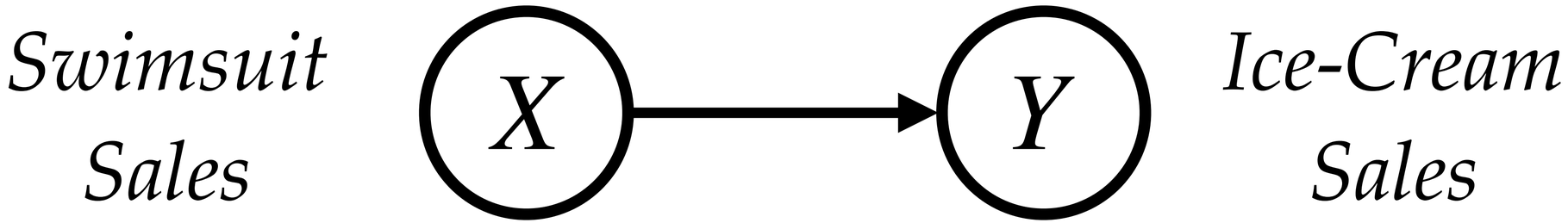

X
Y


Swimsuit Sales
X
Y
Ice-Cream Sales

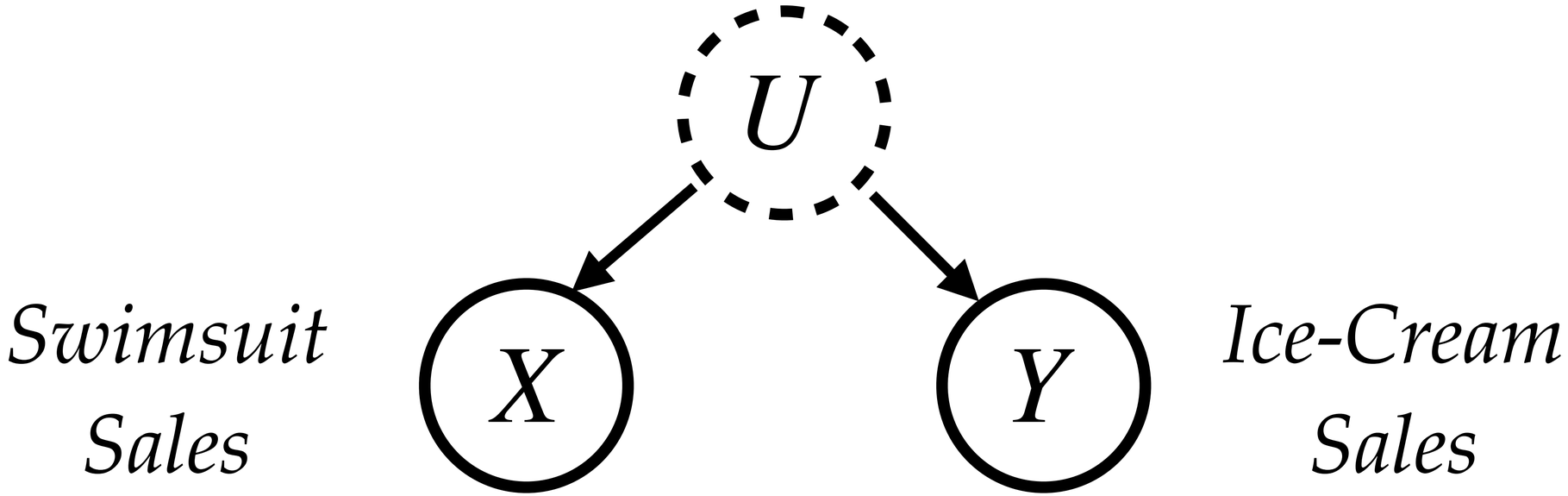
Swimsuit Sales

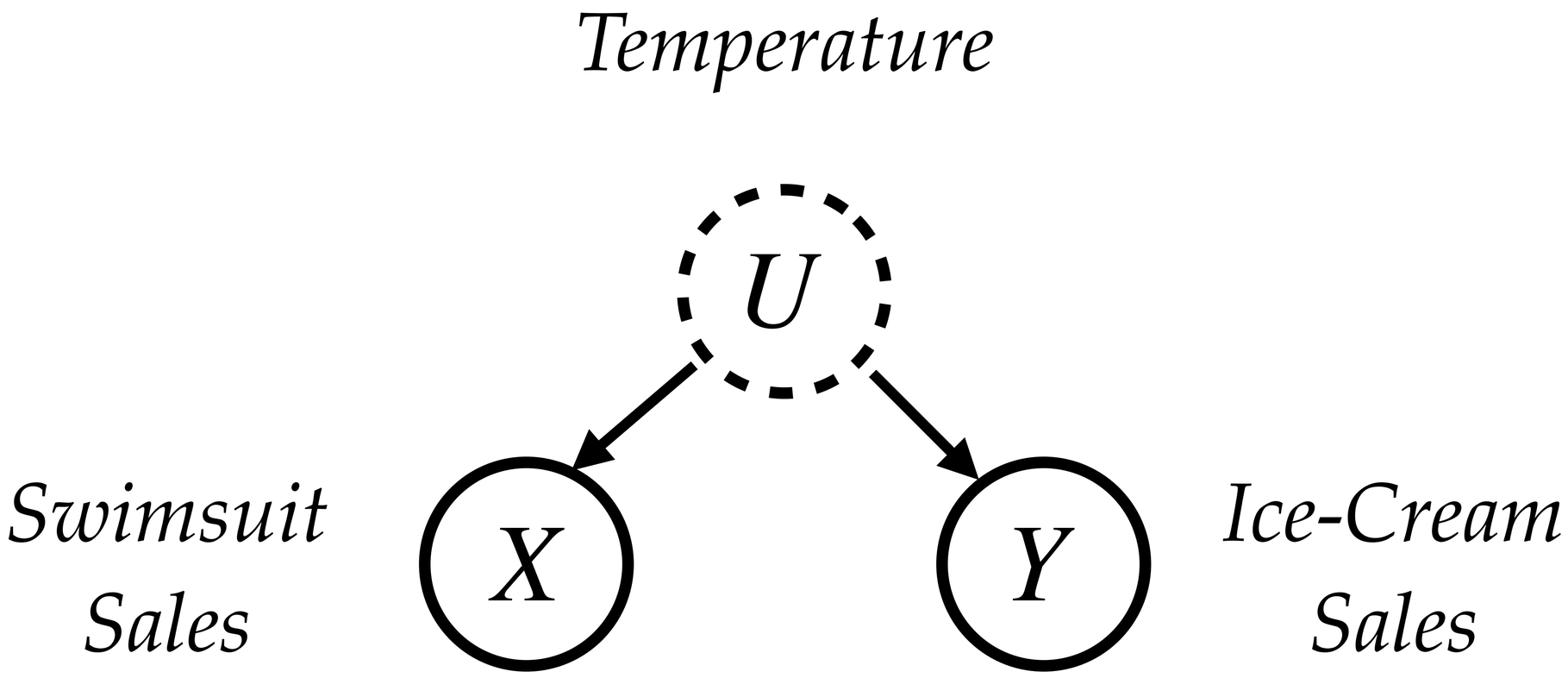


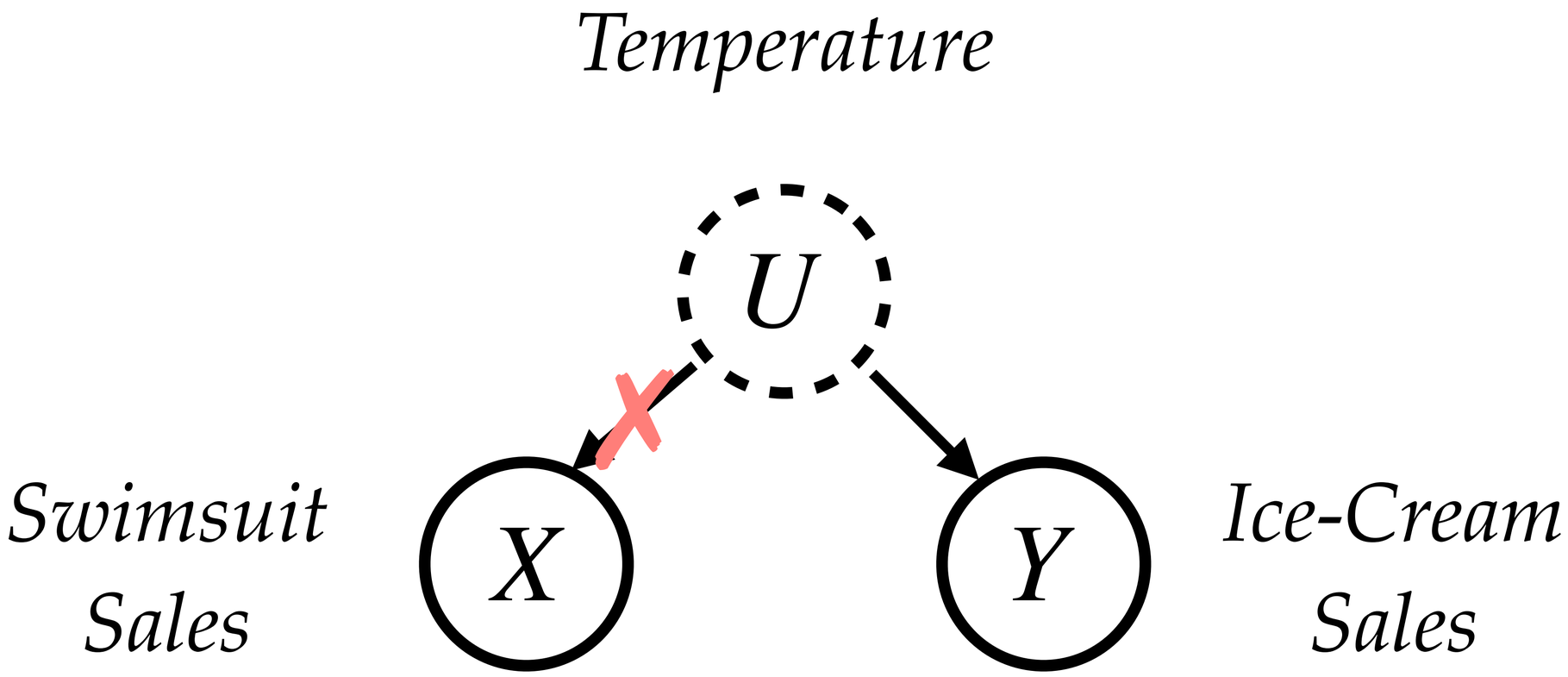
Ice-Cream Sales

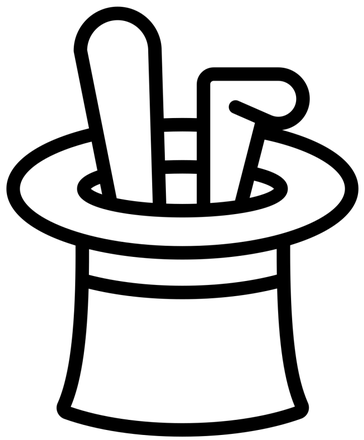

Swimsuit Sales
X
Y
Ice-Cream Sales


Swimsuit Sales
X
Y
Ice-Cream Sales

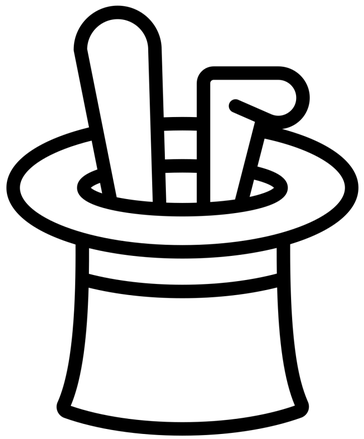

U
X
Y
Swimsuit Sales
Ice-Cream Sales

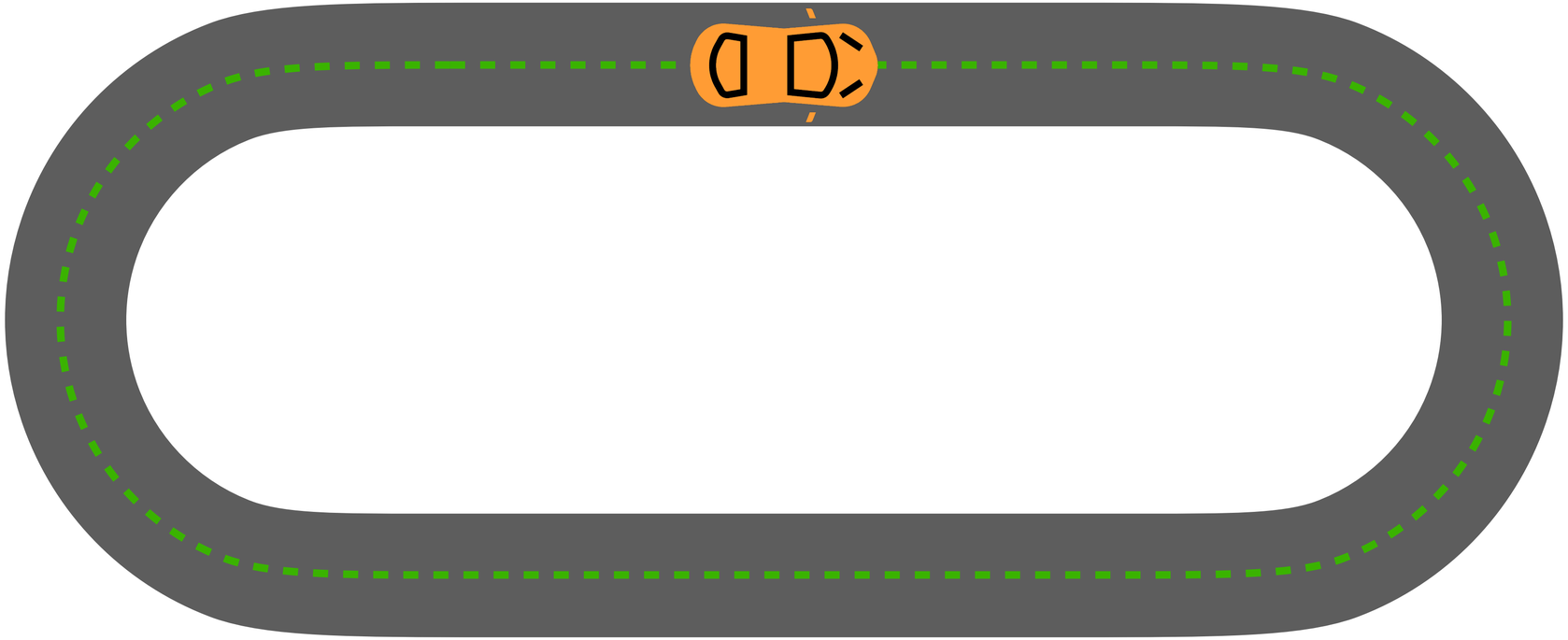

Temperature
U
X
Y
Swimsuit Sales
Ice-Cream Sales

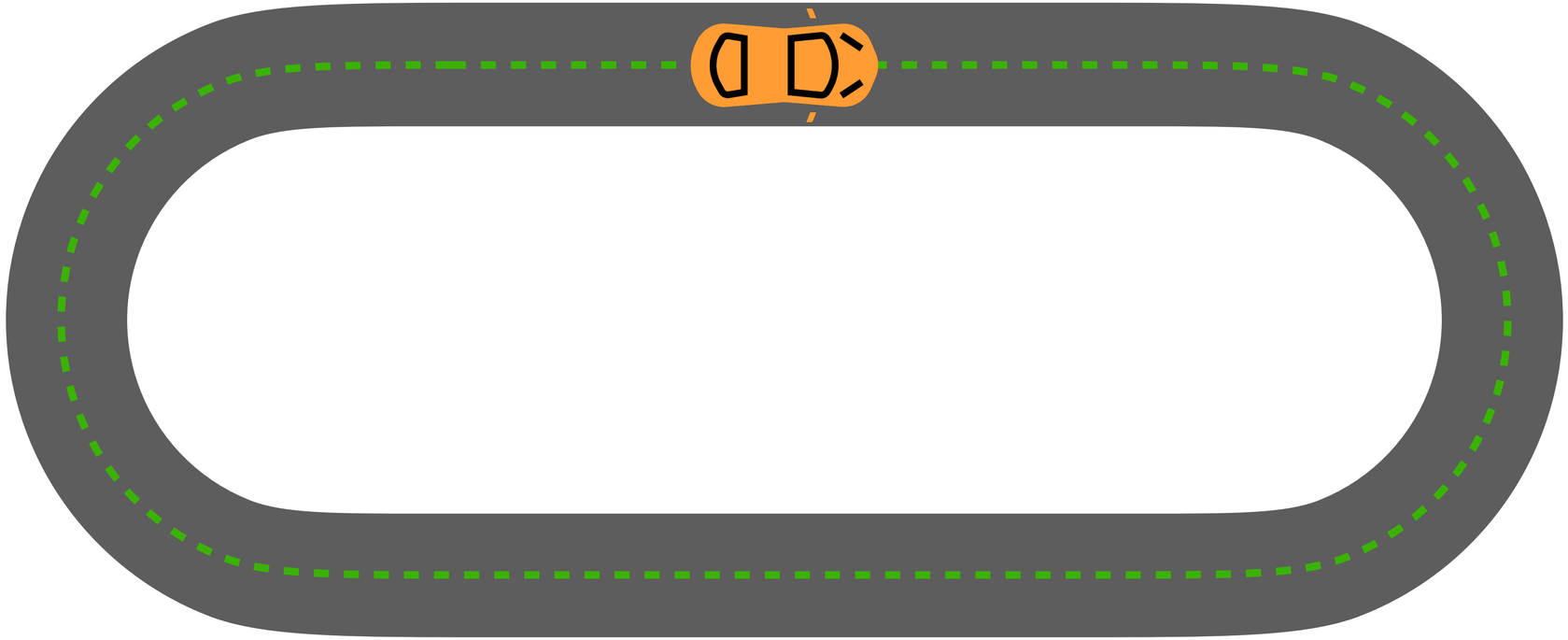

Temperature
U
X
Y
Swimsuit Sales
Ice-Cream Sales



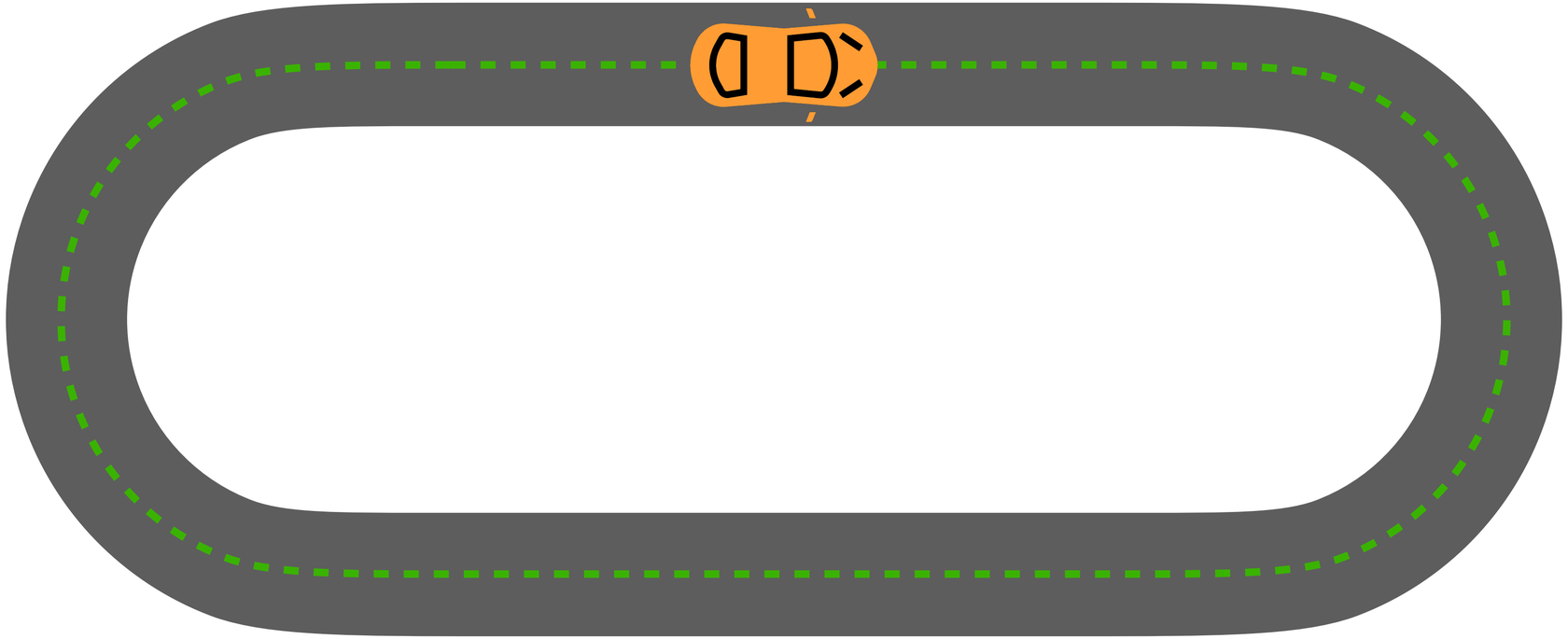
Interventions happen via interaction with the environment in sequential decision making.



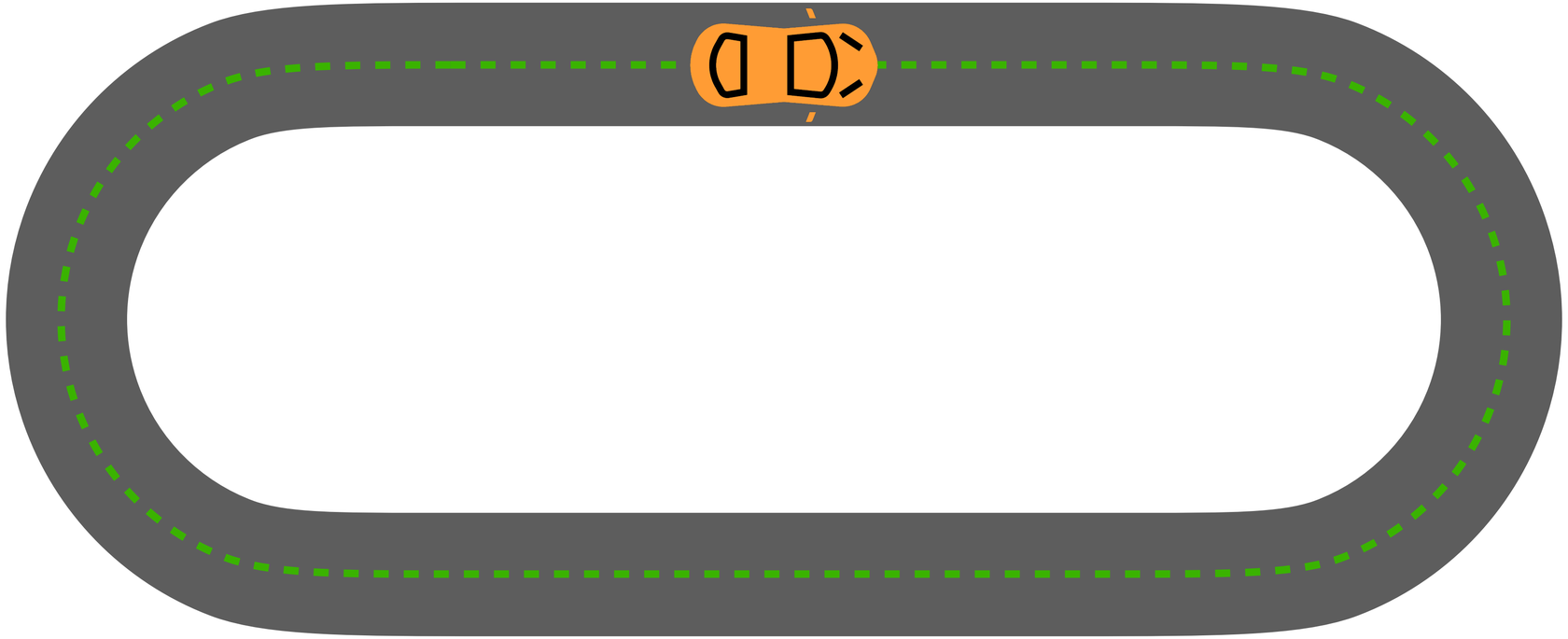




$\{s_1 \ldots s_n\}$ $\{a_1 \ldots a_n\}$

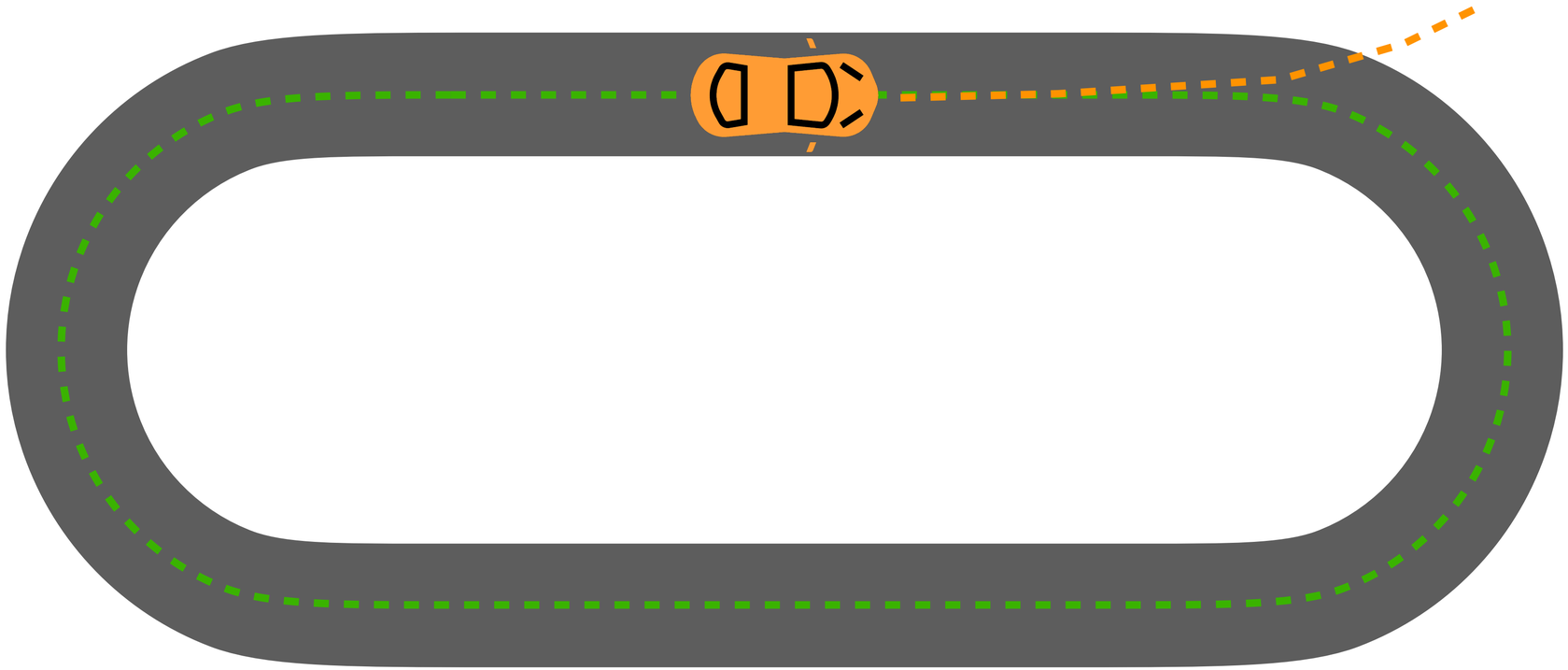


$$\{s_1 \ldots s_n\} \mapsto \{a_1 \ldots a_n\}$$



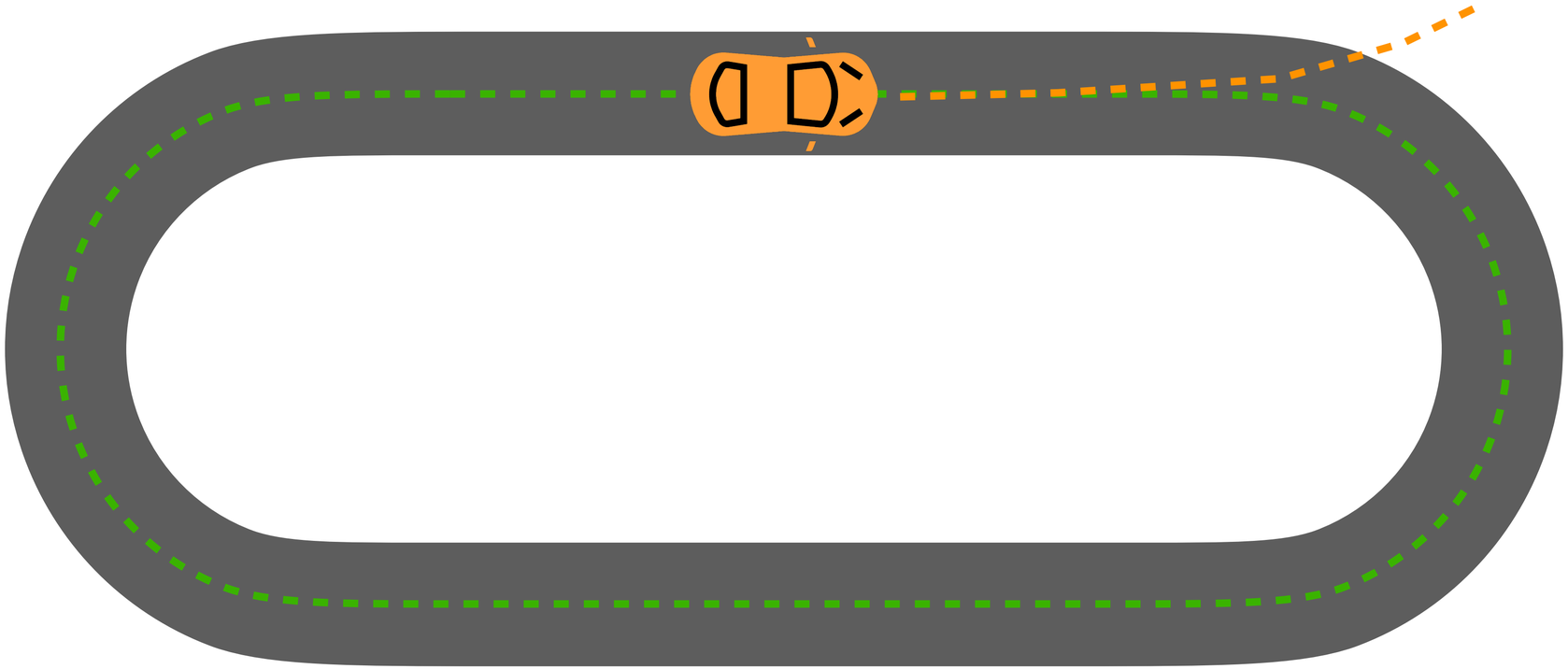
$$\{s_1 \ldots s_n\} \mapsto \{a_1 \ldots a_n\}$$

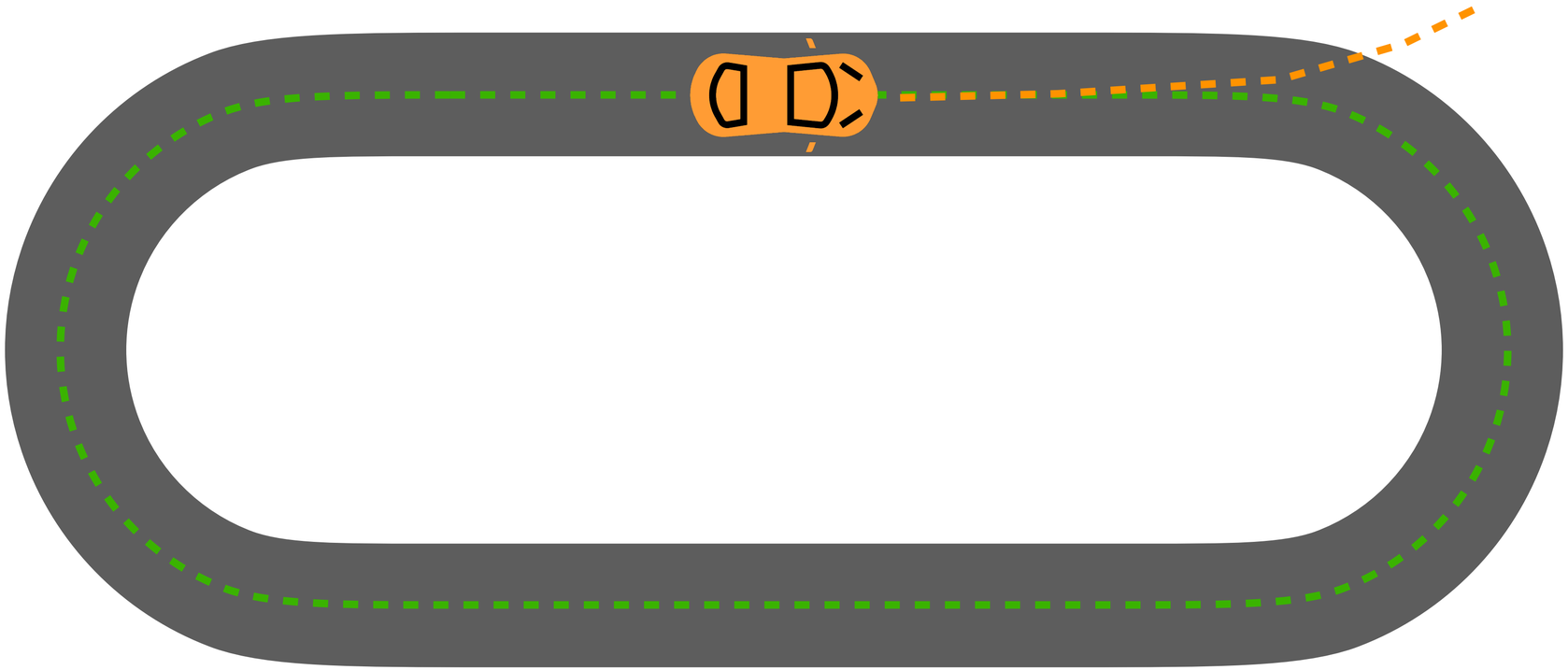
Behavioral Cloning



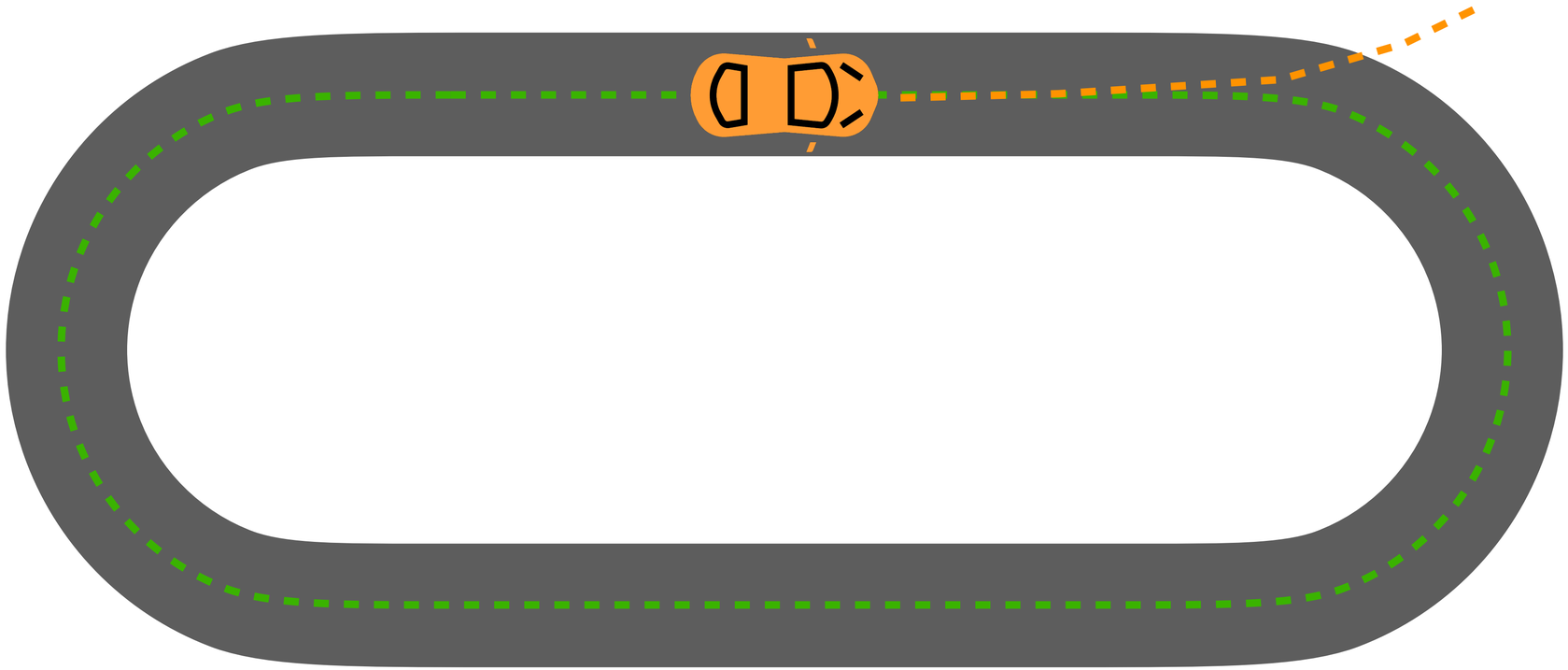
$$\{s_1 \ldots s_n\} \mapsto \{a_1 \ldots a_n\}$$

Behavioral Cloning



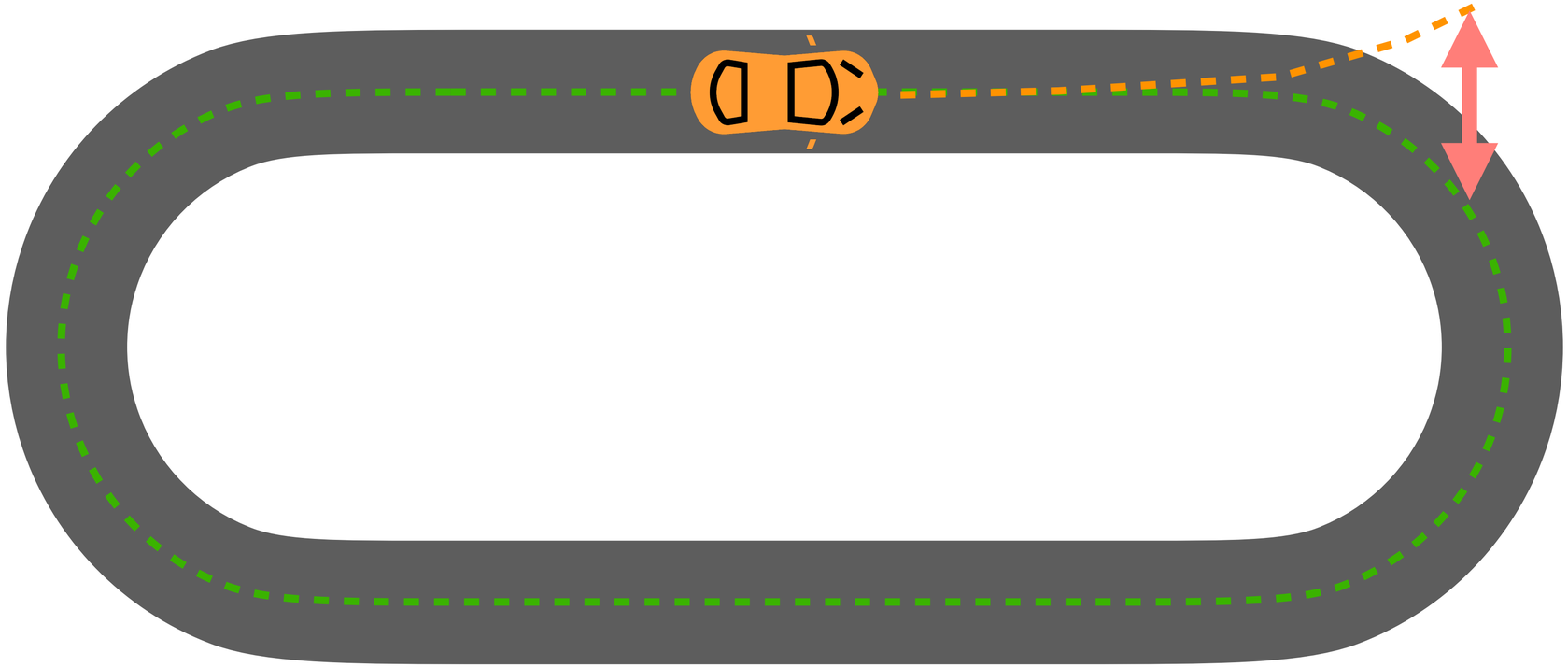


MaxEnt IRL / GAIL



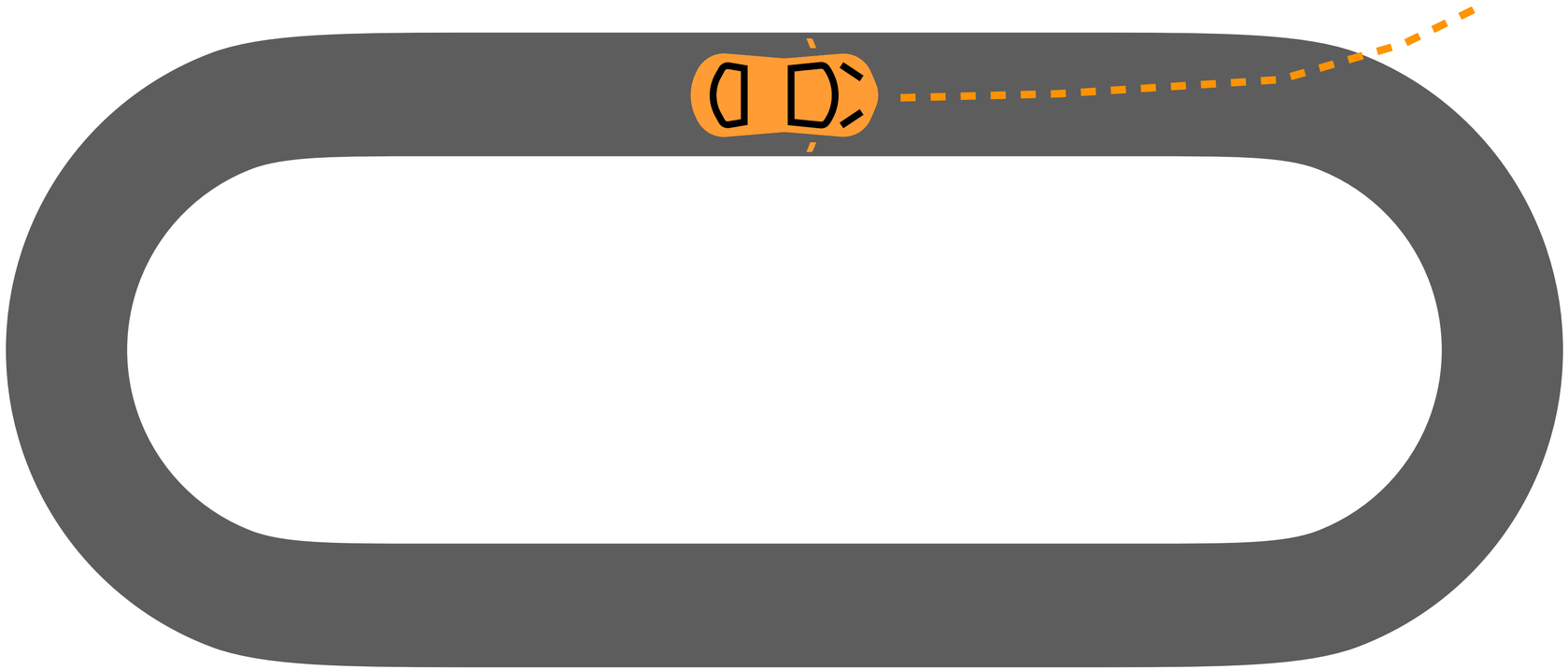
$\{s_1 \ldots s_n\}$
$\{a_1 \ldots a_n\}$

MaxEnt IRL / GAIL



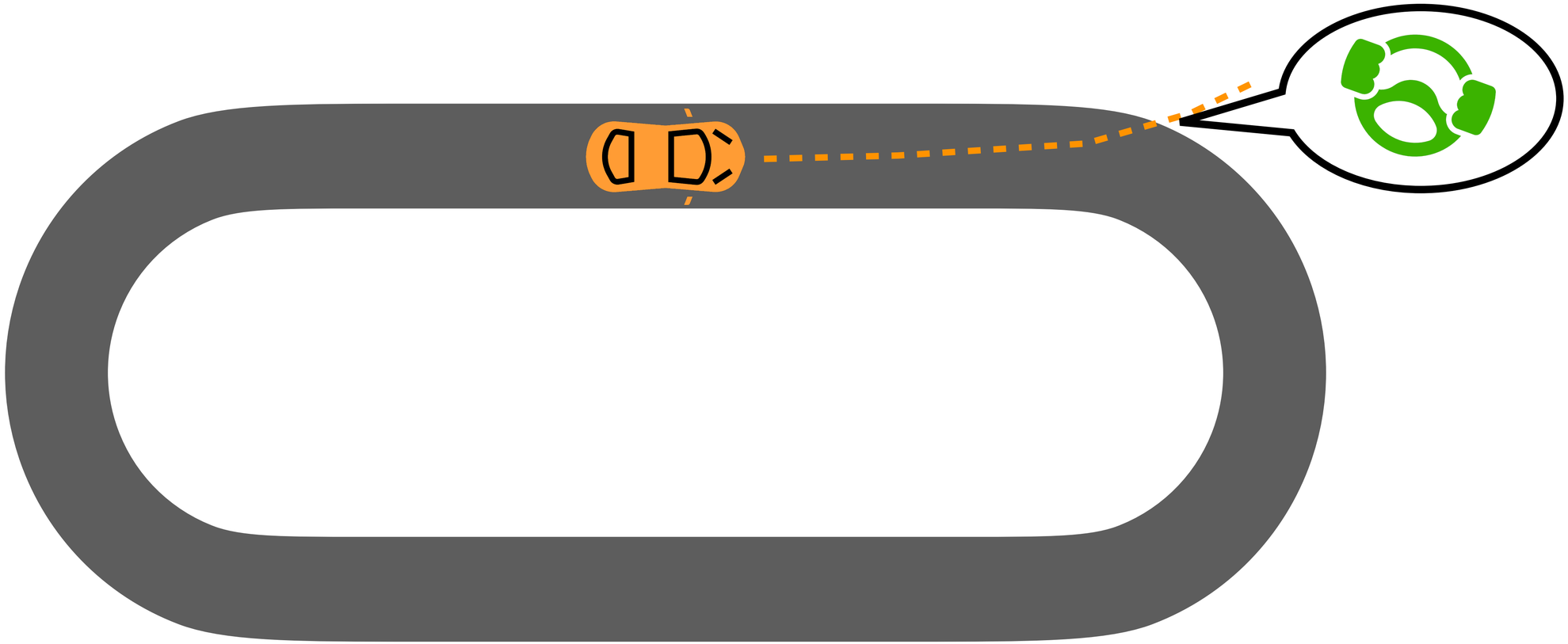
$$\begin{matrix}\{s_1 \ldots s_n\} \\ \{a_1 \ldots a_n\}\end{matrix} \longleftrightarrow \begin{matrix}\{s_1 \ldots s_n\} \\ \{a_1 \ldots a_n\}\end{matrix}$$

MaxEnt IRL / GAIL



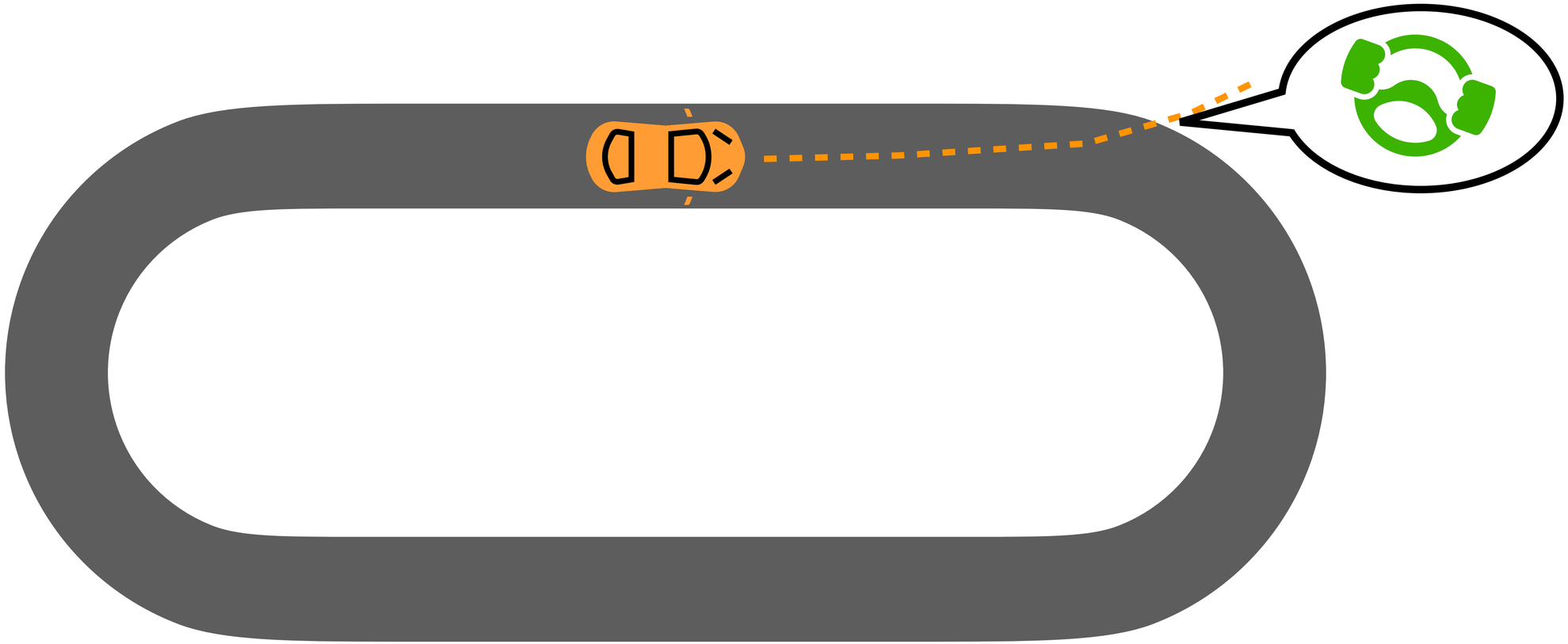
$\{s_1 \ldots s_n\}$

DAgger



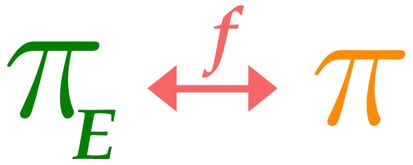
$\{s_1 \ldots s_n\}$

DAgger

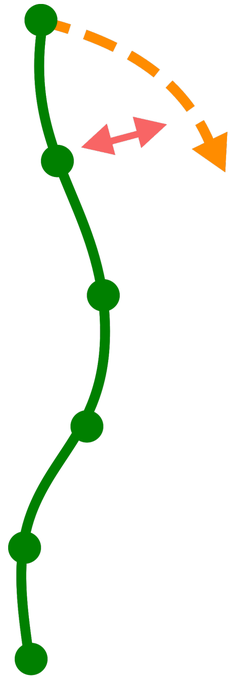


$$\{s_1 \ldots s_n\} \mapsto \{a_1 \ldots a_n\}$$

DAgger

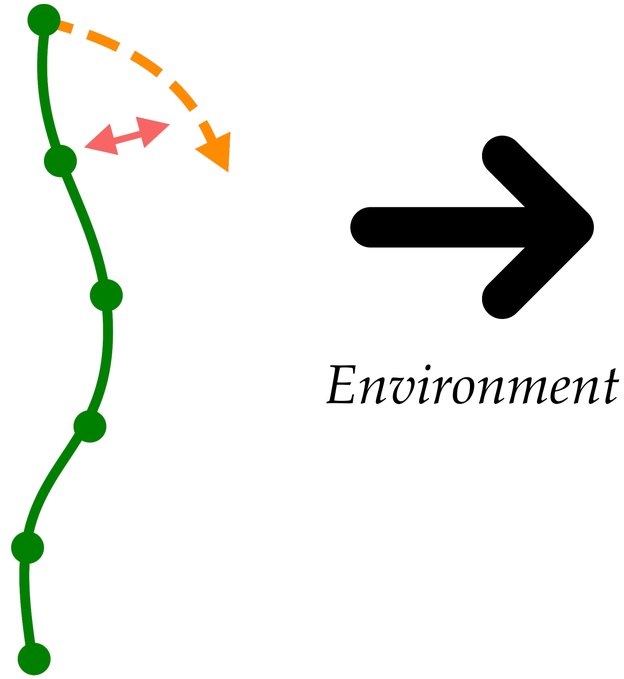

π_E
f
π

$$\pi_E \overset{f}{\longleftrightarrow} \pi$$



Behavioral Cloning

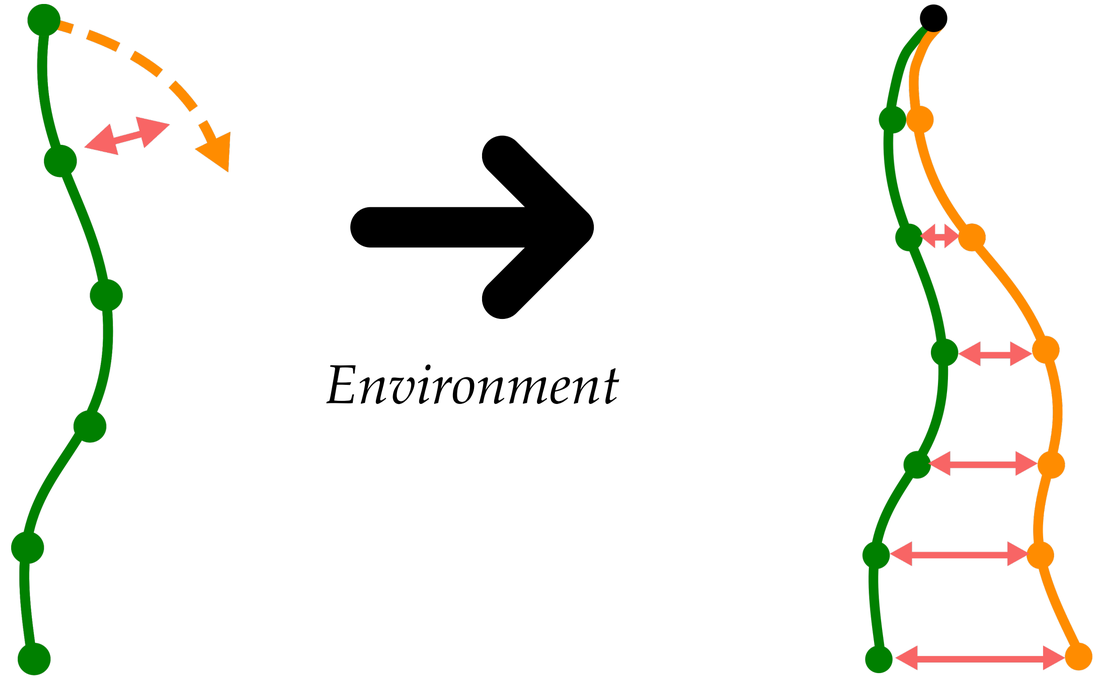
$\pi_E \overset{f}{\longleftrightarrow} \pi$



Environment

Behavioral Cloning

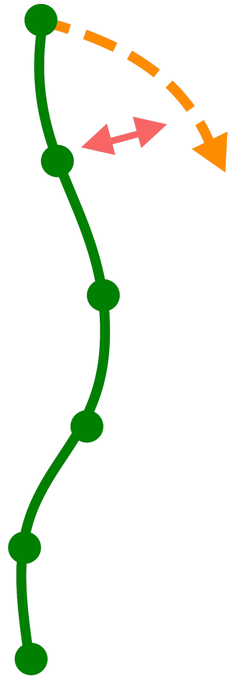
$\pi_E \overset{f}{\longleftrightarrow} \pi$



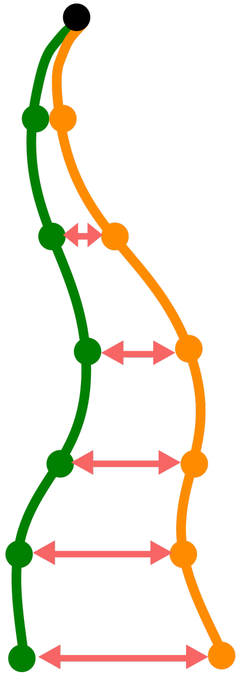
Behavioral Cloning

GAIL

$\pi_E \overset{f}{\longleftrightarrow} \pi$



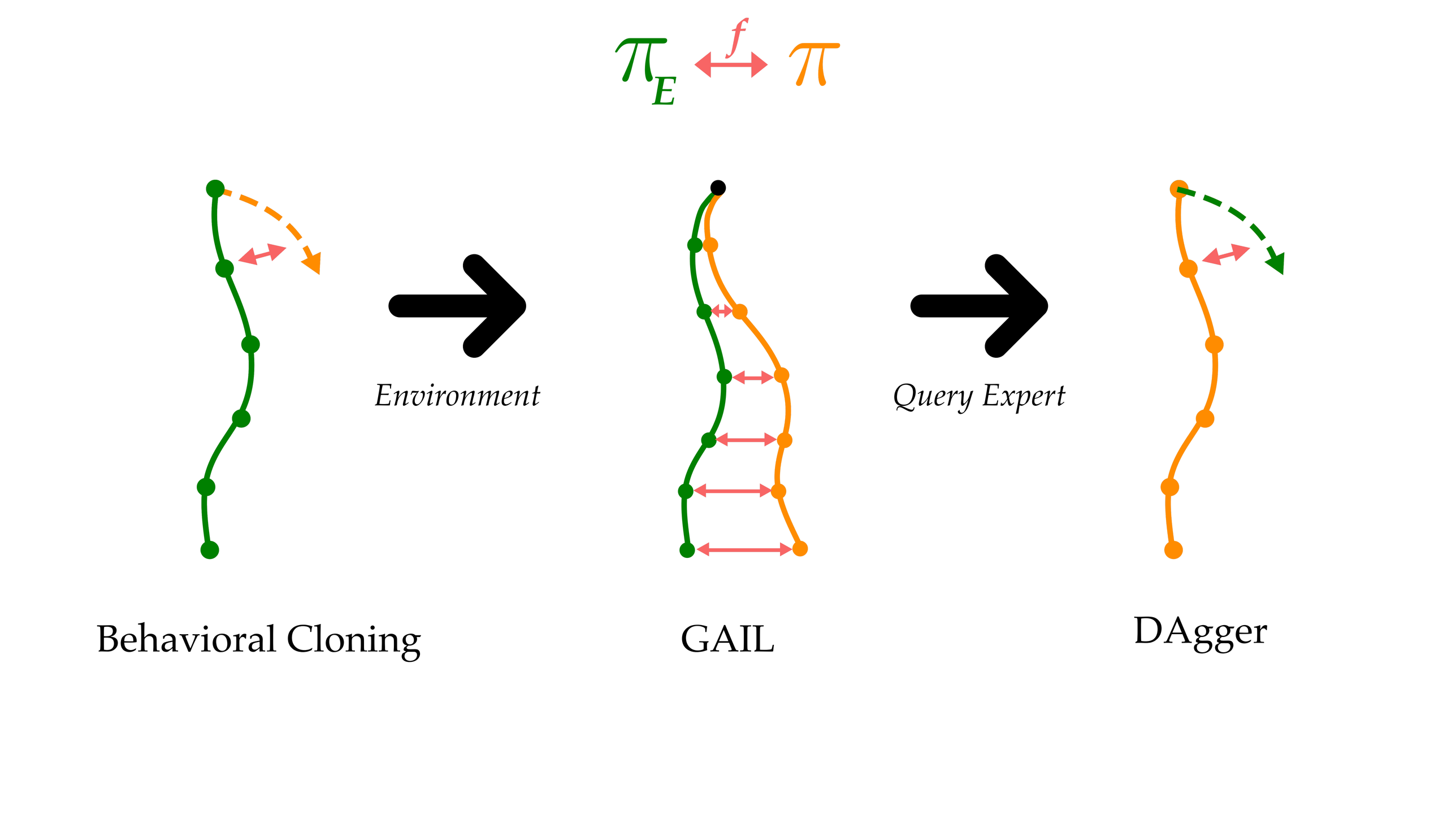
Environment

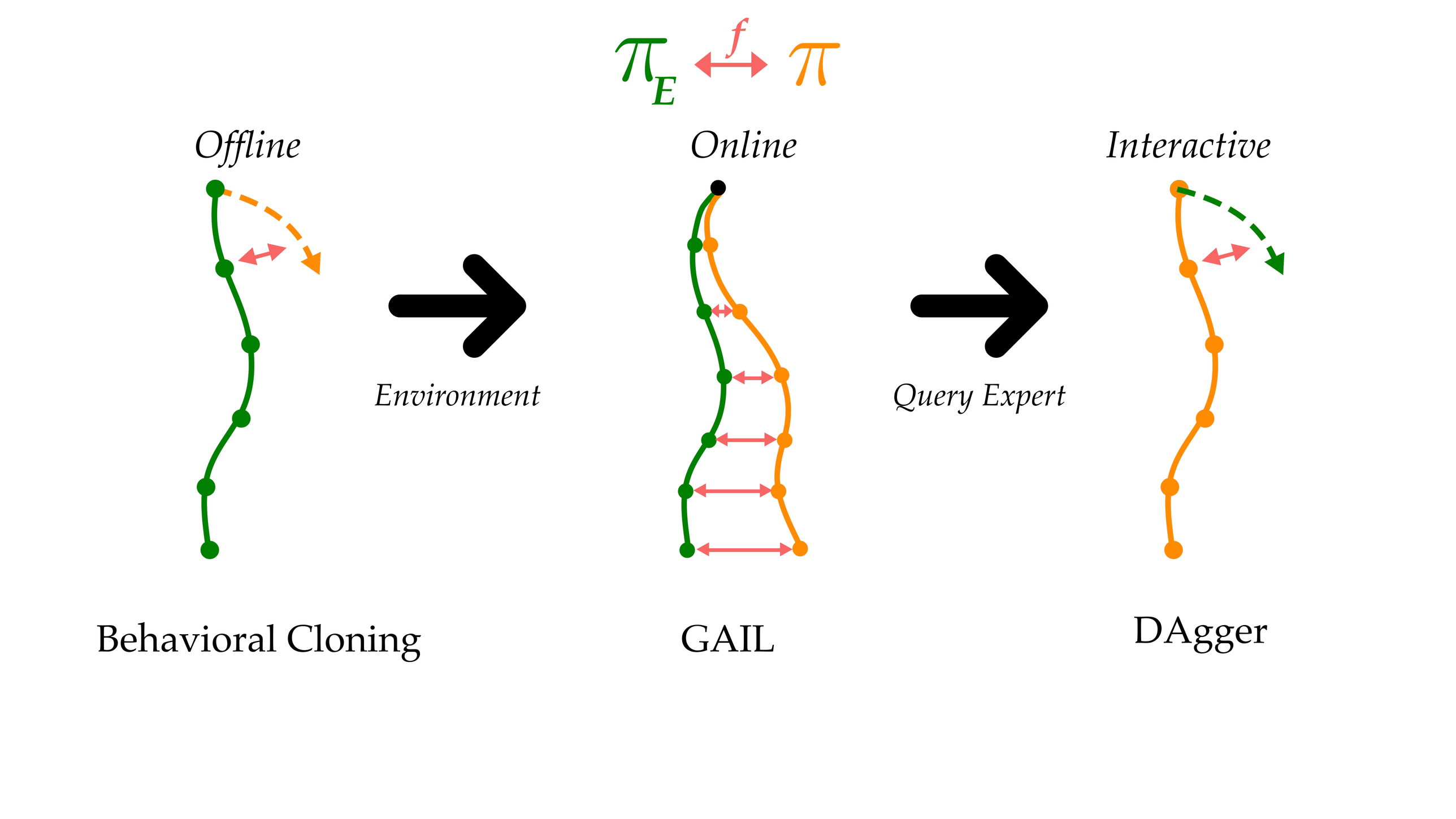


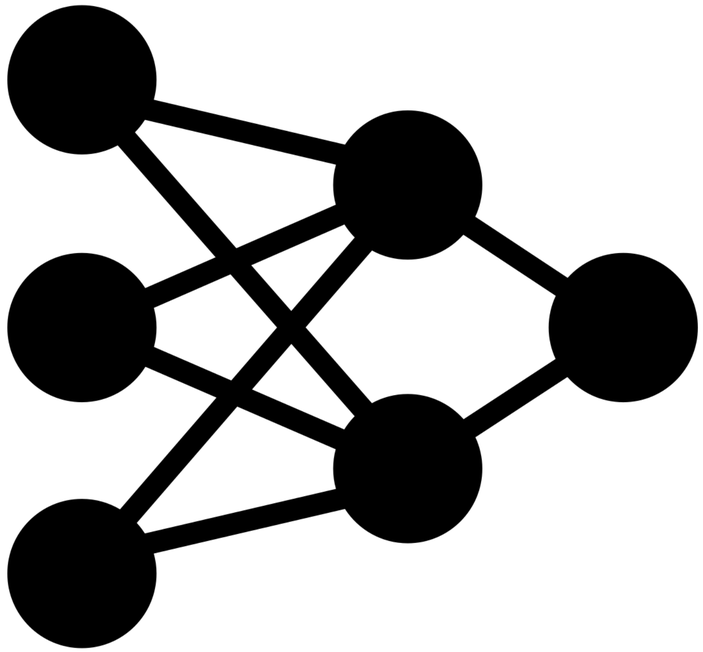
Query Expert

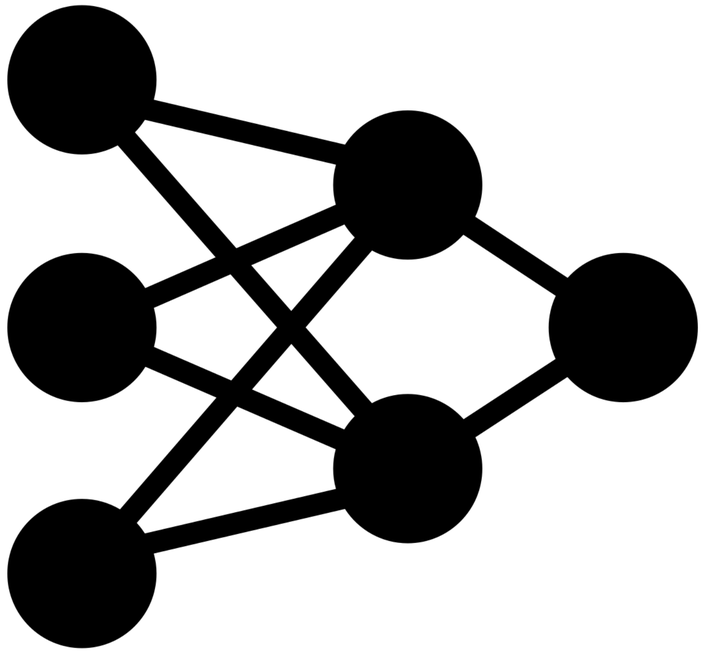
Behavioral Cloning

GAIL

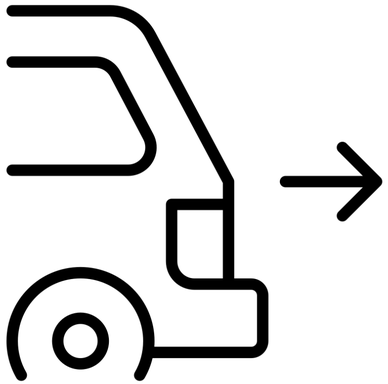

π_E
f
π
Environment
Query Expert
Behavioral Cloning
GAIL
DAgger

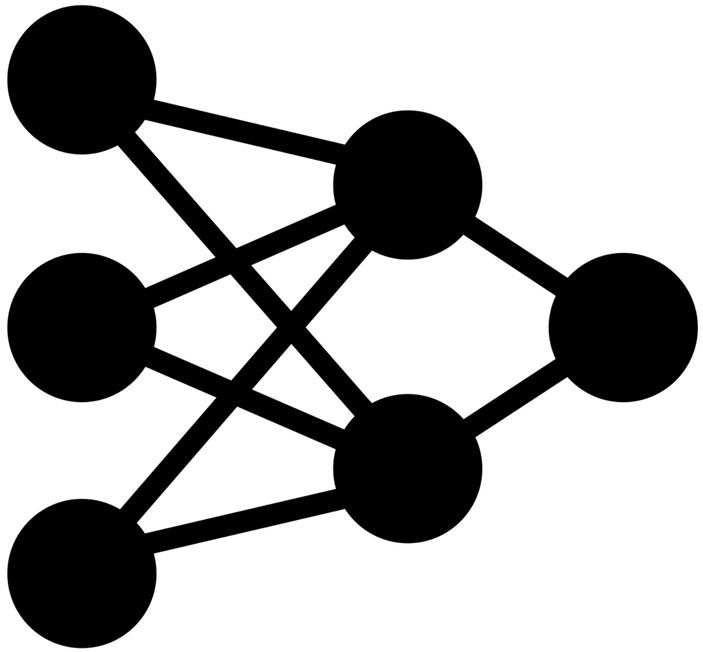

$\pi_E \overset{f}{\longleftrightarrow} \pi$
Offline
Online
Interactive
Environment
Query Expert
Behavioral Cloning
GAIL
DAgger



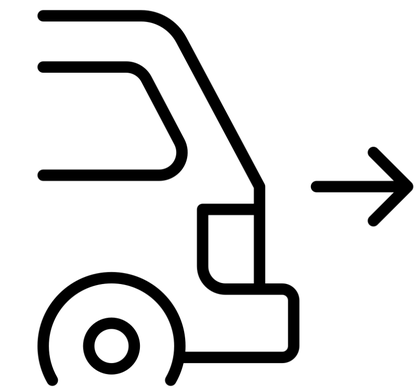


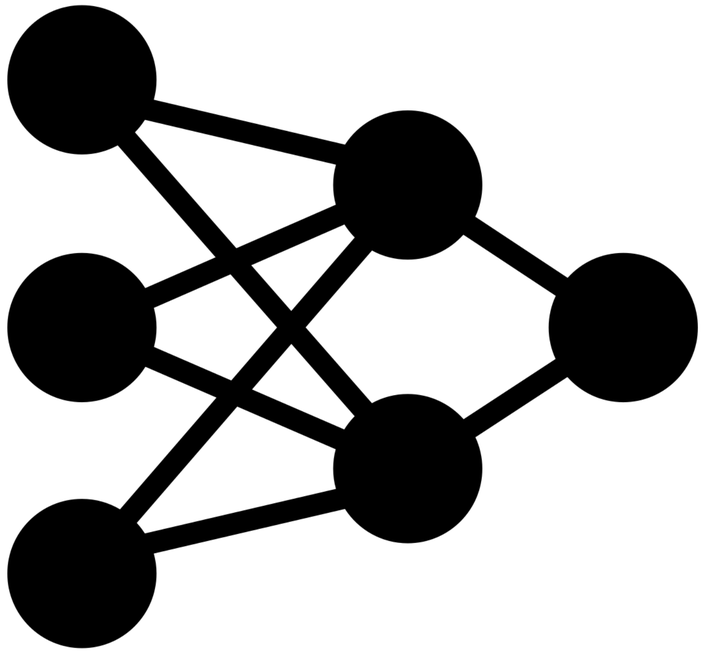
Brake?





Brake?





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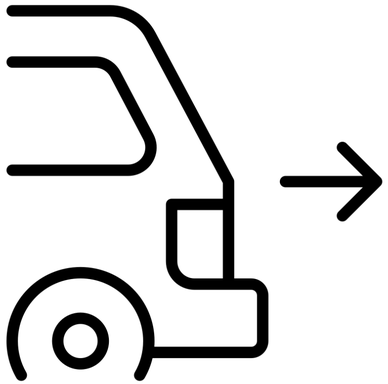
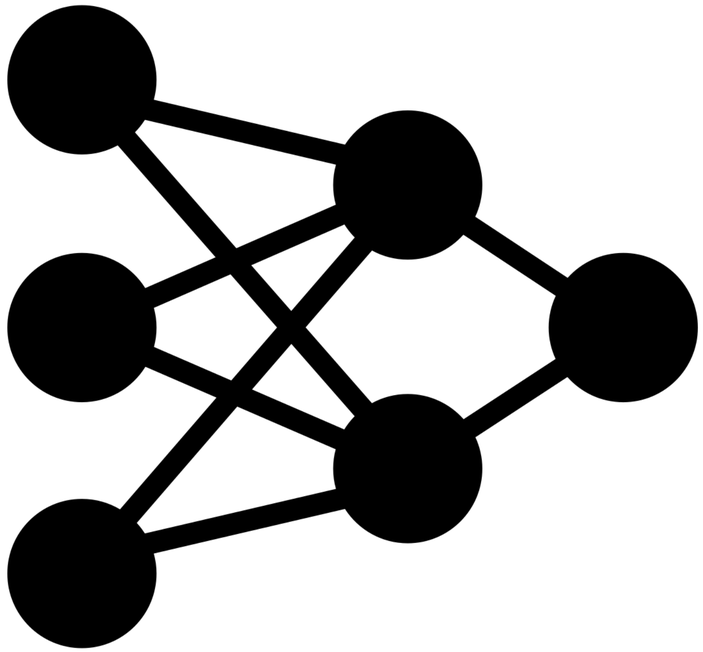
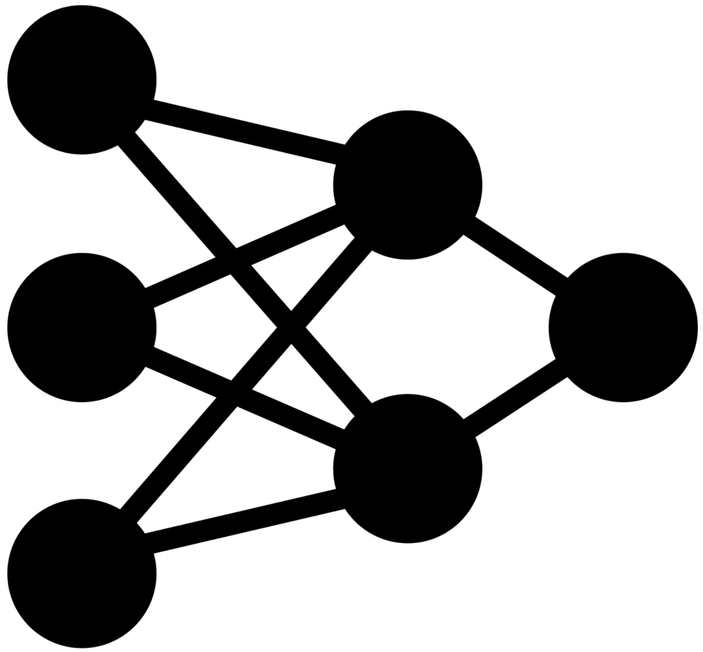
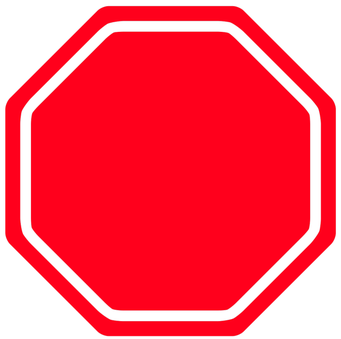
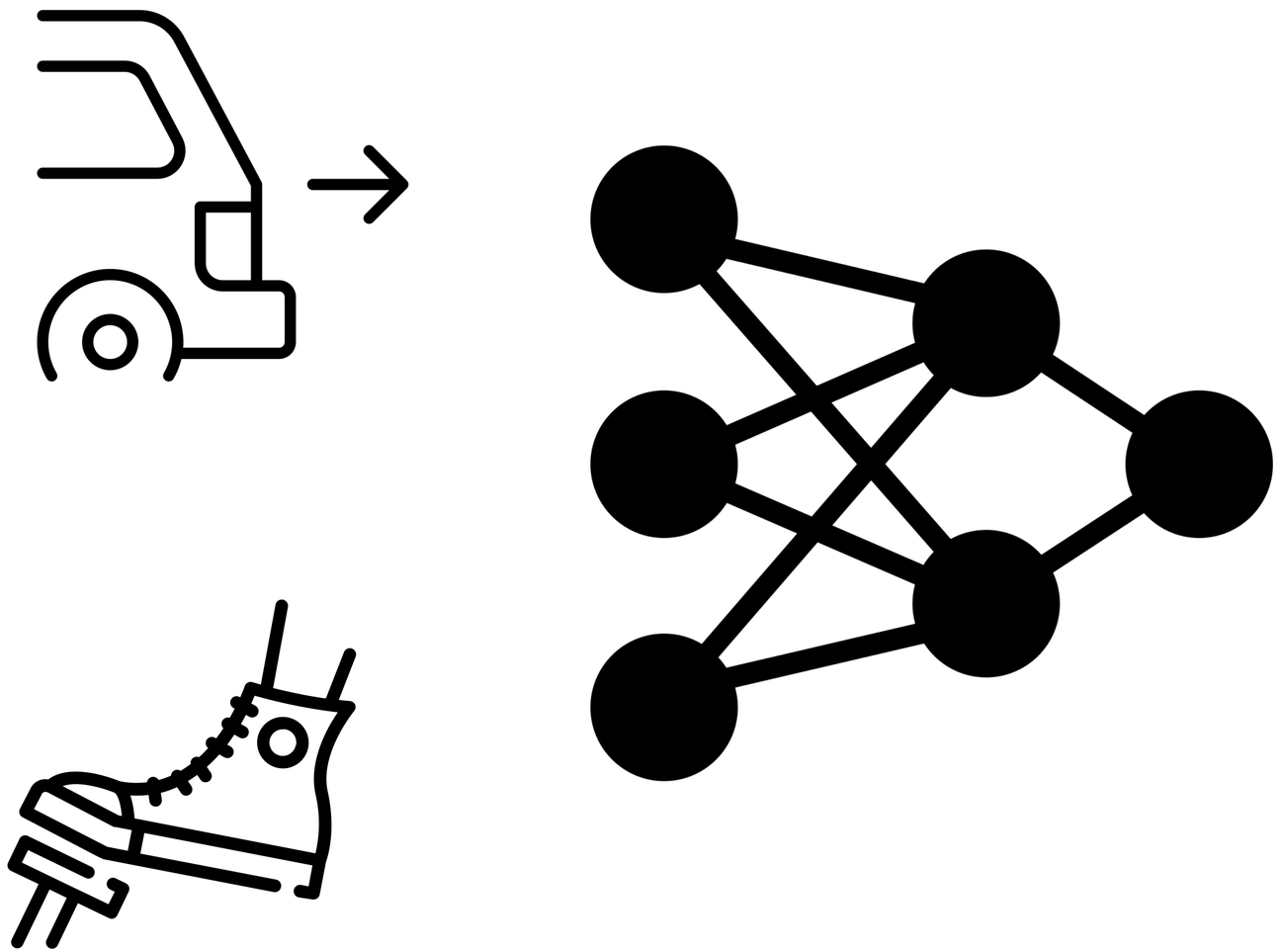
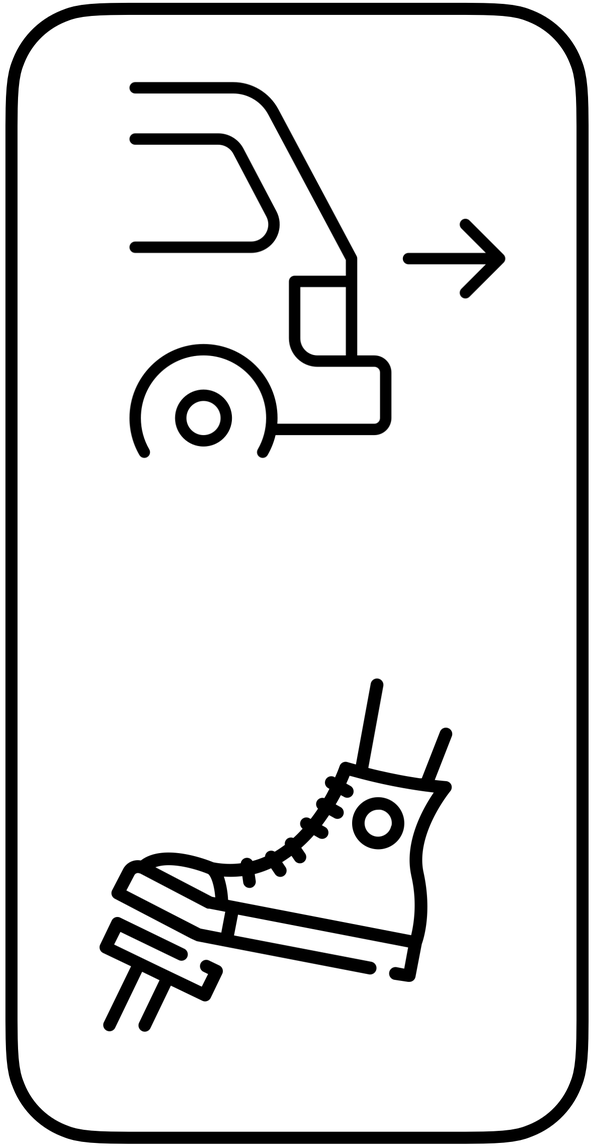


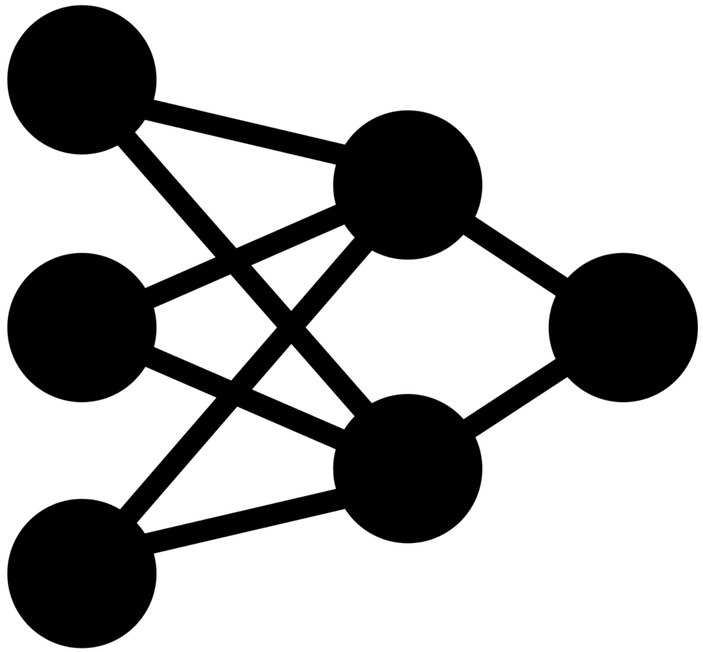












Q: Would DAgger fix this problem?

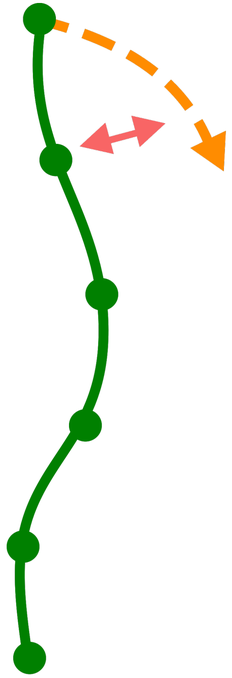
Q: Would DAgger fix this problem?

A: Yes, it's just covariate shift?

	Offline	Online	Interactive
Covariate Shift	✘	✔	✔
Hidden Context			
TCN			

$\pi_E \overset{f}{\leftrightarrow} \pi$

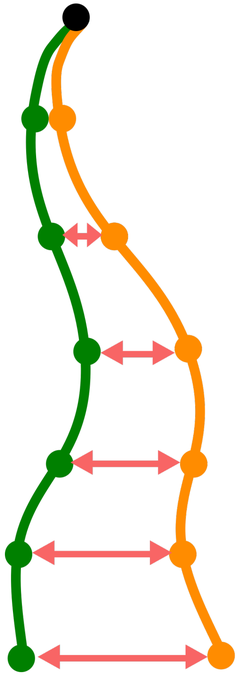
Offline



$J(\pi_E) - J(\pi) \leq O(\epsilon T^2)$

Behavioral Cloning …

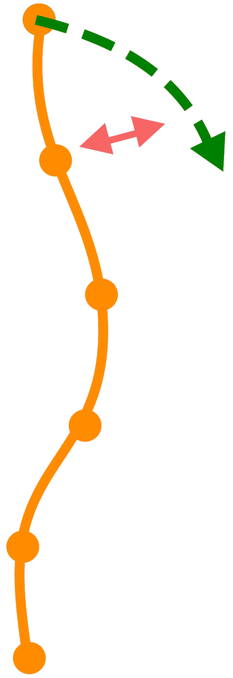
Online



$J(\pi_E) - J(\pi) \leq O(\epsilon T)$

GAIL, MaxEnt IRL …

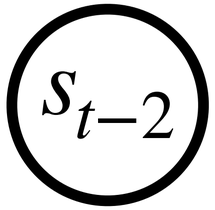
Interactive

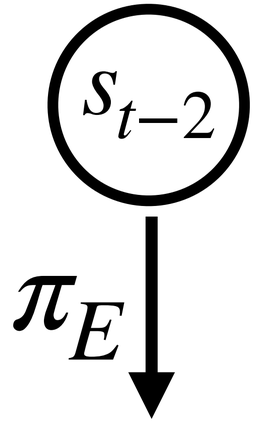


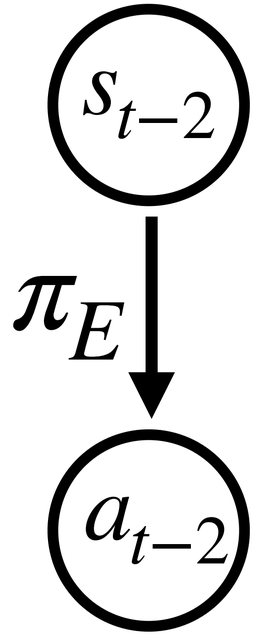
$J(\pi_E) - J(\pi) \leq O(\epsilon H T)$

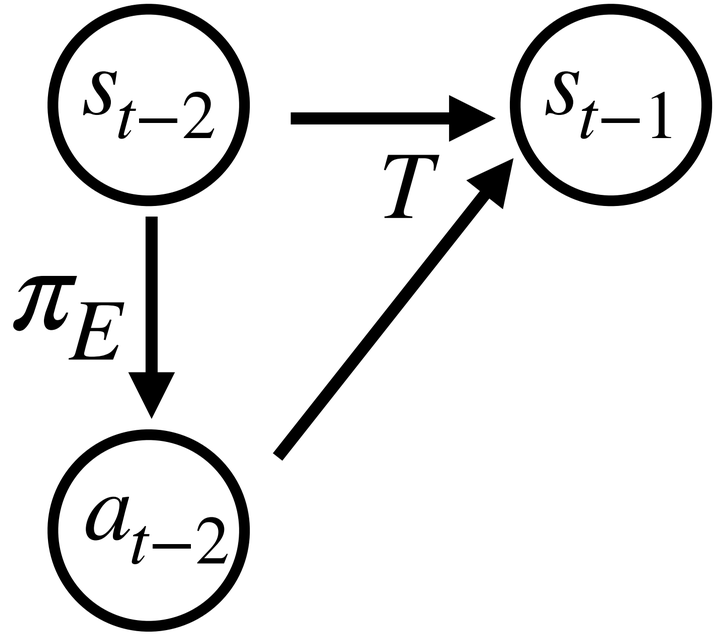
DAgger …

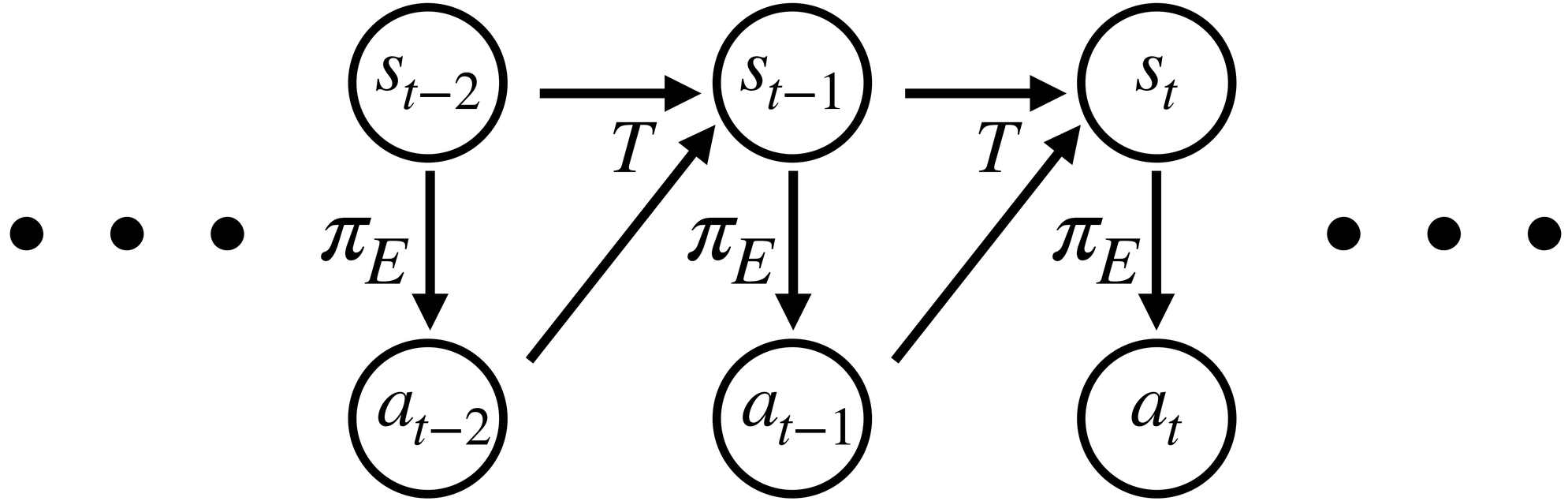
"Hence, a system trained with multiple frames would merely predict a steering angle equal to the current rate of turn as observed through the camera. This would lead to catastrophic behavior in test mode. The robot would simply turn in circles."
— Muller et al., 2006

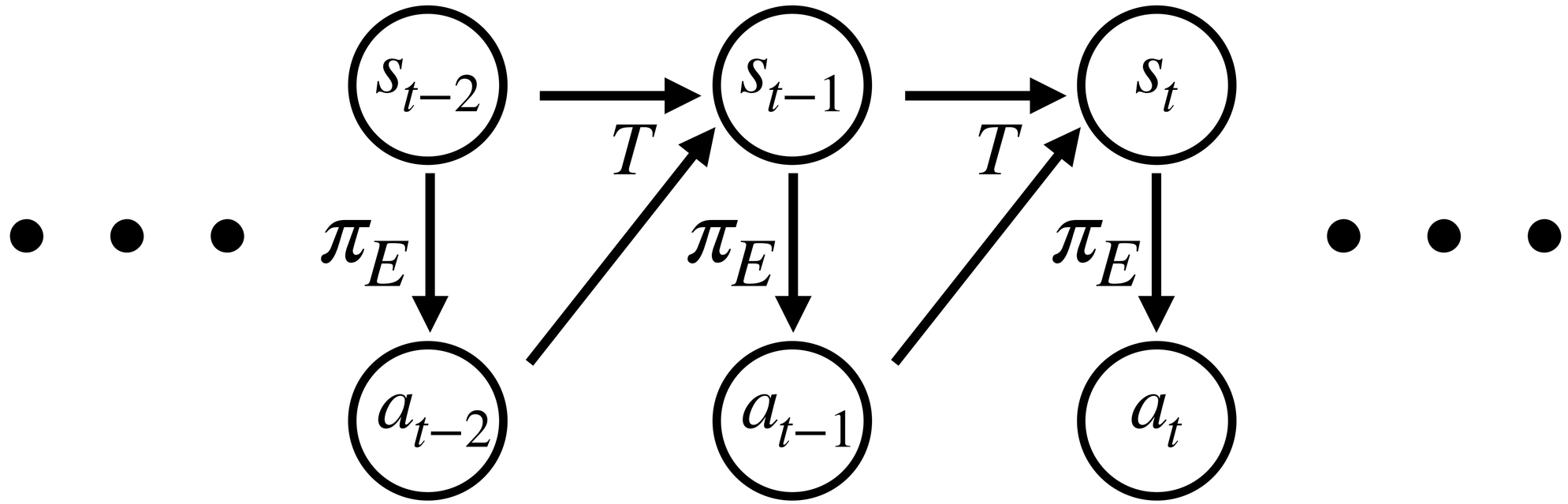

s_{t-2}


s_{t-2}
π_E

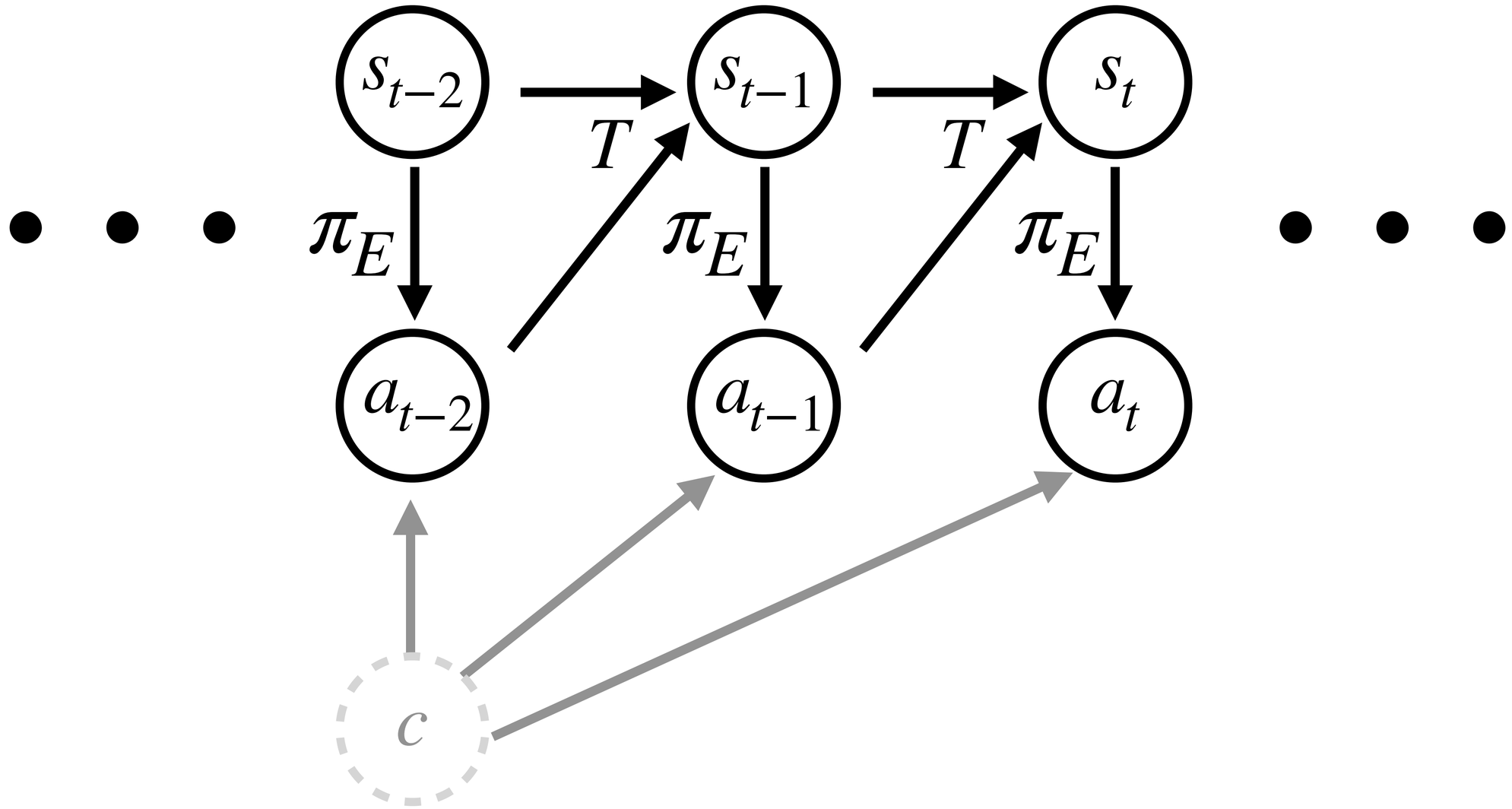

s_{t-2}
π_E
a_{t-2}

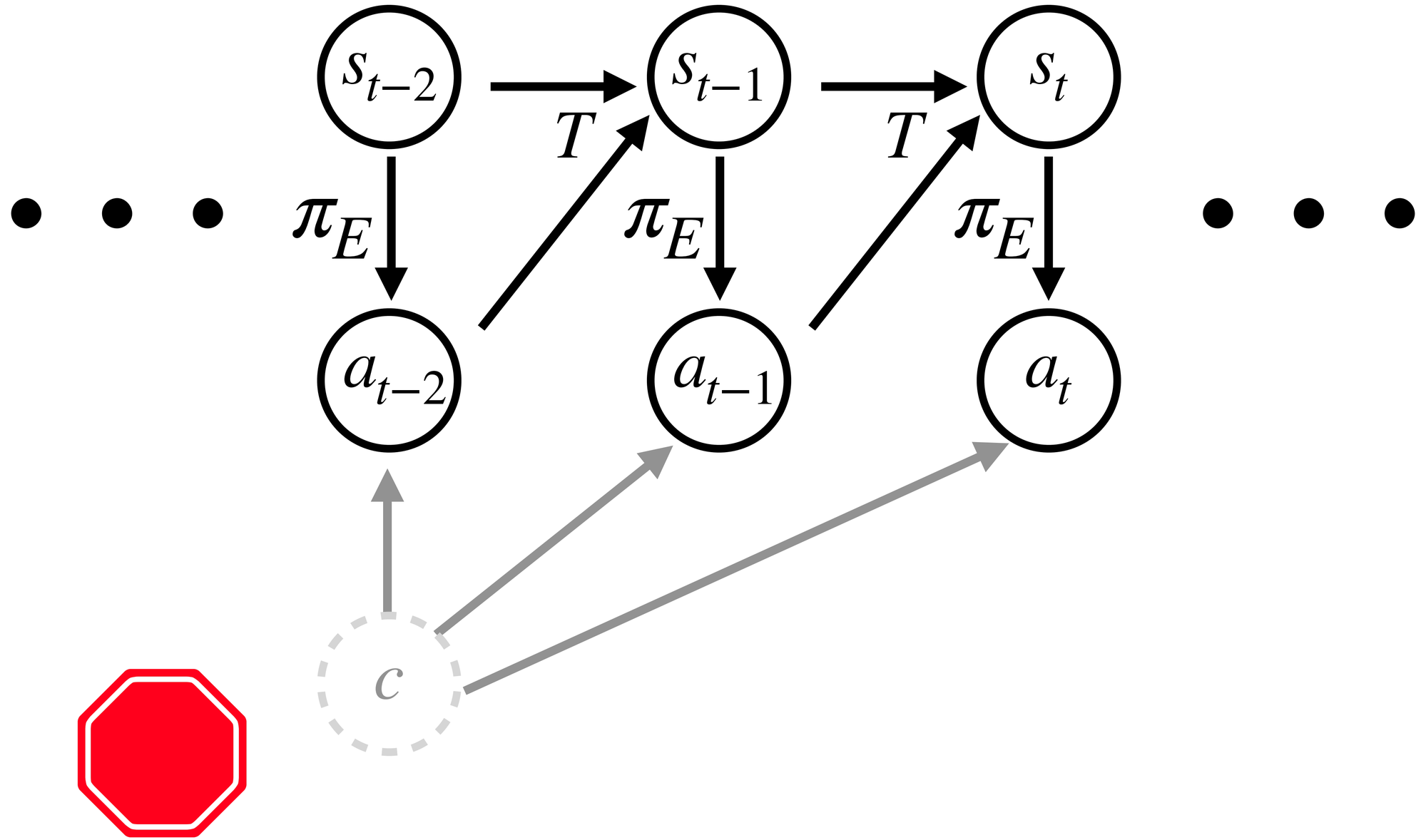

s_{t-2}
s_{t-1}
T
π_E
a_{t-2}

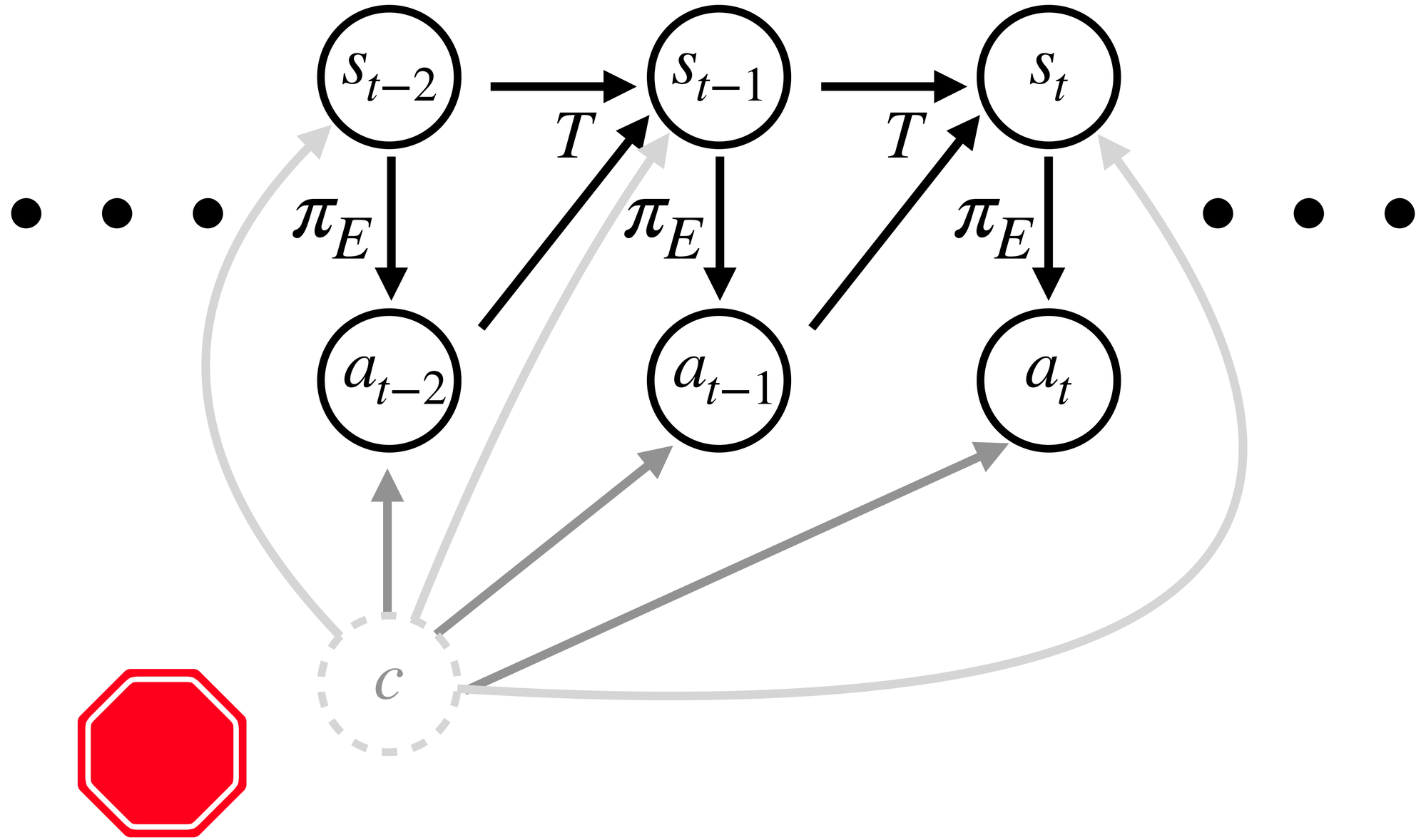

s_{t-2}
s_{t-1}
s_t
T
T
π_E
π_E
π_E
a_{t-2}
a_{t-1}
a_t

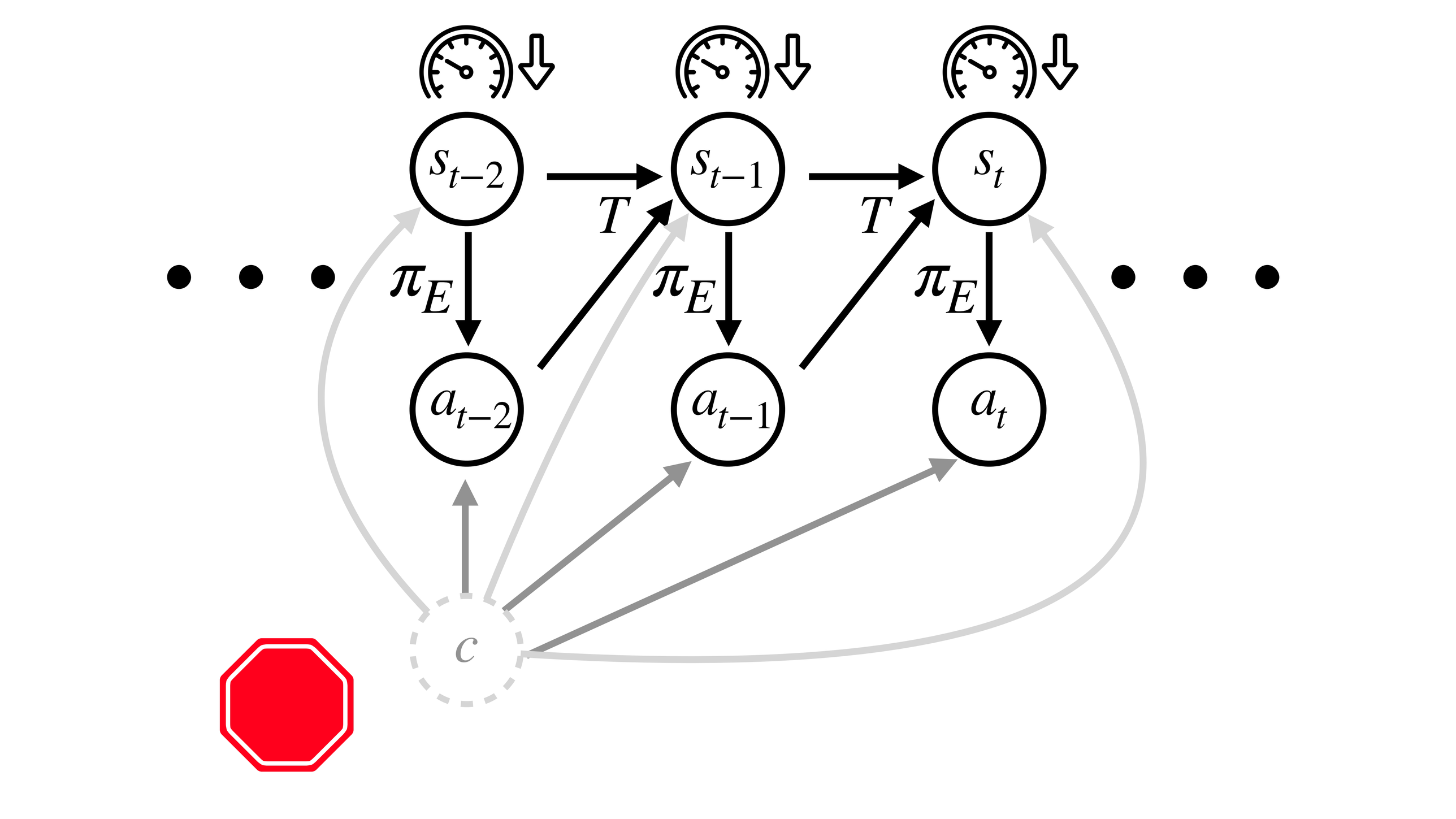


c


s_{t-2}
s_{t-1}
s_t
T
T
π_E
π_E
π_E
a_{t-2}
a_{t-1}
a_t
c


s_{t-2}
s_{t-1}
s_t
T
T
π_E
π_E
π_E
a_{t-2}
a_{t-1}
a_t
c

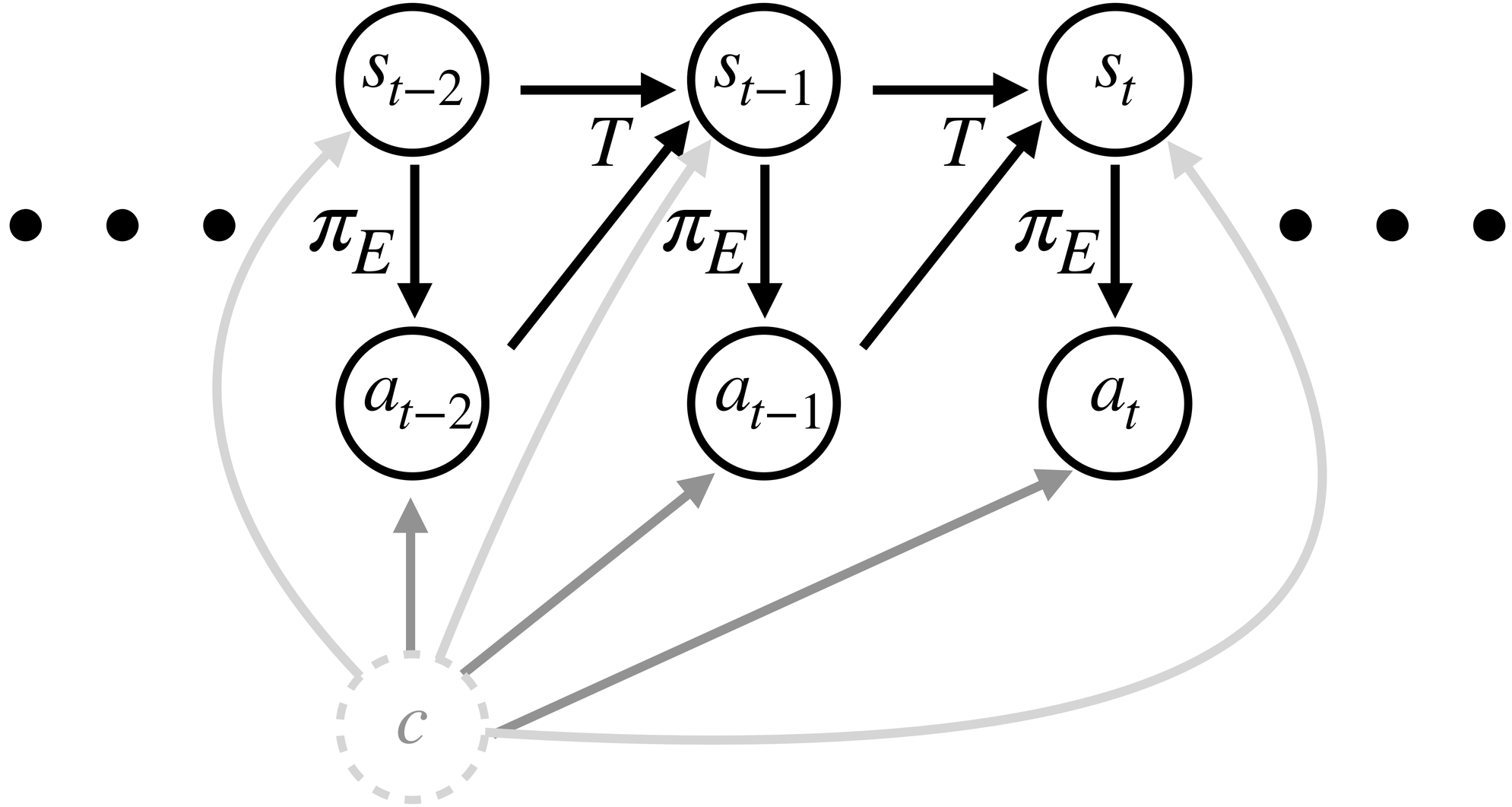

s_{t-2}
s_{t-1}
s_t
T
T
π_E
π_E
π_E
a_{t-2}
a_{t-1}
a_t
c

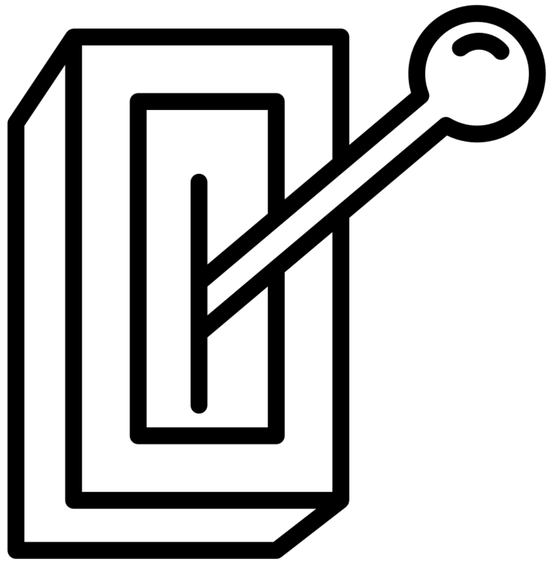

s_{t-2}
s_{t-1}
s_t
T
T
π_E
π_E
π_E
a_{t-2}
a_{t-1}
a_t
c

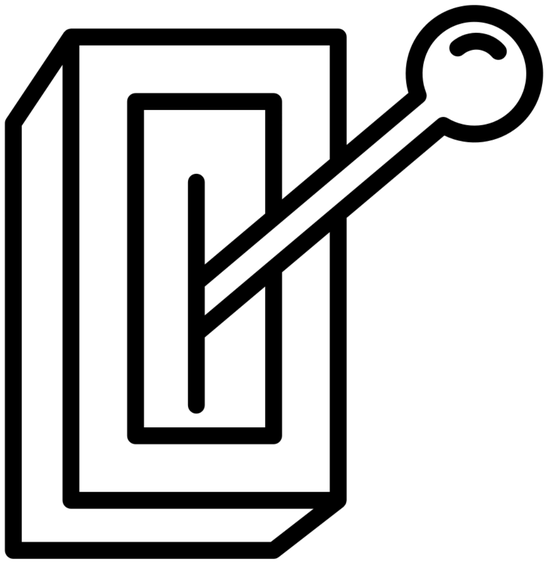
	MDP	POMDP
State	s_t	
Policy	$\pi(\cdot \mid s_t)$	

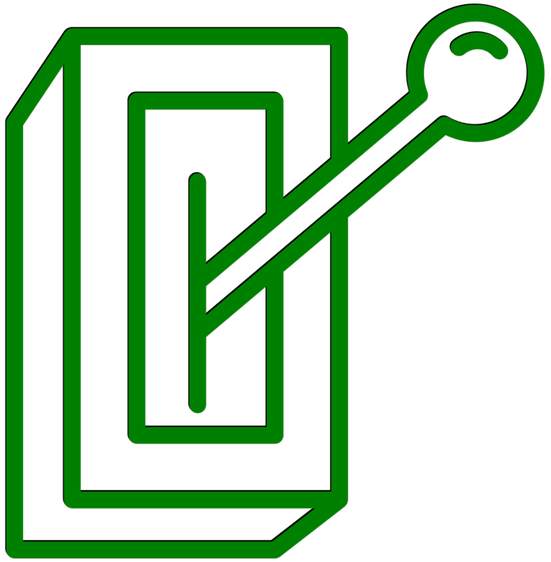
	MDP	POMDP
State	s_t	$p(s_t, c \mid s_1, a_1 \ldots s_{t-1}, a_{t-1})$
Policy	$\pi(\cdot \mid s_t)$	

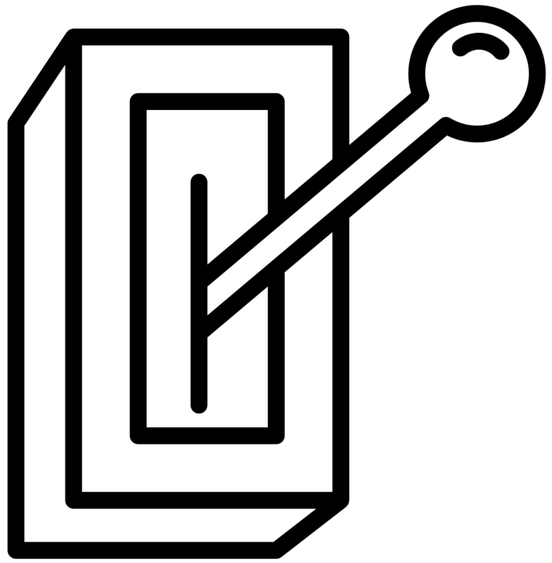
	MDP	POMDP
State	s_t	$p(s_t, c \mid s_1, a_1 \ldots s_{t-1}, a_{t-1})$
Policy	$\pi(\cdot \mid s_t)$	$\pi(\cdot \mid s_1, a_1 \ldots s_{t-1}, a_{t-1})$

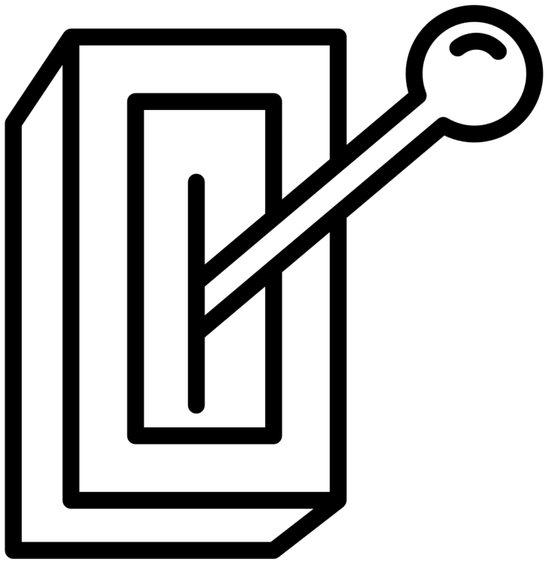

s_{t-2}
s_{t-1}
s_t
T
T
π_E
π_E
π_E
a_{t-2}
a_{t-1}
a_t
c

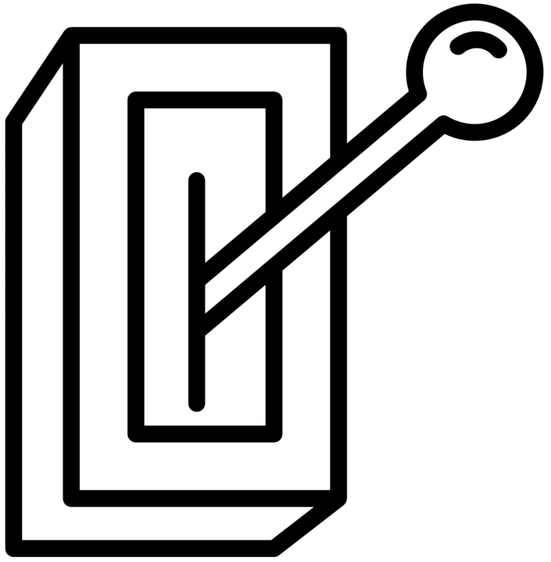


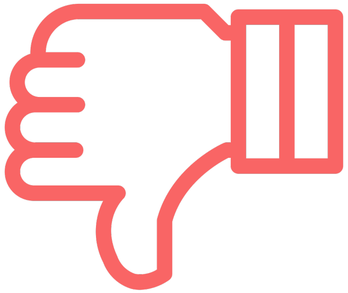


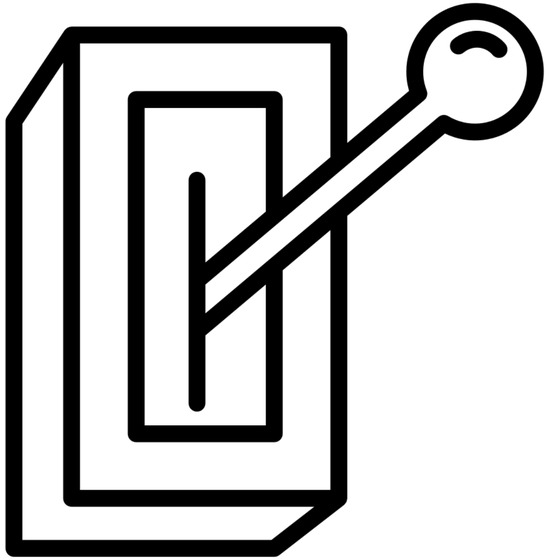


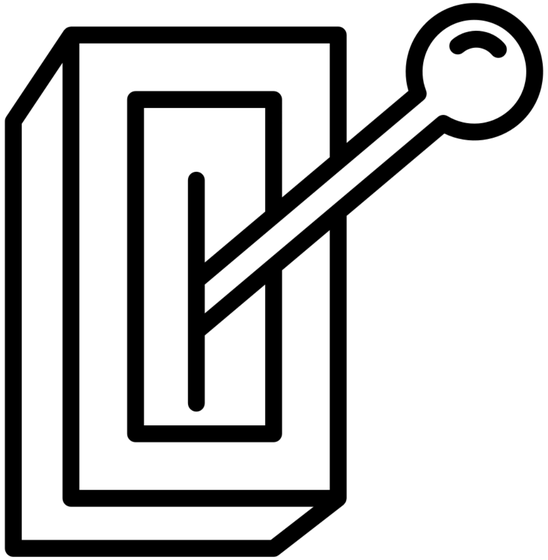


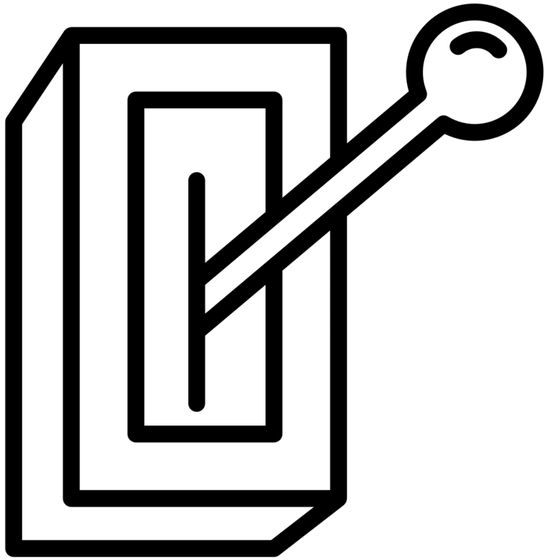


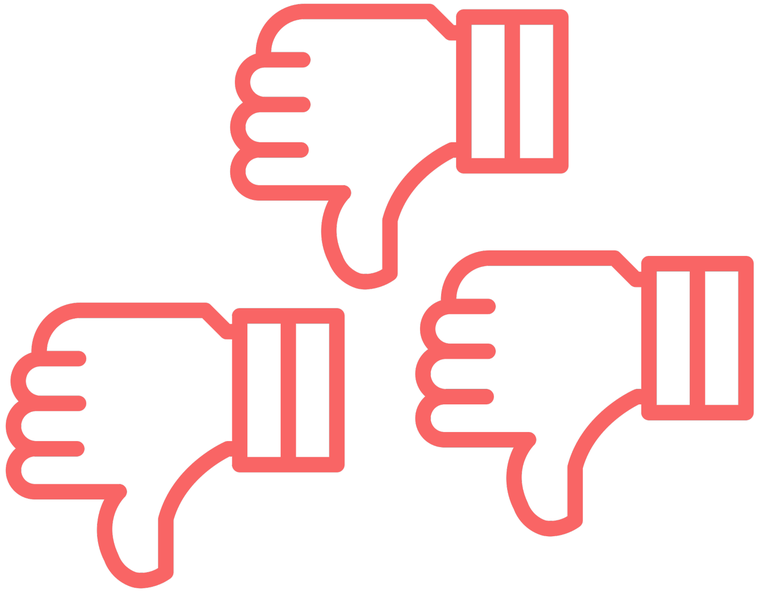


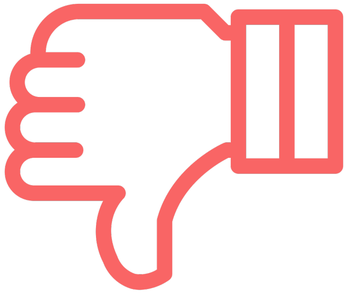


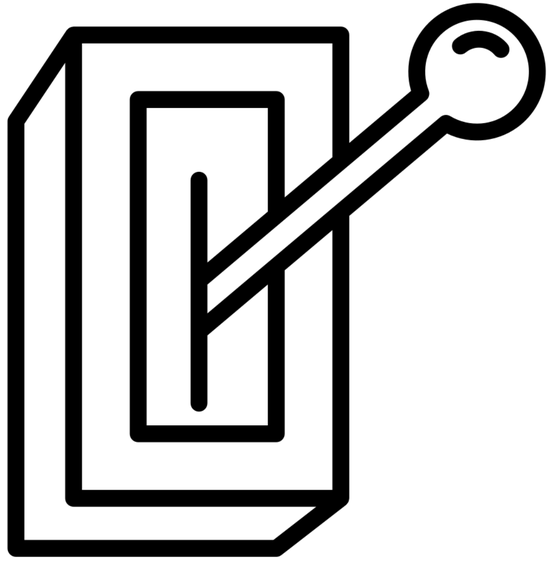


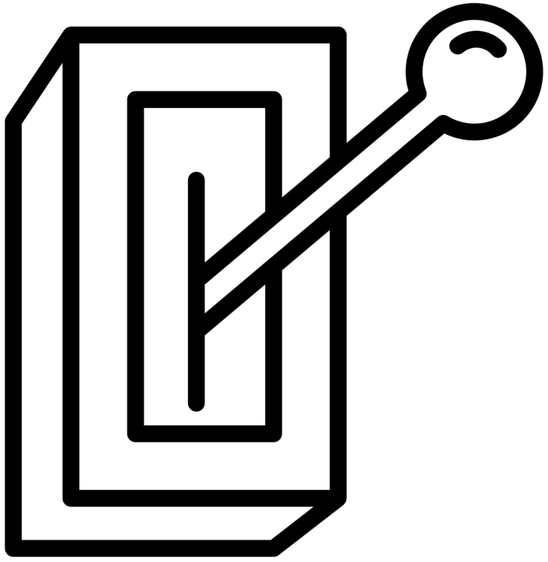


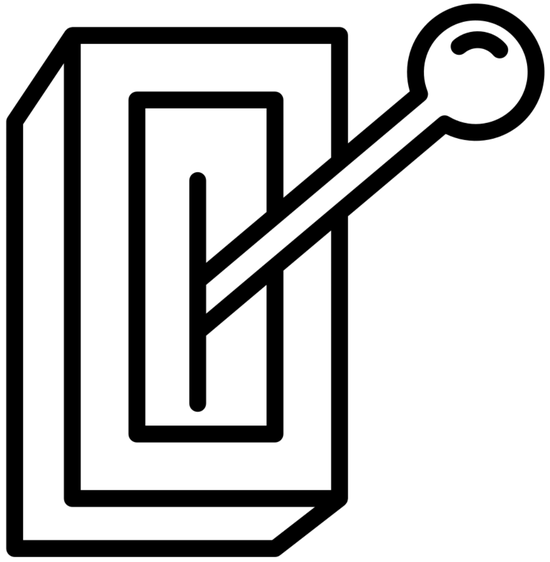


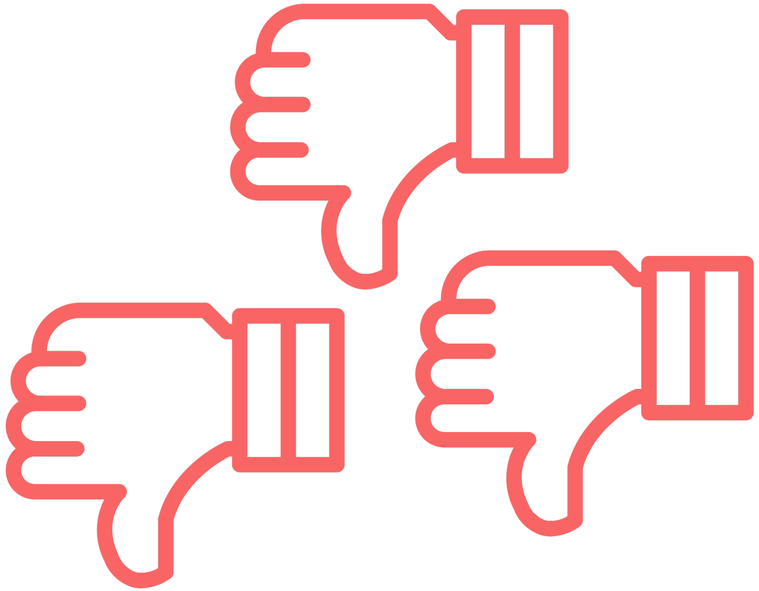


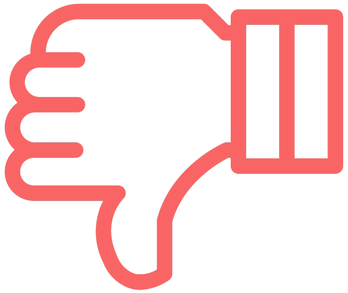


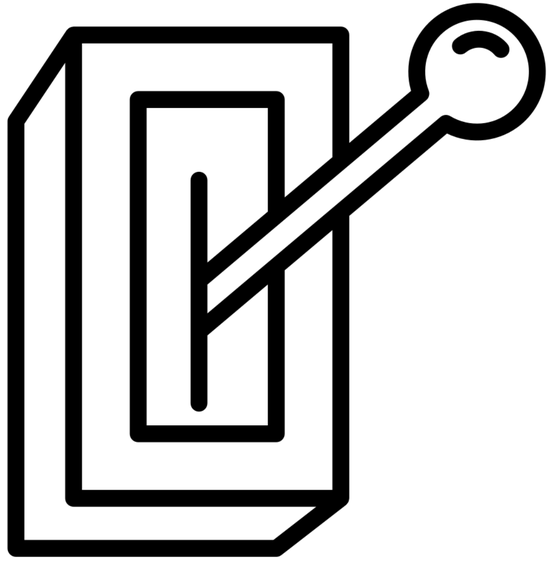


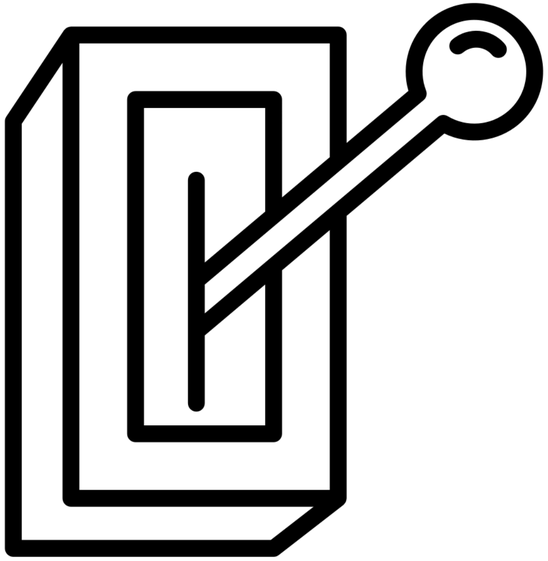


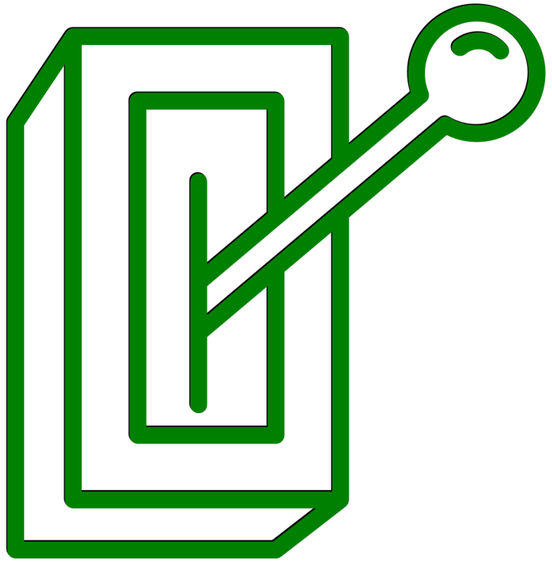










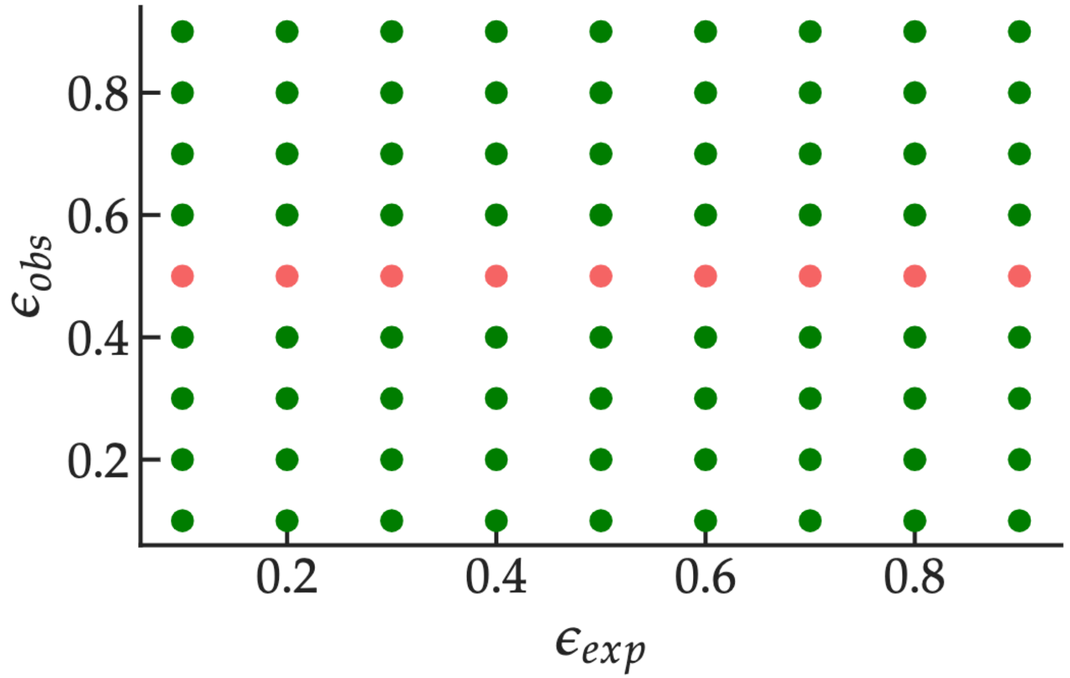


On-Policy (e.g. DAgger):

On-Policy (e.g. DAgger):

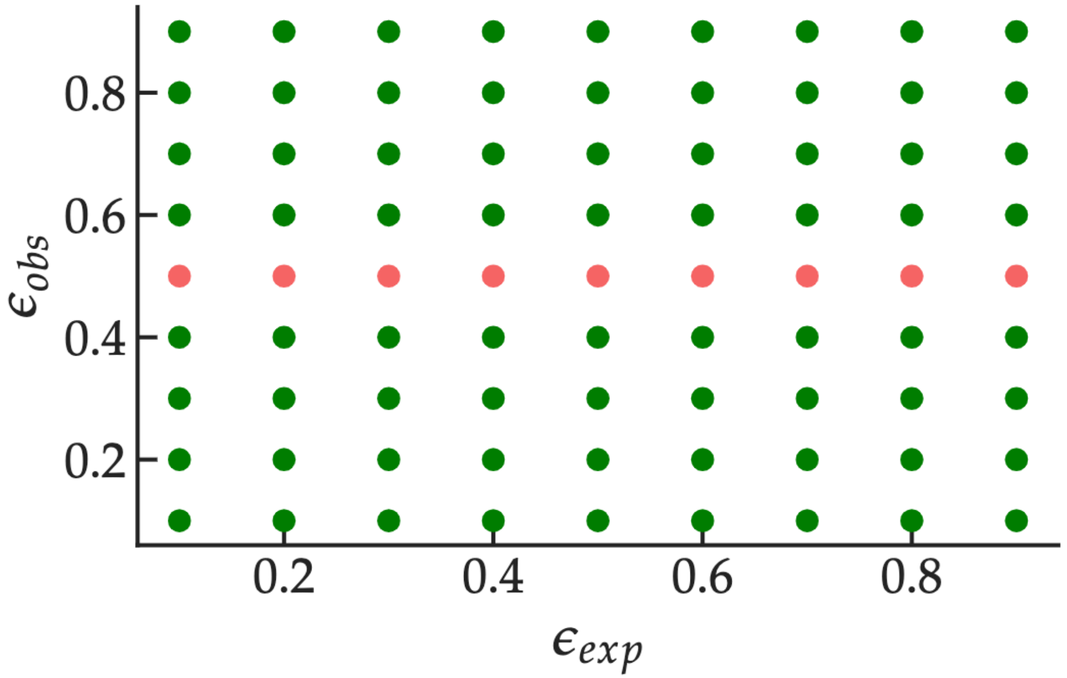
Off-Policy (e.g. BC):

On-Policy (e.g. DAgger):

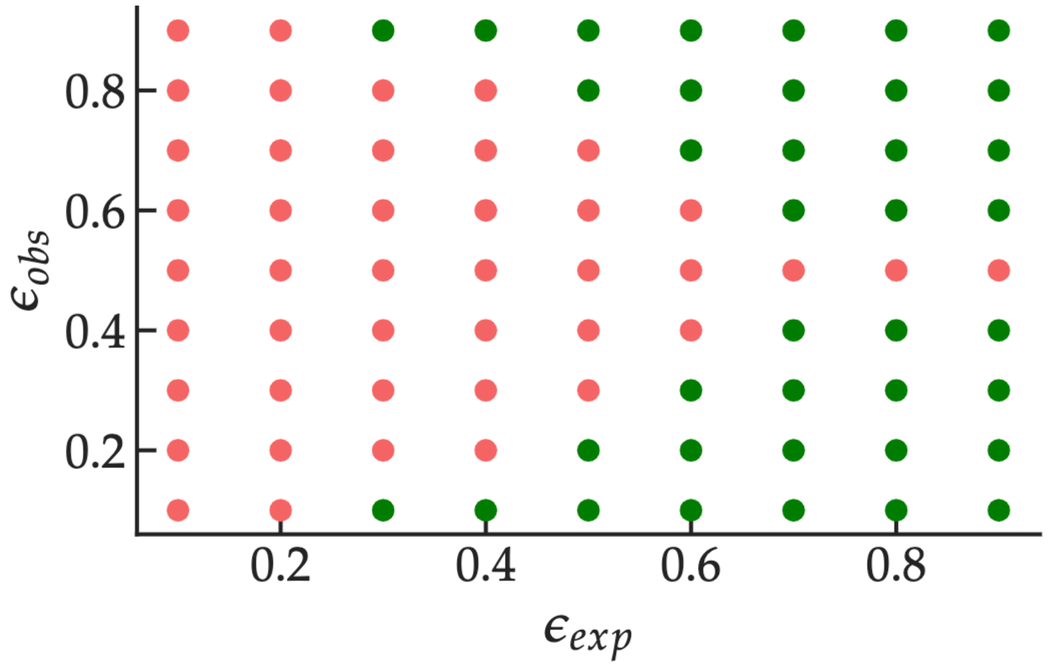


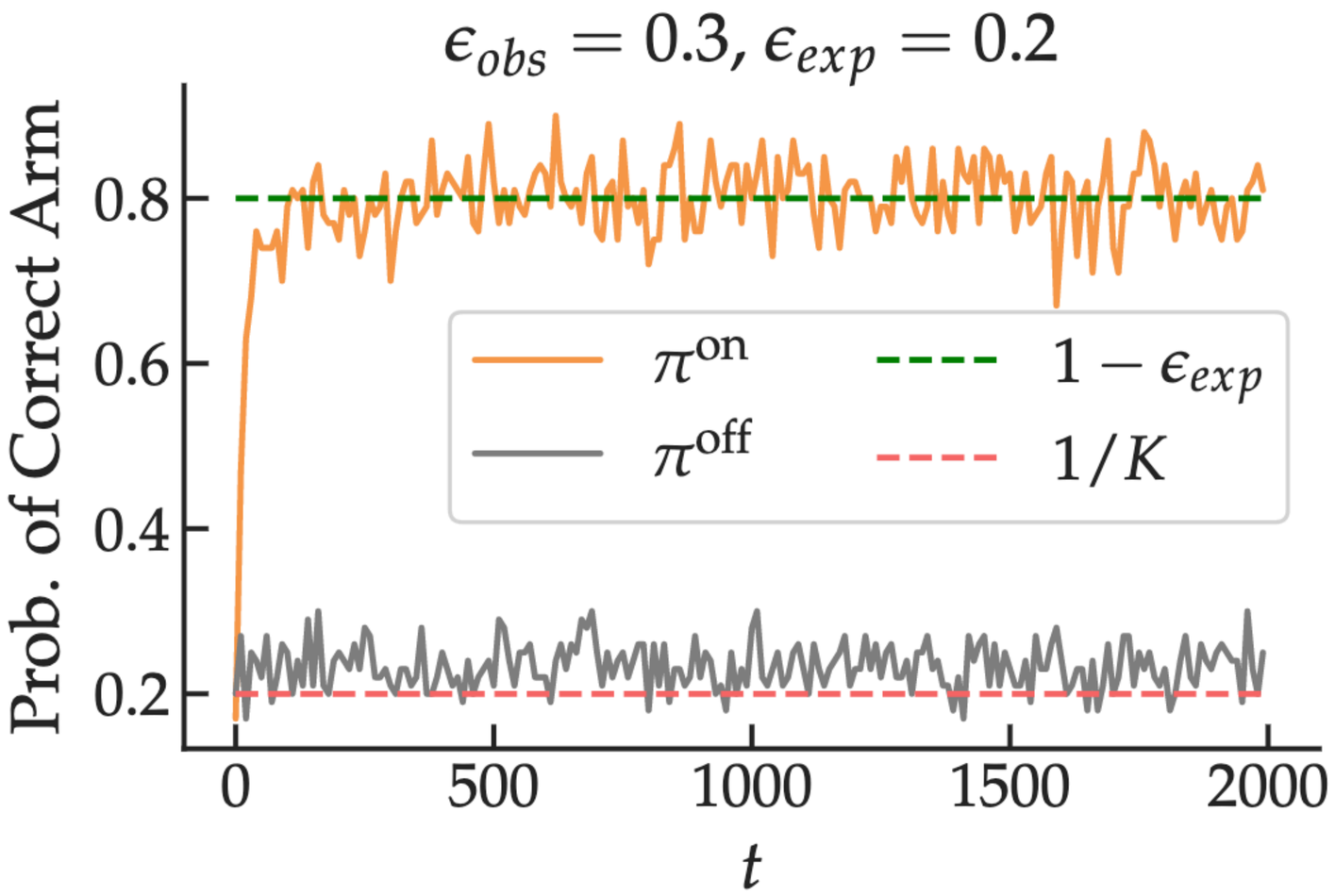
Off-Policy (e.g. BC):

On-Policy (e.g. DAgger):



Off-Policy (e.g. BC):




$\epsilon_{obs} = 0.3, \epsilon_{exp} = 0.2$
Prob. of Correct Arm
0.8
0.6
0.4
0.2
0
500
1000
1500
2000
t
π^{on}
π^{off}
$1 - \epsilon_{exp}$
$1/K$


$t = 0$
$t = \infty$

On-Policy:

On-Policy:

Off-Policy:

Train-time:

Train-time: $\pi(a_t \mid h_t) \approx p(a_t^E \mid s_1^E, a_1^E, \ldots, s_t^E)$

Train-time: $\pi(a_t \mid h_t) \approx p(a_t^E \mid s_1^E, a_1^E, \ldots, s_t^E)$

Test-time:

Train-time: $\pi(a_t \mid h_t) \approx p(a_t^E \mid s_1^E, a_1^E, \ldots, s_t^E)$

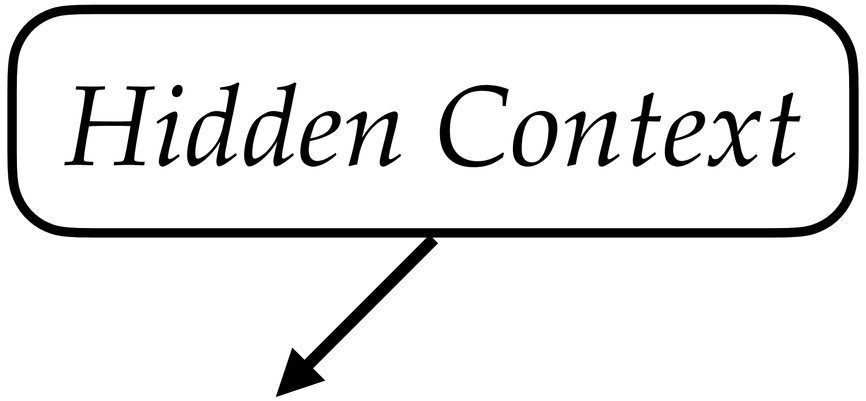
Test-time: $p(a_t^E \mid s_1, a_1, \ldots, s_t)$

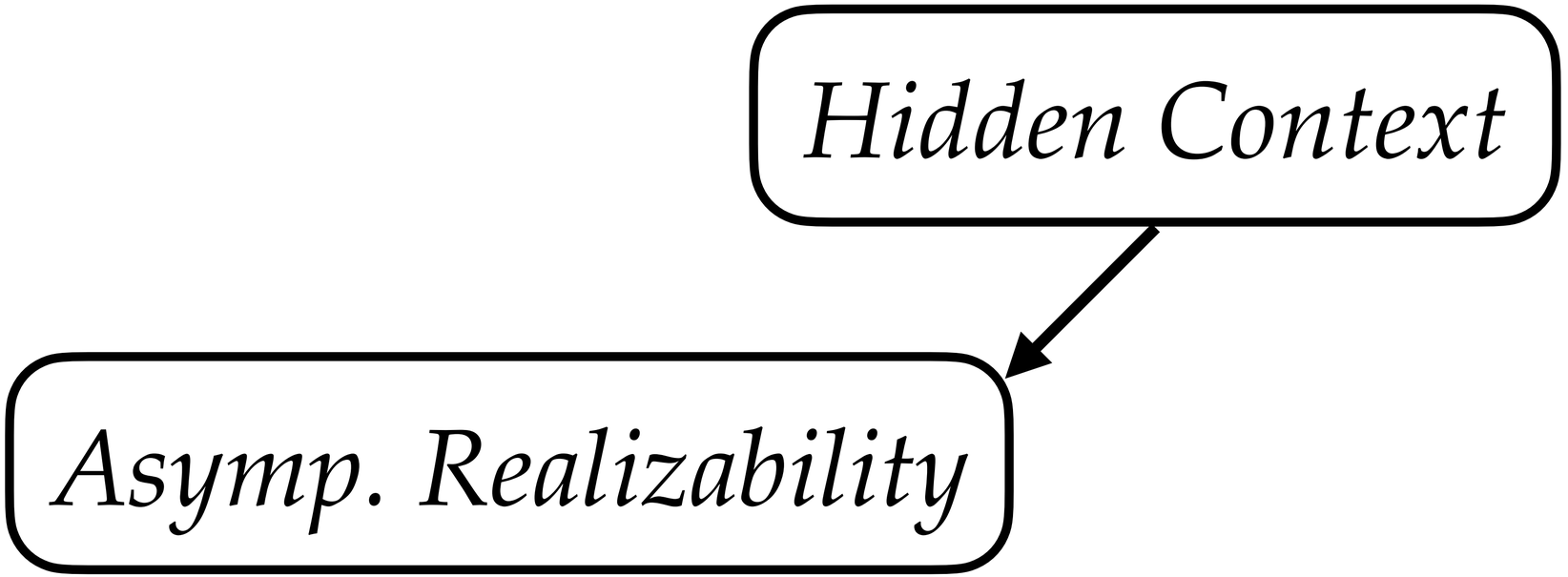
Train-time: $\pi(a_t \mid h_t) \approx p(a_t^E \mid s_1^E, a_1^E, \ldots, s_t^E)$

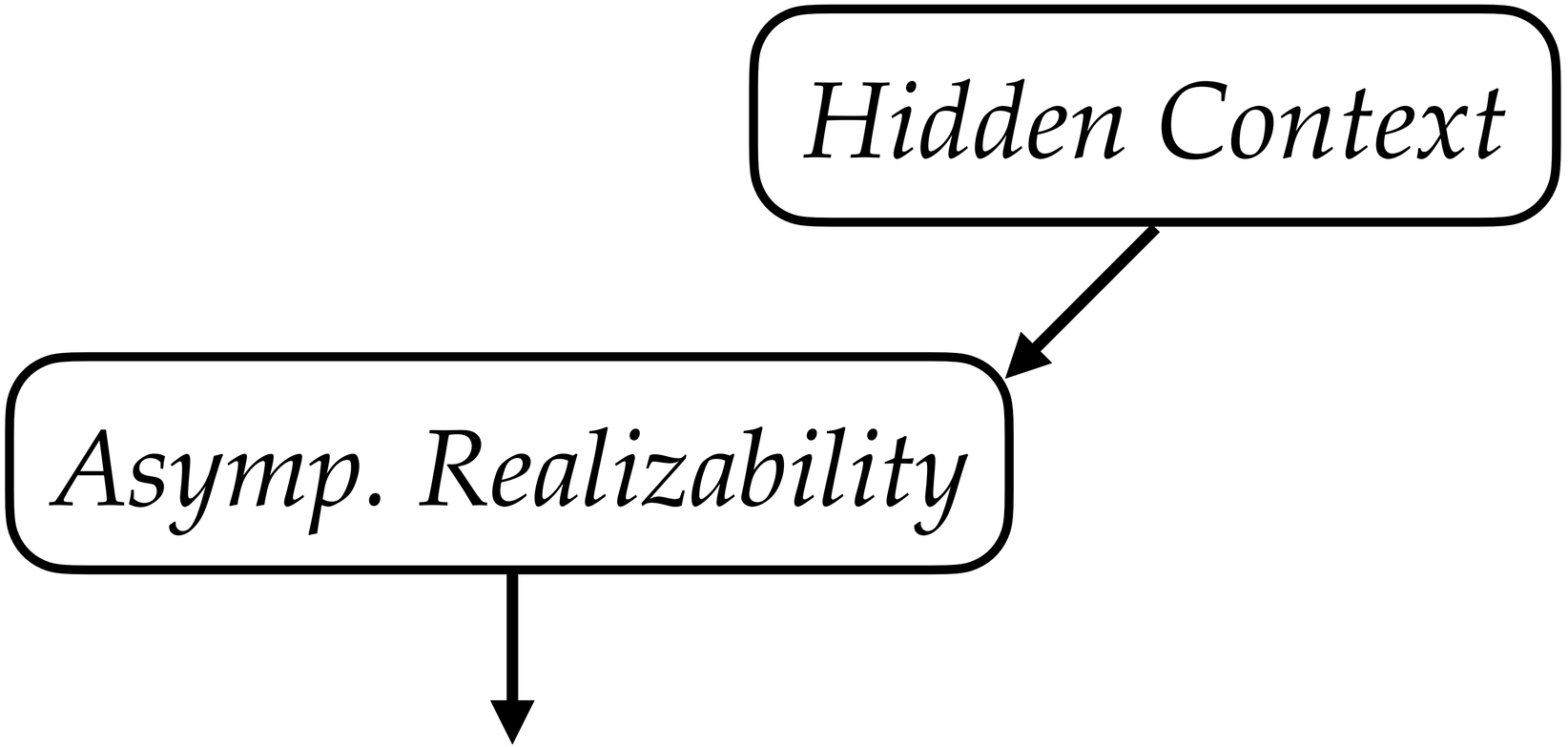
Test-time: $p(a_t^E \mid s_1, a_1, \ldots, s_t)$

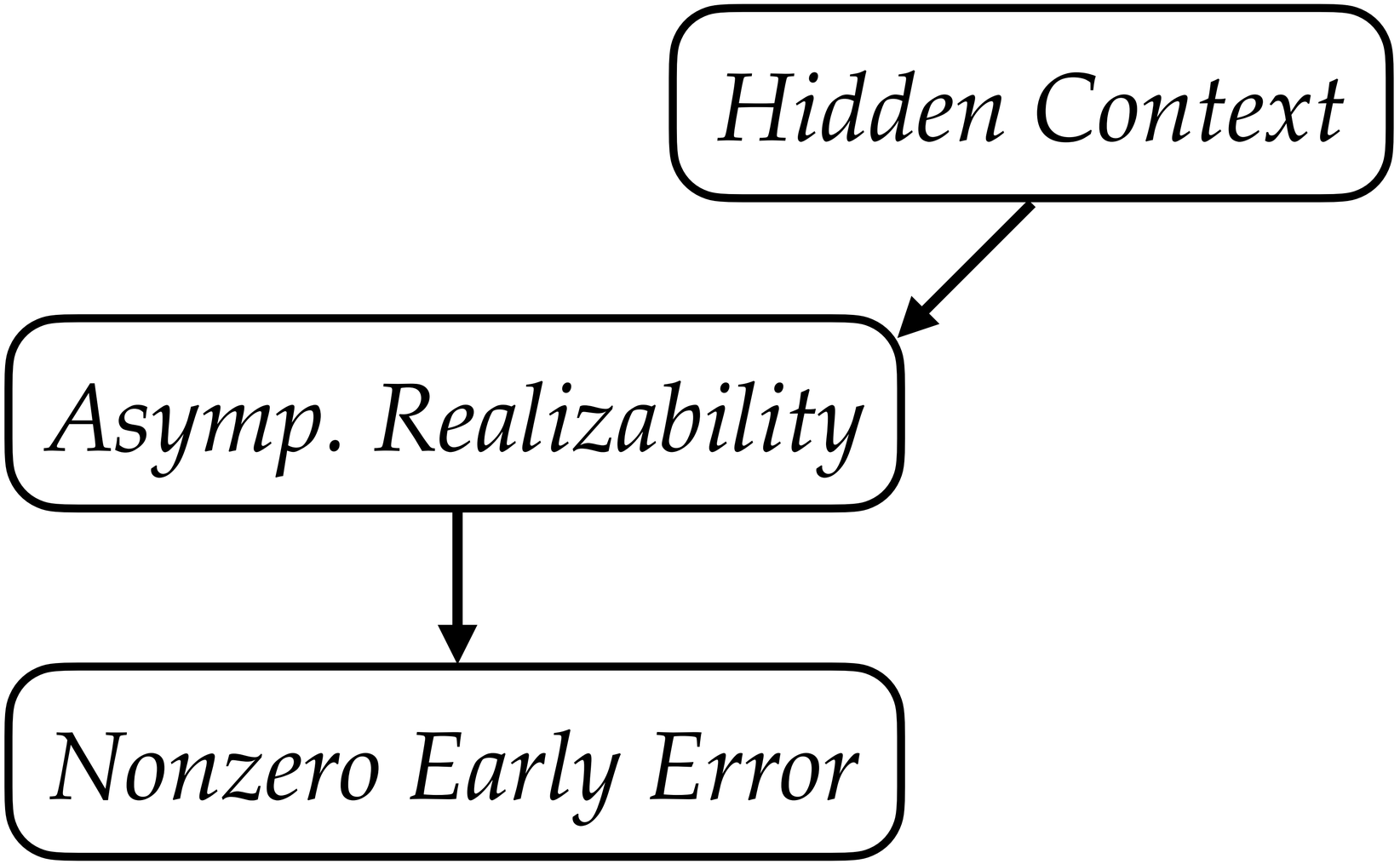
It's just covariate shift in the space of histories!

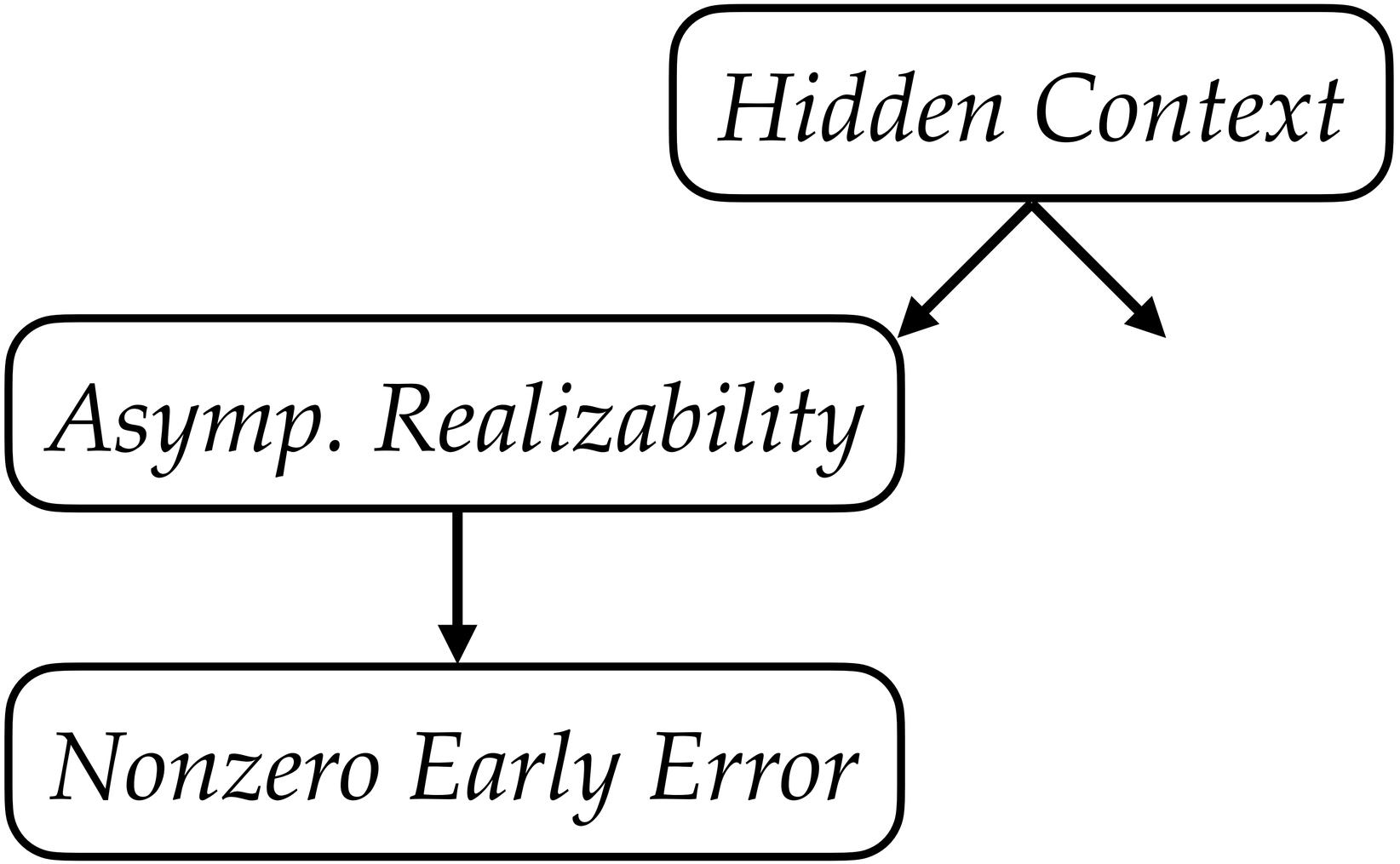
Hidden Context

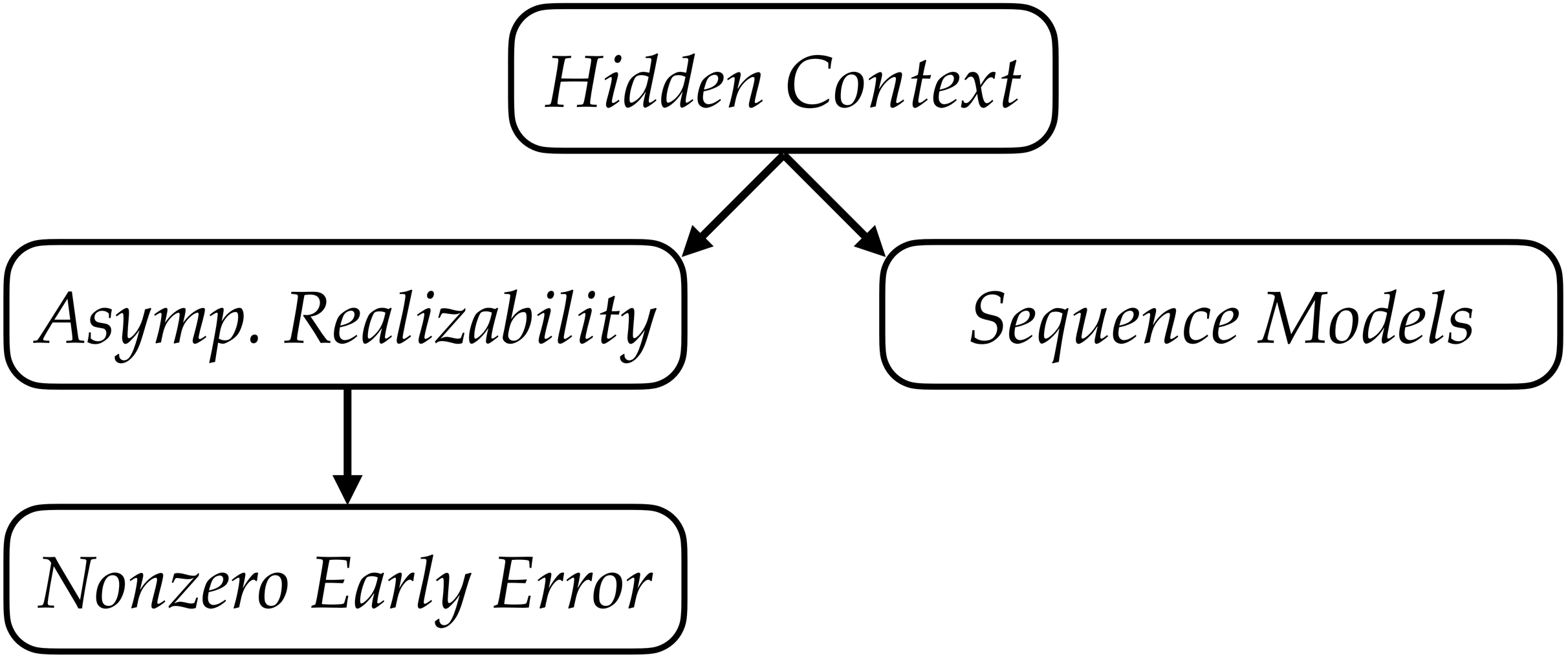

Hidden Context

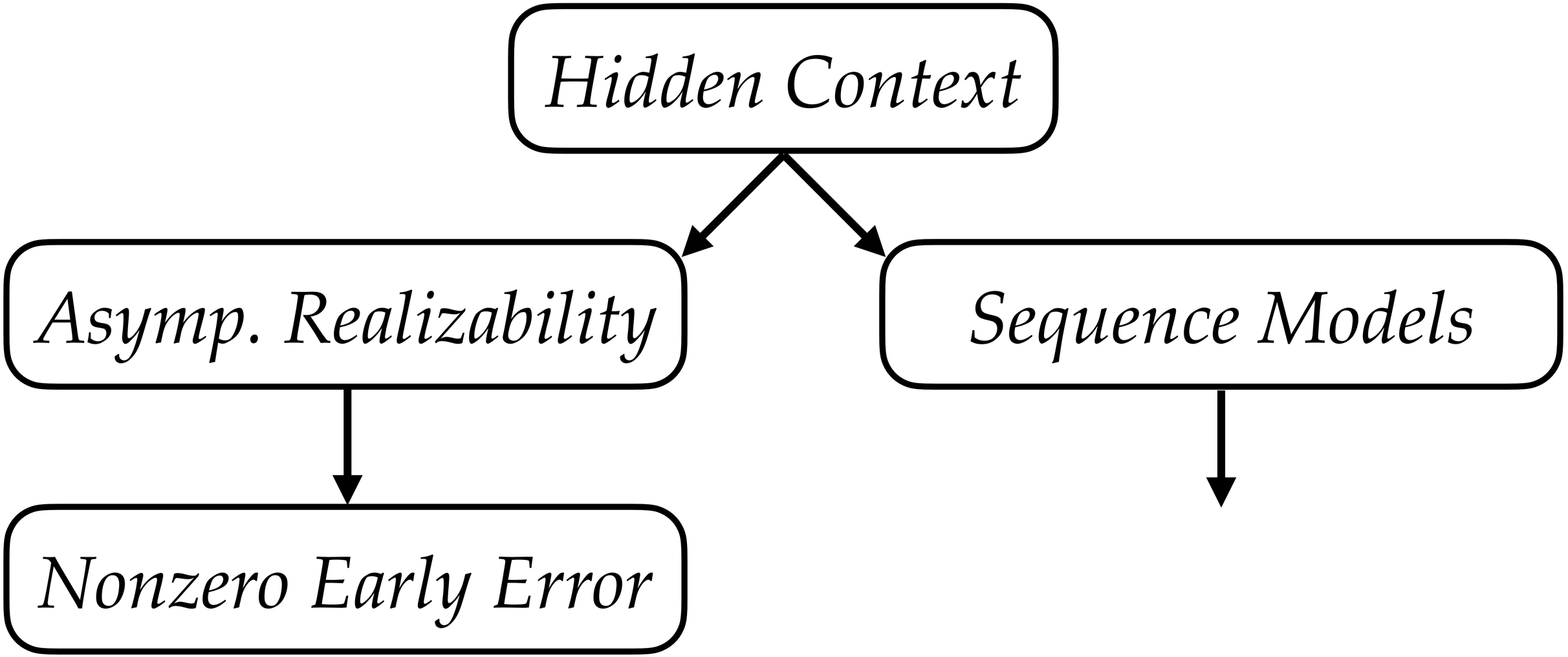

Hidden Context
Asymp. Realizability

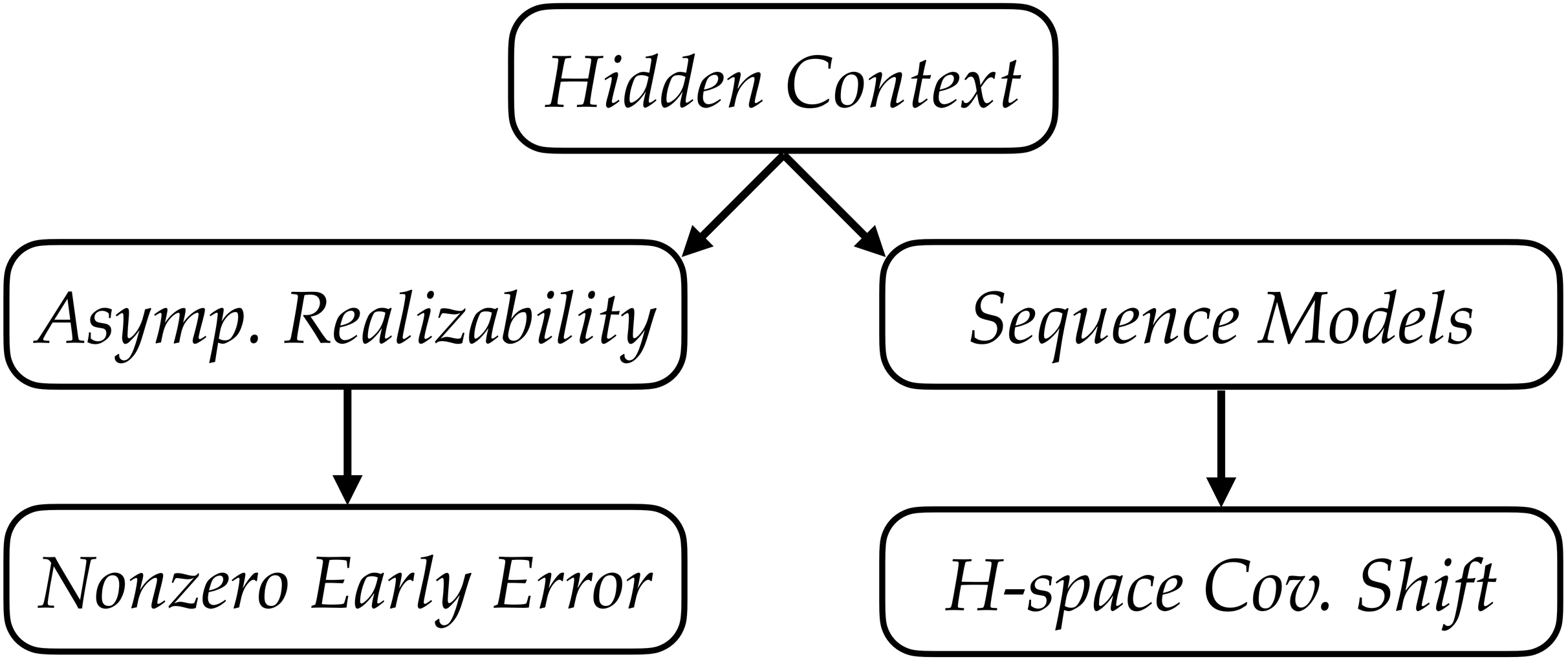

Hidden Context
Asymp. Realizability

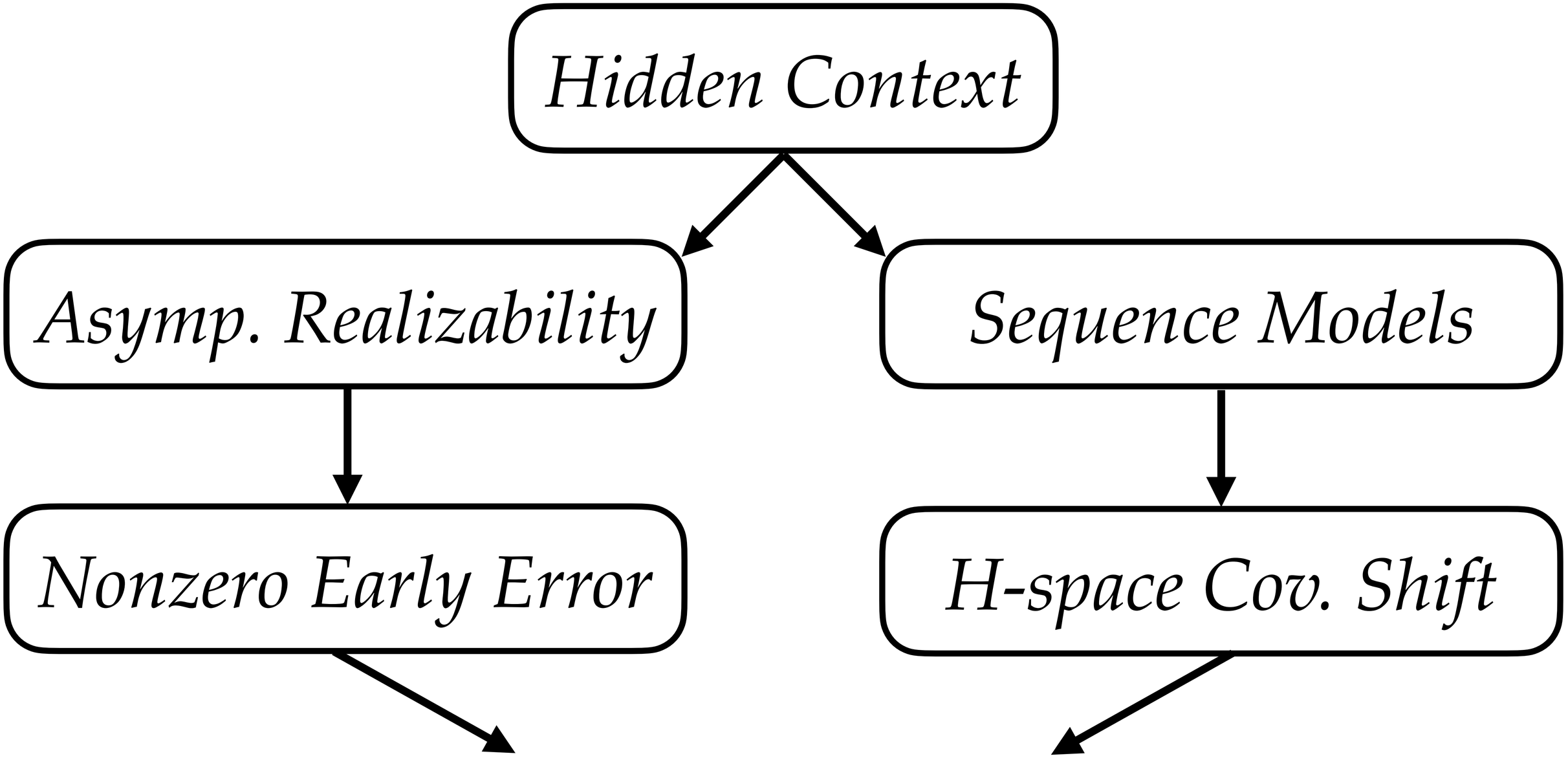

Hidden Context
Asymp. Realizability
Nonzero Early Error

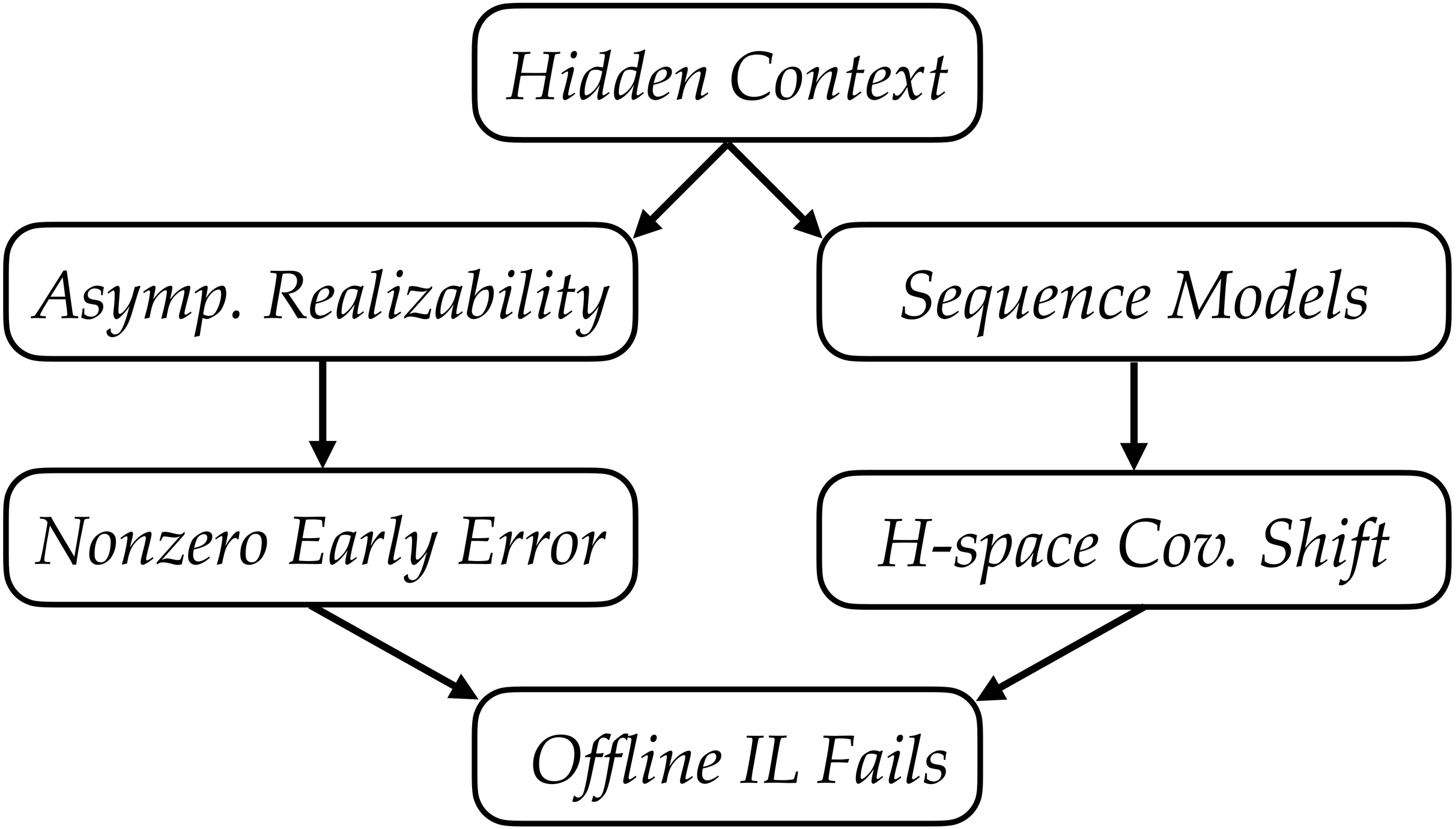

Hidden Context
Asymp. Realizability
Nonzero Early Error

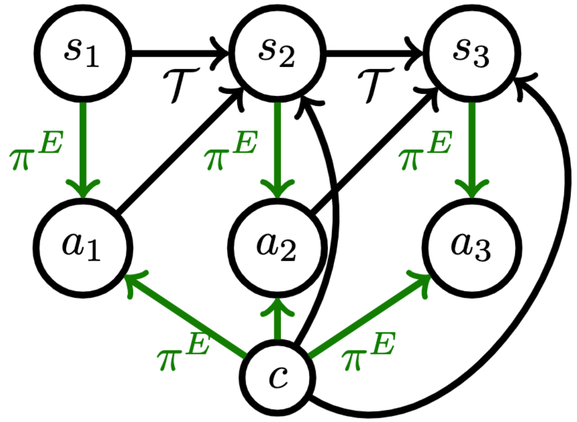

Hidden Context
Asymp. Realizability
Sequence Models
Nonzero Early Error


Hidden Context
Asymp. Realizability
Sequence Models
Nonzero Early Error

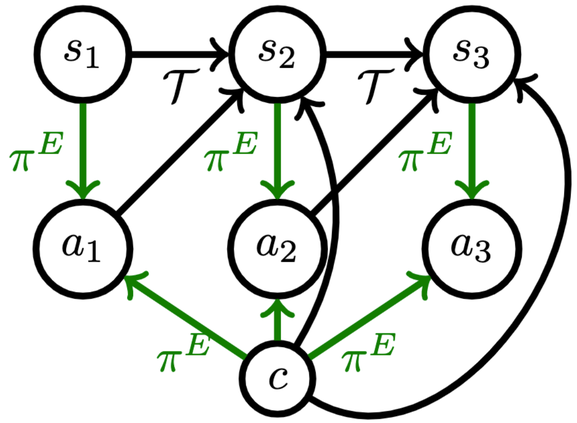

Hidden Context
Asymp. Realizability
Sequence Models
Nonzero Early Error
H-space Cov. Shift


Hidden Context
Asymp. Realizability
Sequence Models
Nonzero Early Error
H-space Cov. Shift

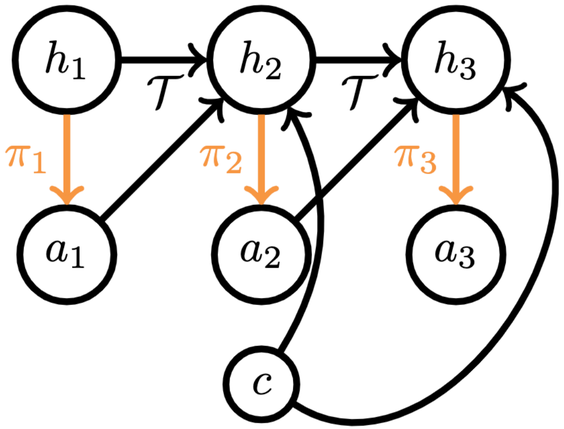

Hidden Context
Asymp. Realizability
Sequence Models
Nonzero Early Error
H-space Cov. Shift
Offline IL Fails



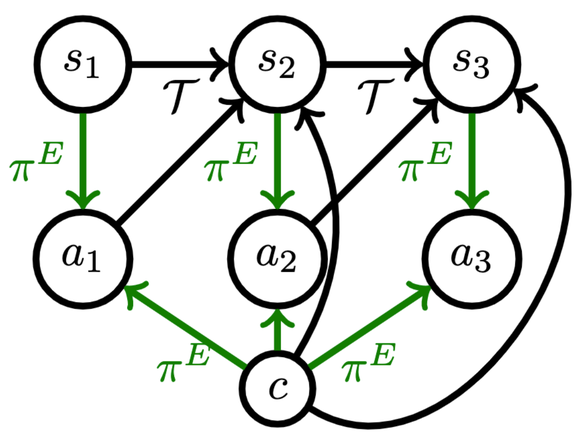
(a) $\tau \sim \pi^E$



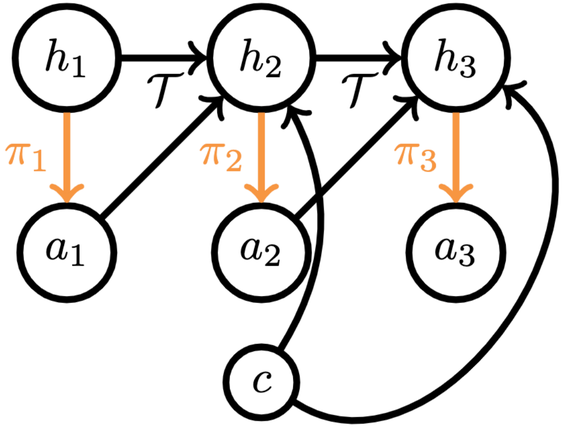
(a) $\tau \sim \pi^E$



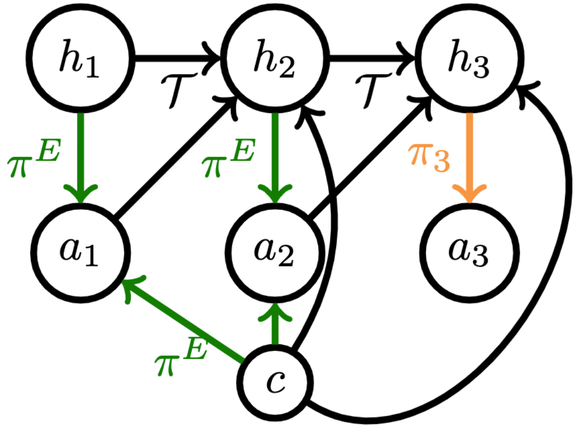
(b) On-Policy



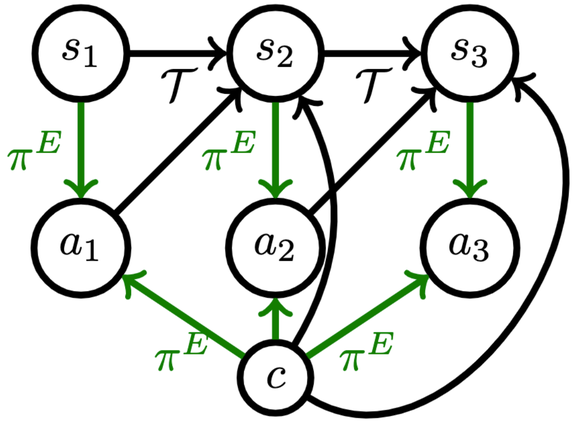
(a) $\tau \sim \pi^E$



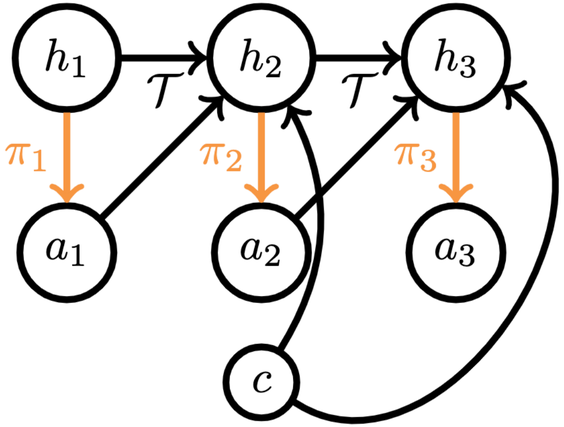
(b) On-Policy



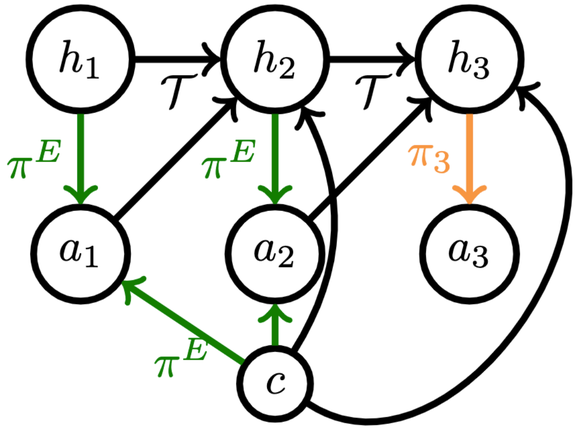
(c) Off-Policy



(a) $\tau \sim \pi^E$



(b) On-Policy



(c) Off-Policy

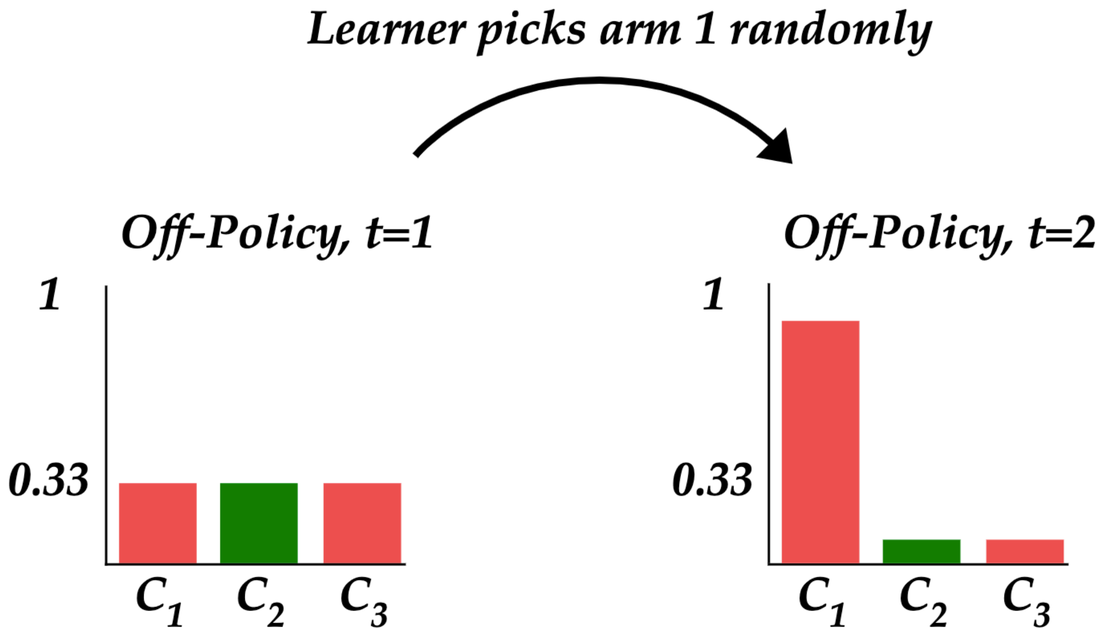
$$p_{\text{on}}(c, h_t) \propto p(\tau; \pi) \propto p(c)p(s_1)\prod_{i=1}^{t-1} \mathcal{T}(s_{i+1} \mid s_i, a_i, c)$$

$$p_{\text{on}}(c, h_t) \propto p(\tau; \pi) \propto p(c)p(s_1)\prod_{i=1}^{t-1} \mathcal{T}(s_{i+1} \mid s_i, a_i, c)$$

$$p_{\text{off}}(c, h_t) \propto p(\tau; \pi^E) \propto p(c)p(s_1)\prod_{i=1}^{t-1} \pi^E(a_i \mid c, s_i)\mathcal{T}(s_{i+1} \mid s_i, a_i, c)$$

$$p_{\text{on}}(c, h_t) \propto p(\tau; \pi) \propto p(c)p(s_1)\prod_{i=1}^{t-1} \mathcal{T}(s_{i+1} \mid s_i, a_i, c)$$

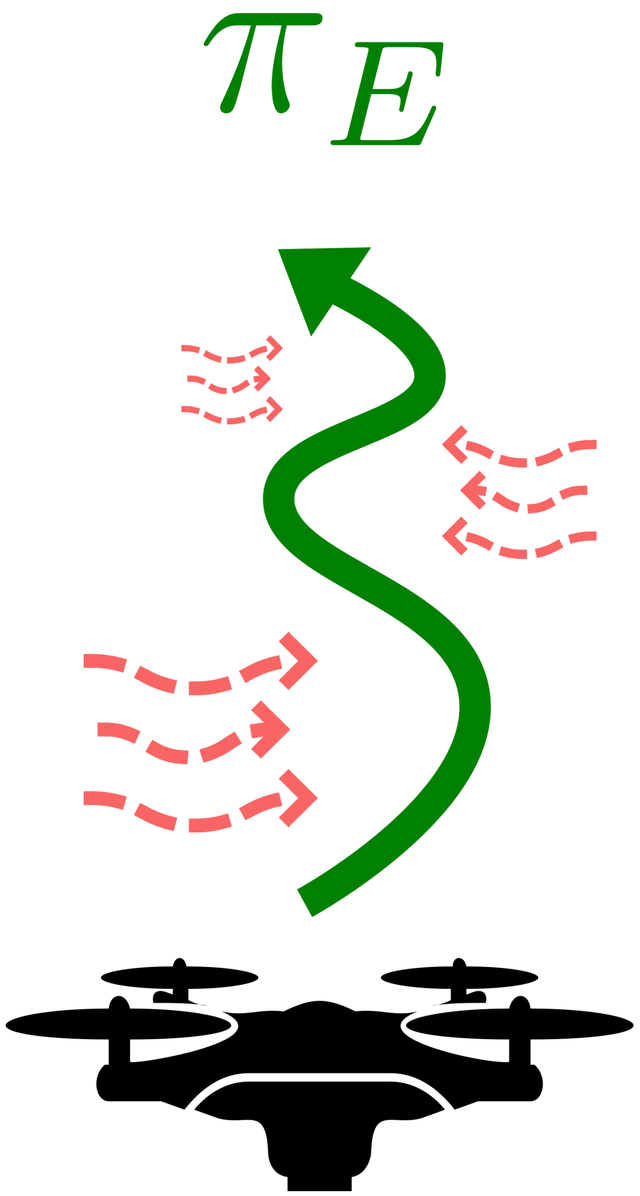
$$p_{\text{off}}(c, h_t) \propto p(\tau; \pi^E) \propto p(c)p(s_1)\prod_{i=1}^{t-1} \pi^E(a_i \mid c, s_i)\mathcal{T}(s_{i+1} \mid s_i, a_i, c)$$

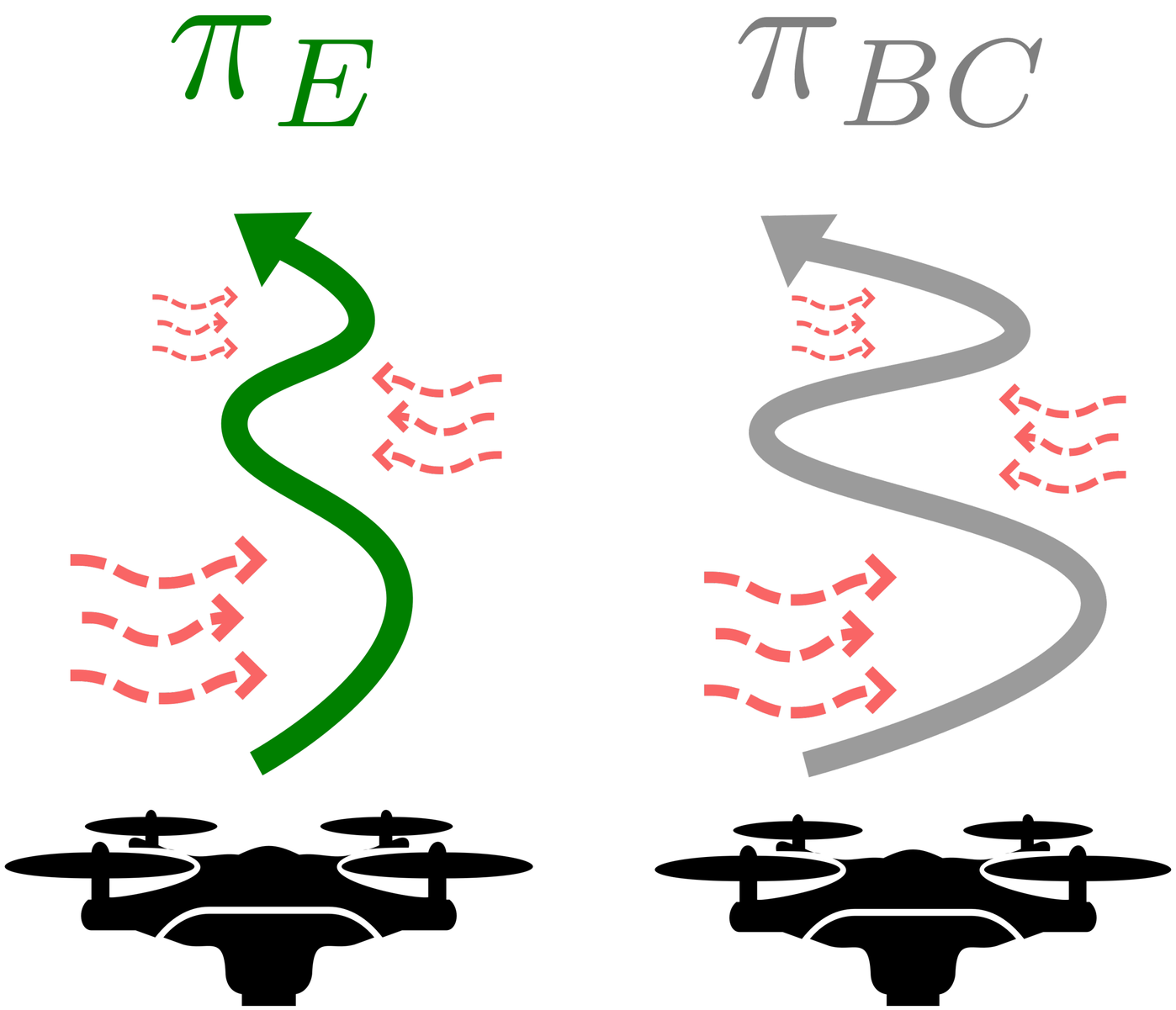


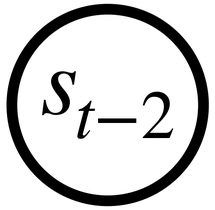
Theorem (informal): Off-policy learners have a value difference to the expert bounded by the sum of their errors (tight) while on-policy learners have one dependent on their asymptotic error.

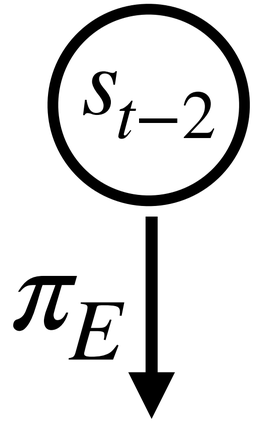
	Offline	Online	Interactive
Covariate Shift	✘	✔	✔
Hidden Context	✘	✔ w/ History	✔ w/ History
TCN			

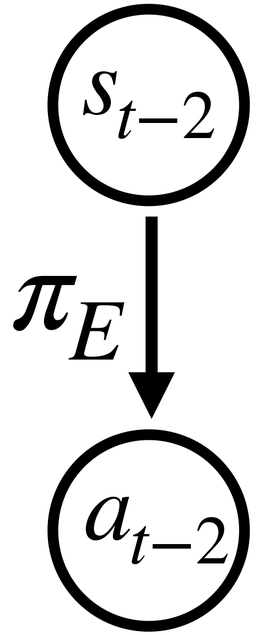
"Actually, since we were fitting a model to a time-series, samples tend to be correlated in time […] Thus, when leaving out a sample in cross validation, we actually left out a large window (16 seconds) of data around that sample, to diminish this bias."
— Ng et al., 2003

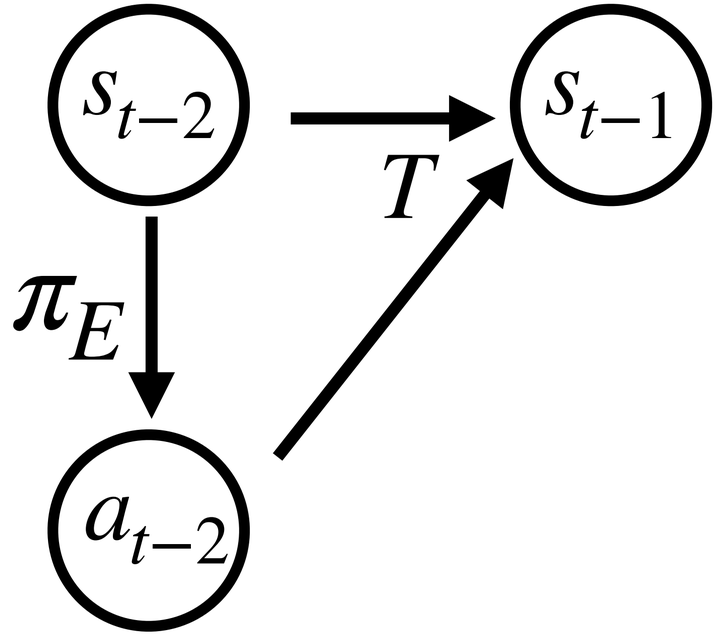

π_E

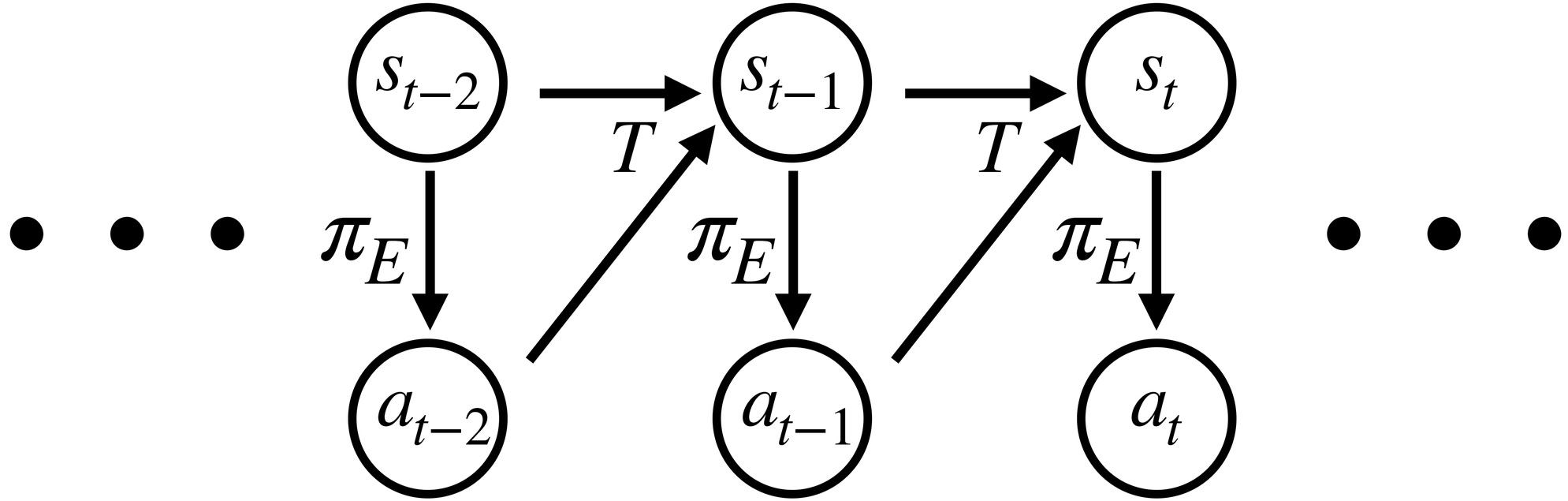

π_E
π_{BC}

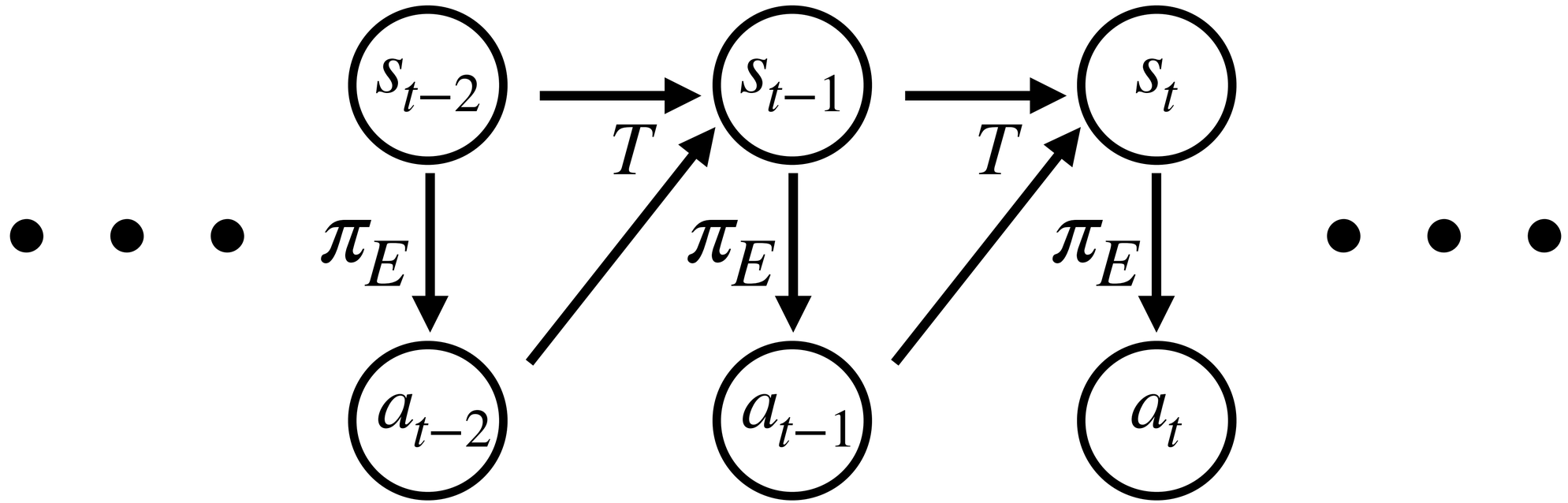

s_{t-2}


s_{t-2}
π_E

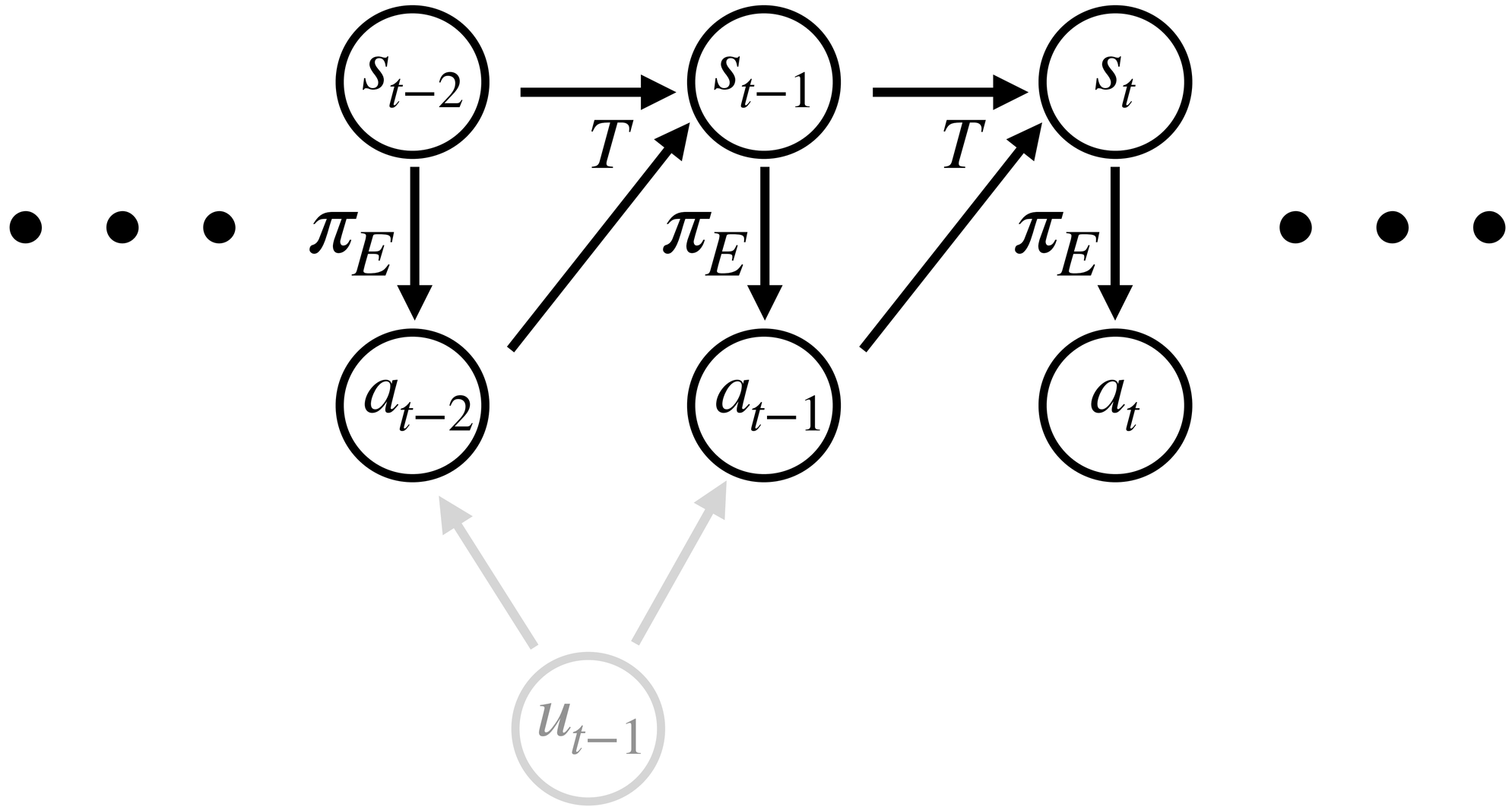

s_{t-2}
π_E
a_{t-2}

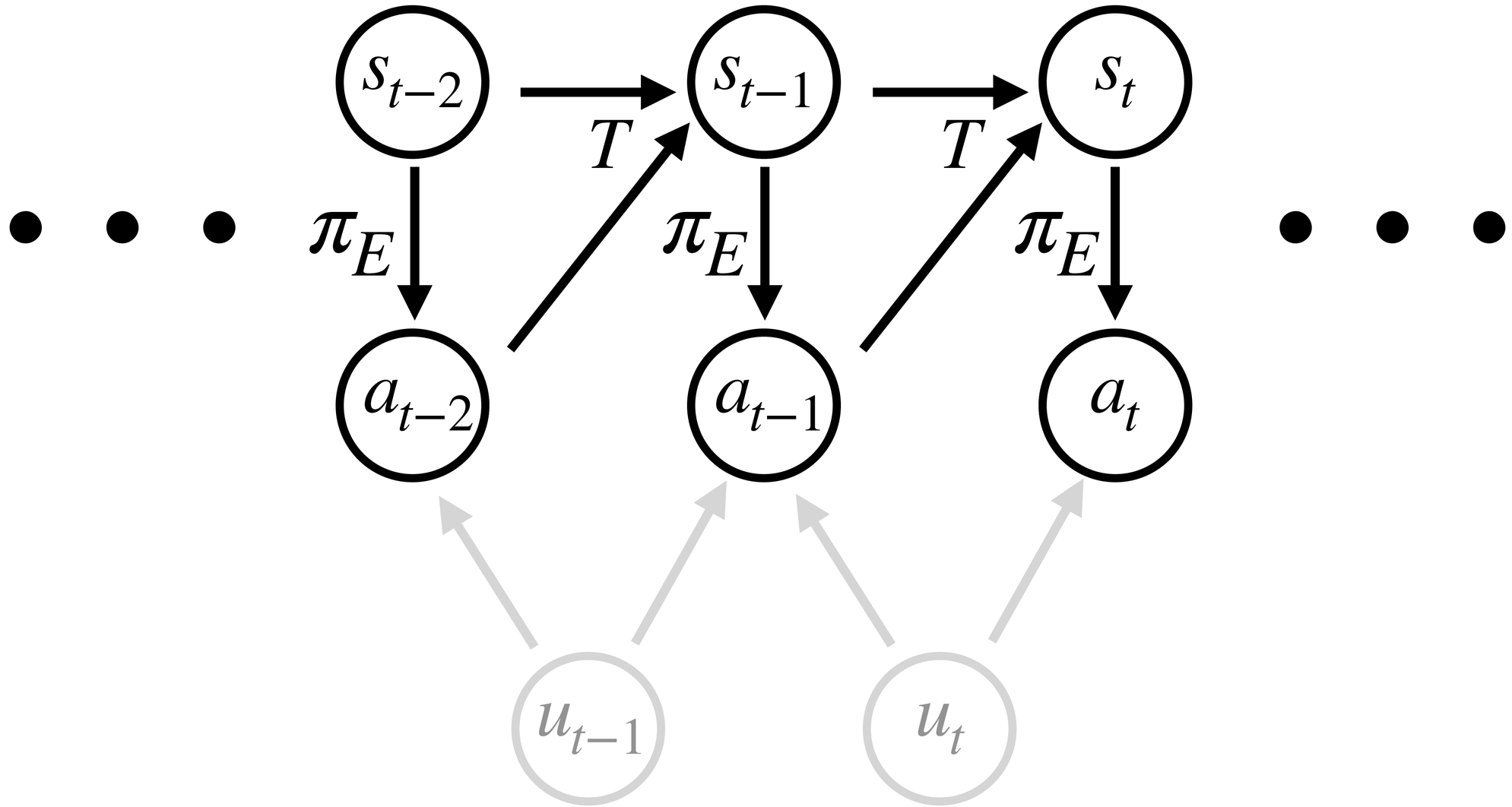

s_{t-2}
s_{t-1}
T
π_E
a_{t-2}

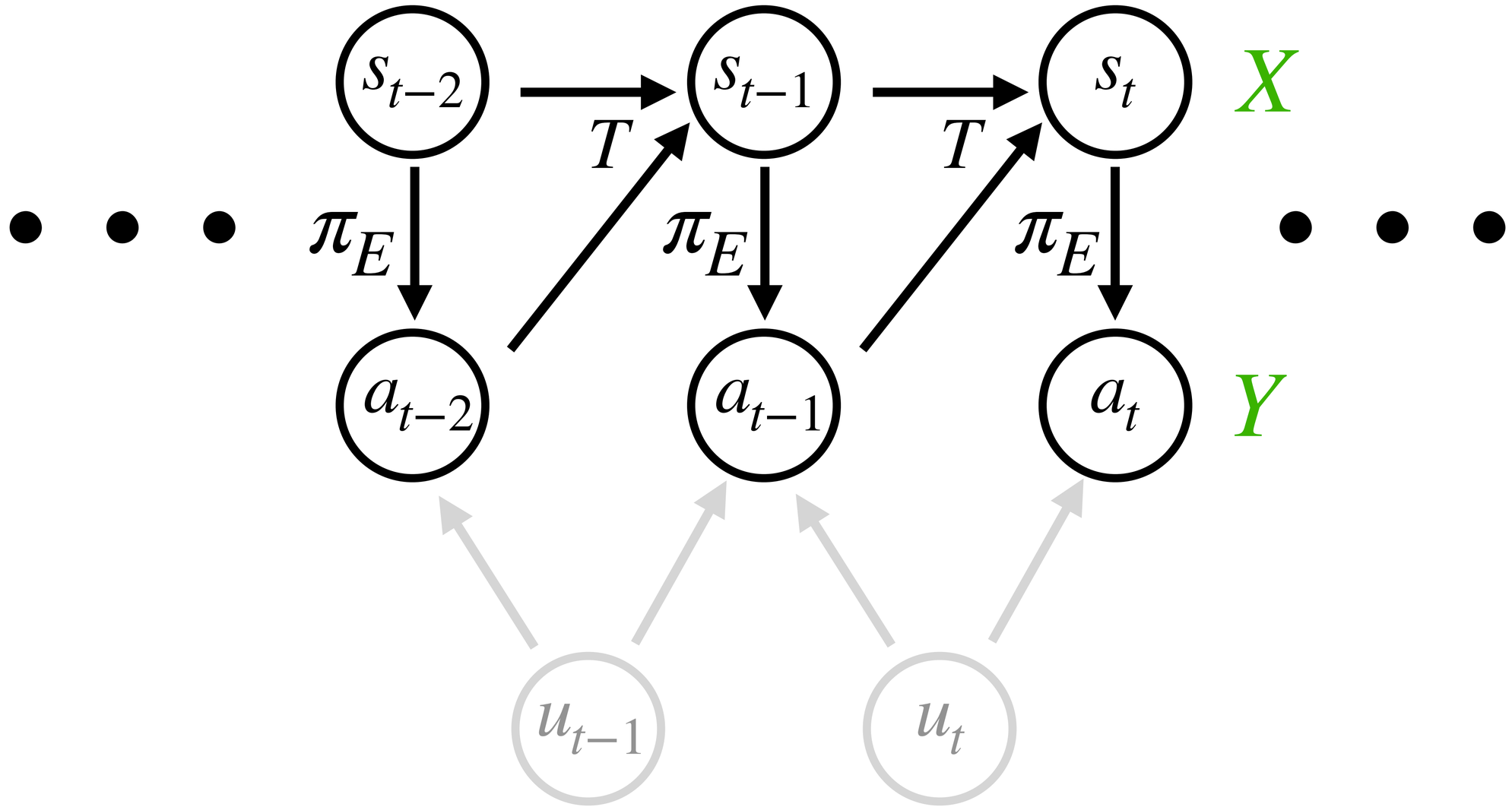

s_{t-2}
s_{t-1}
s_t
T
T
π_E
π_E
π_E
a_{t-2}
a_{t-1}
a_t

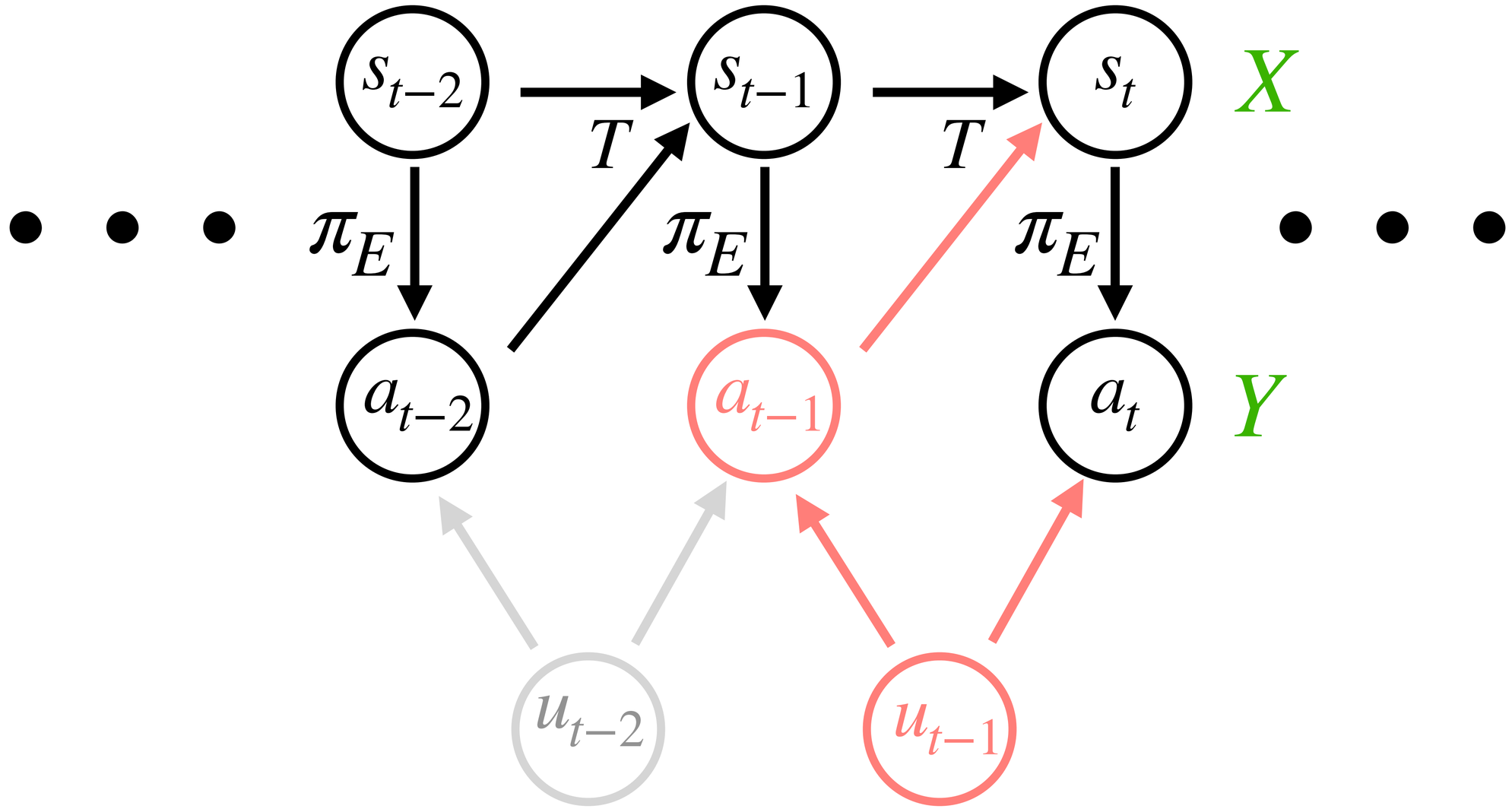


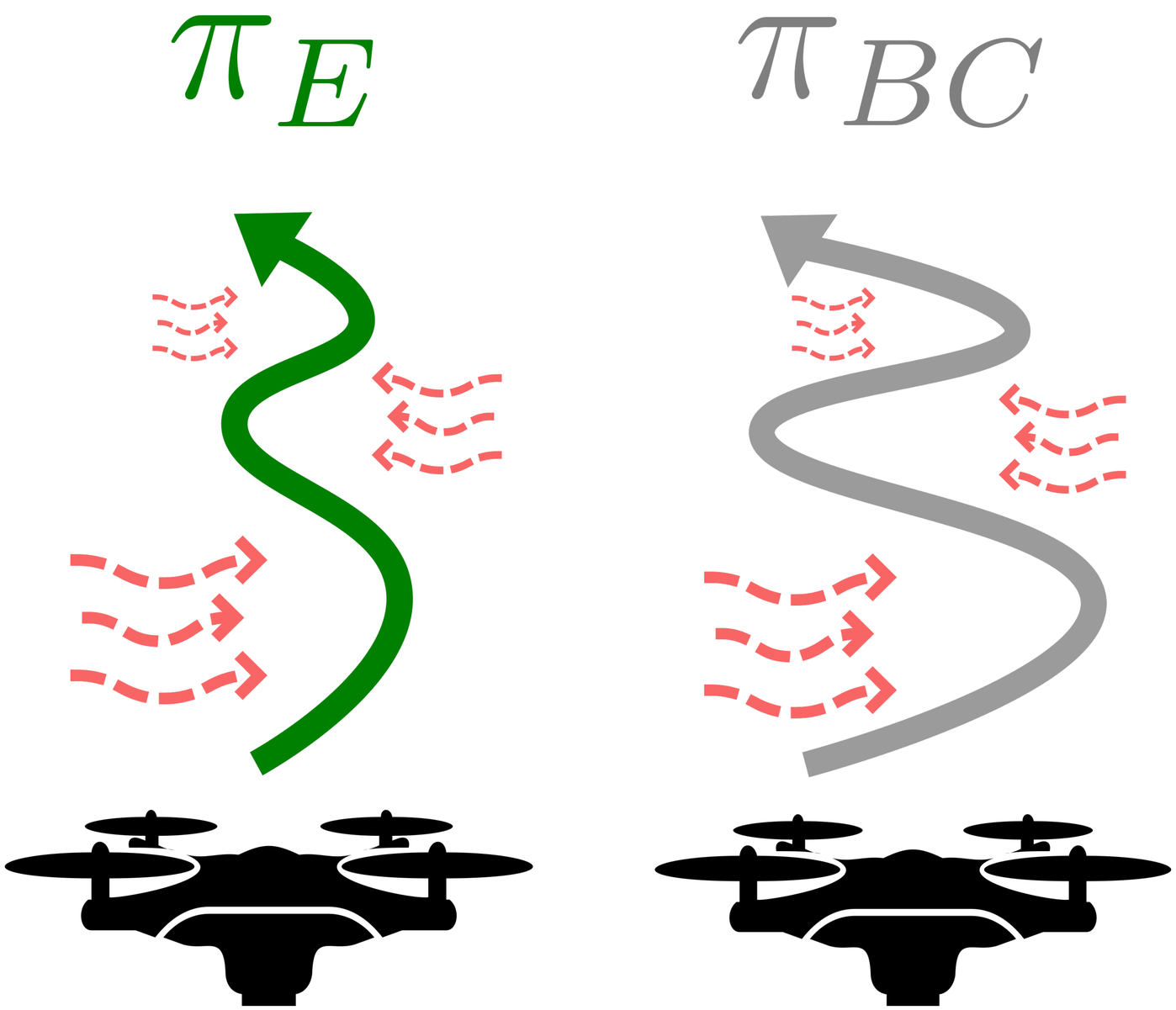
u_{t-1}

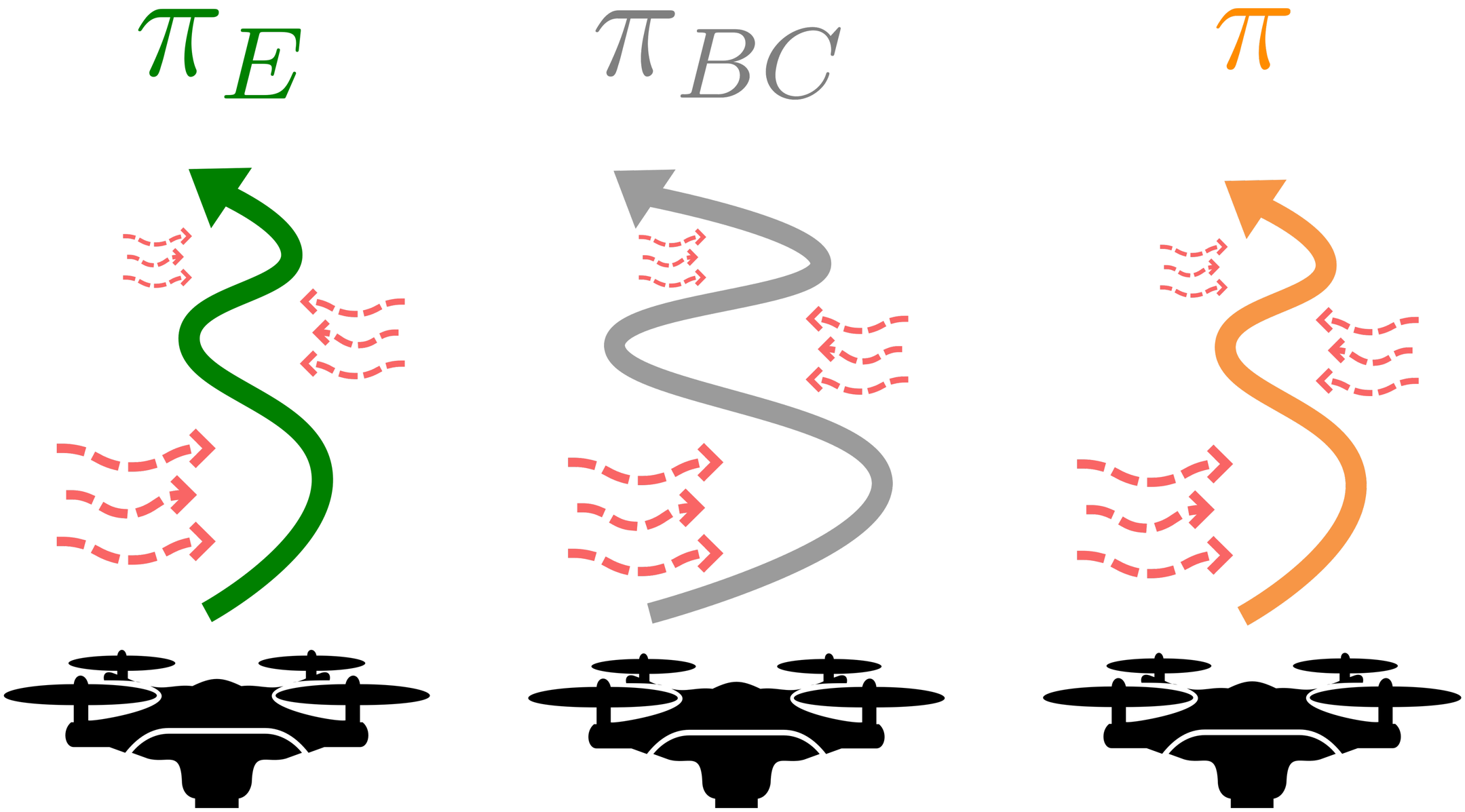

s_{t-2}
s_{t-1}
s_t
T
T
π_E
π_E
π_E
a_{t-2}
a_{t-1}
a_t
u_{t-1}

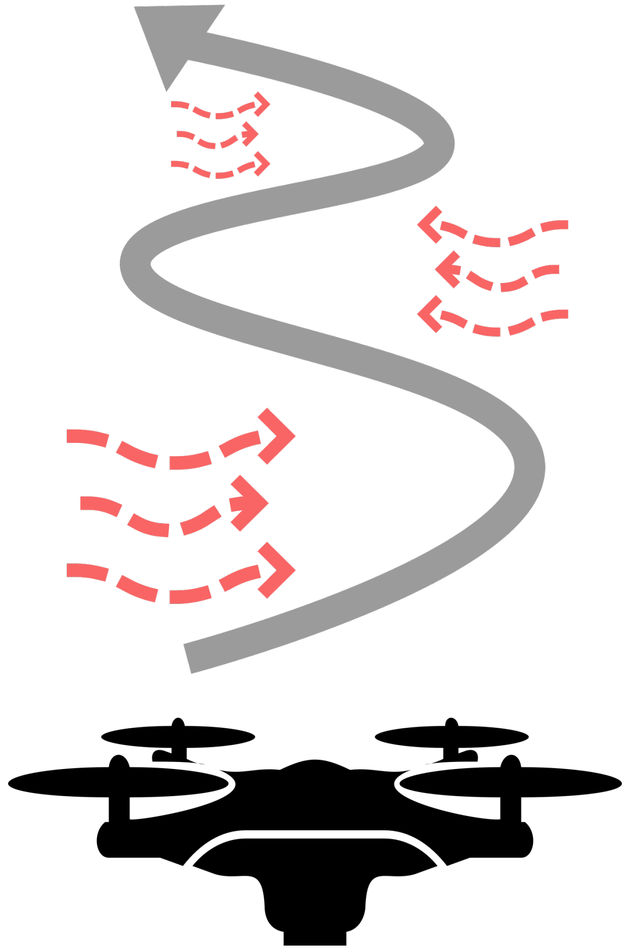

s_{t-2}
s_{t-1}
s_t
T
T
π_E
π_E
π_E
a_{t-2}
a_{t-1}
a_t
u_{t-1}
u_t

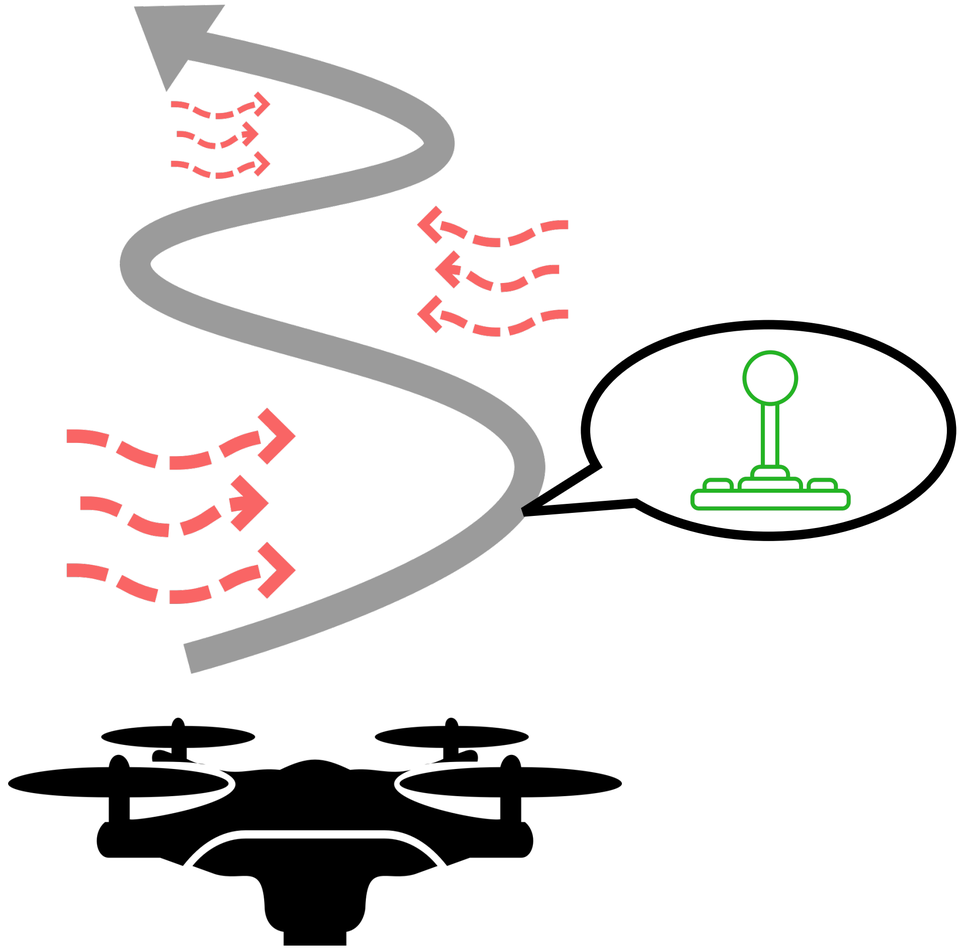

s_{t-2}
s_{t-1}
s_t
X
T
T
π_E
π_E
π_E
a_{t-2}
a_{t-1}
a_t
Y
u_{t-1}
u_t

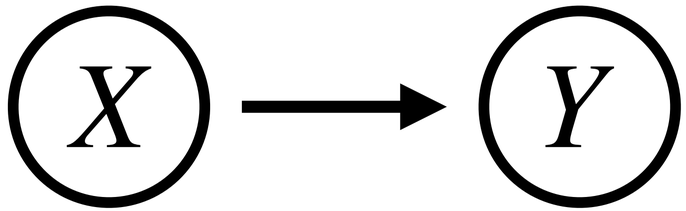

s_{t-2}
s_{t-1}
s_t
X
T
T
π_E
π_E
π_E
a_{t-2}
a_{t-1}
a_t
Y
u_{t-2}
u_{t-1}

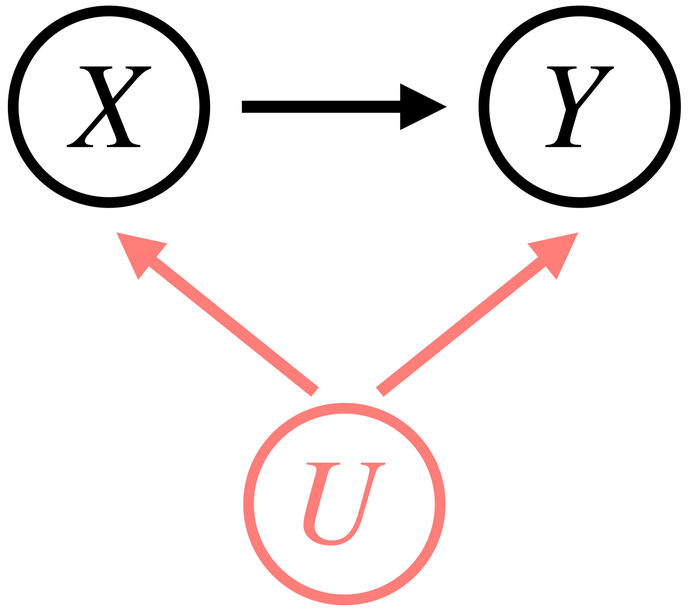

π_E
π_{BC}

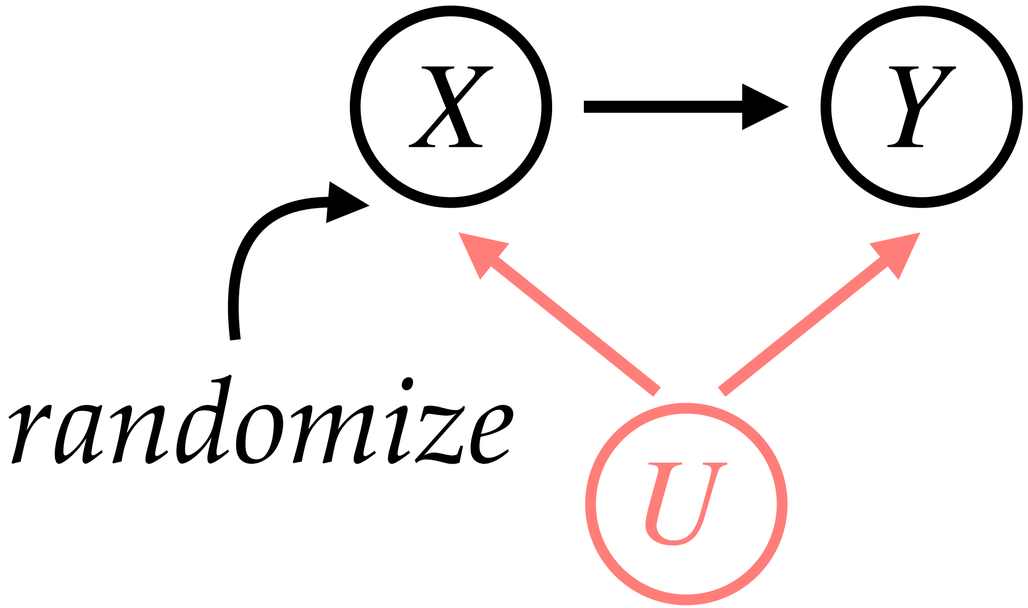

π_E
π_{BC}
π

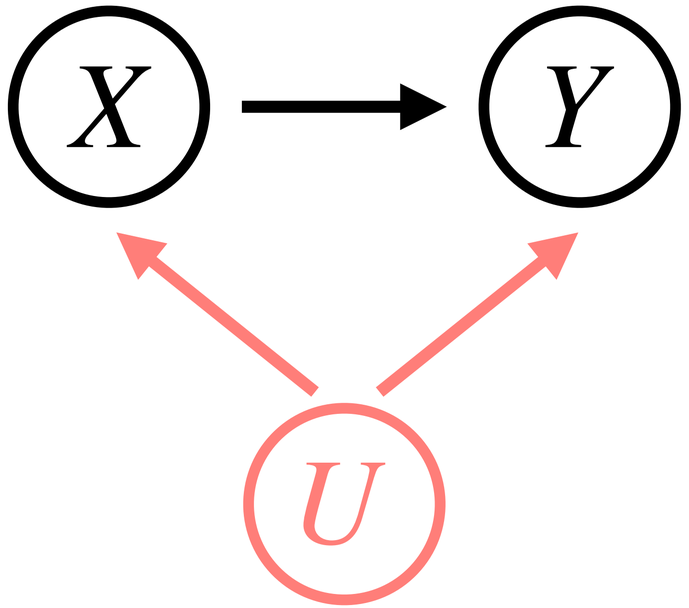


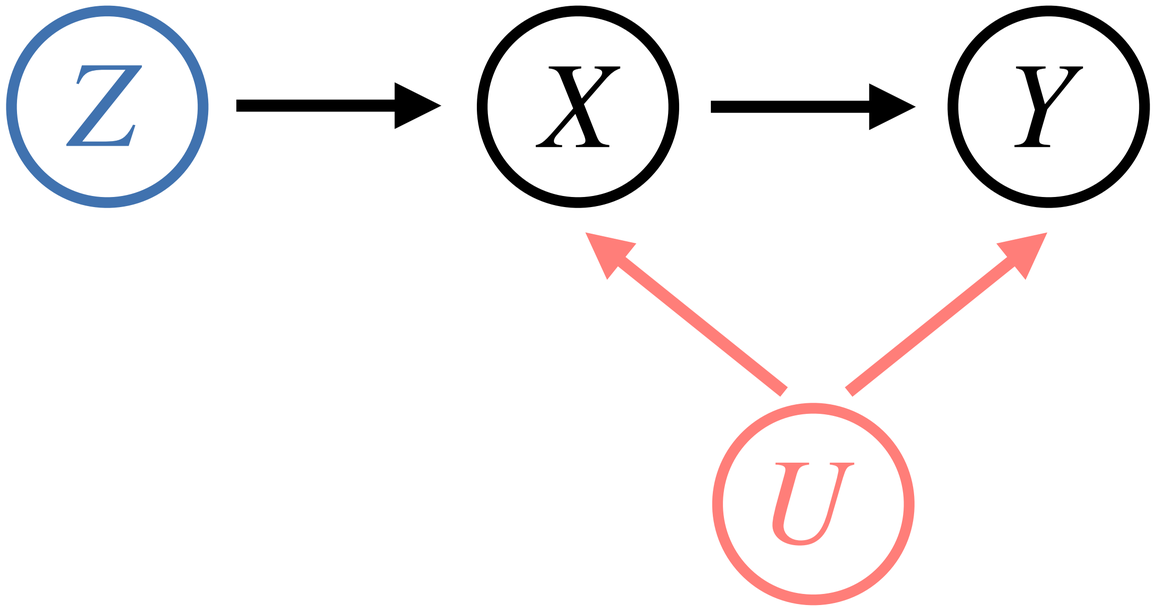


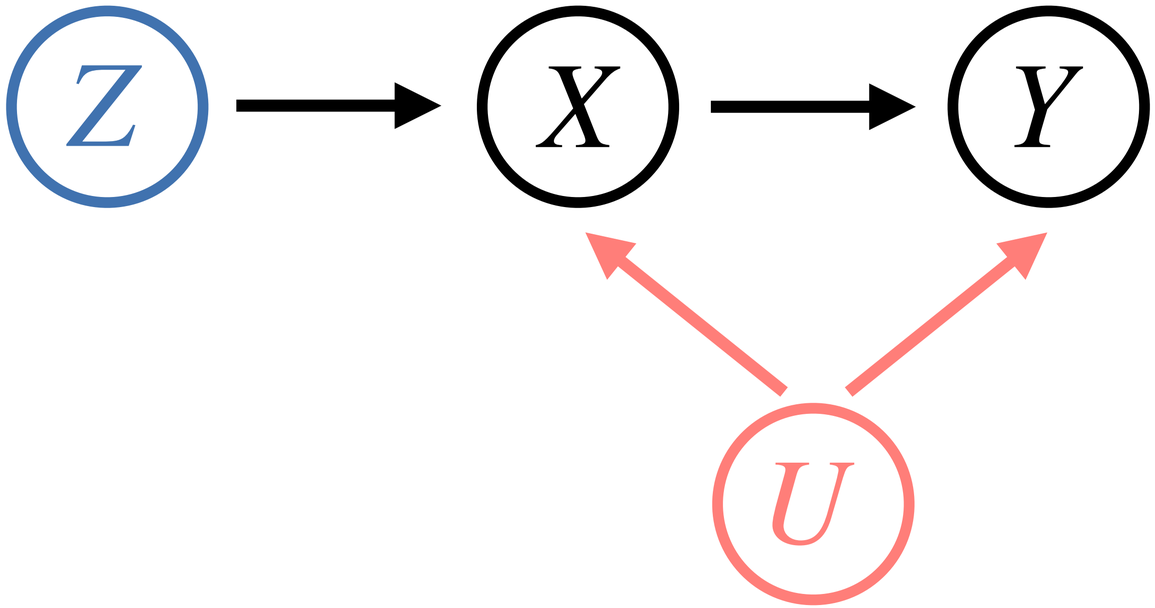

X
Y


X
Y
U

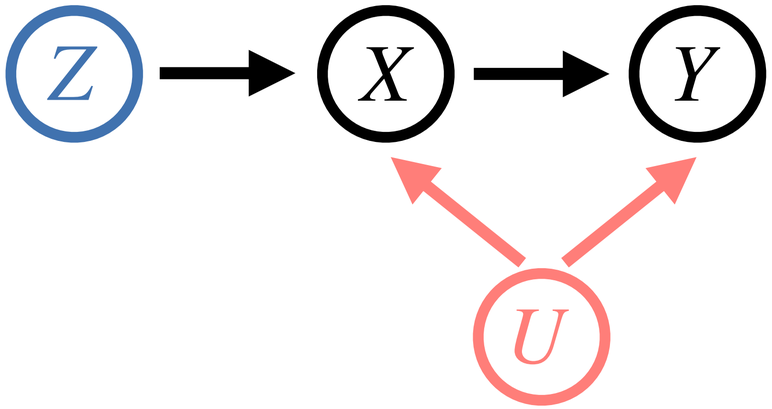

X
Y
randomize
U

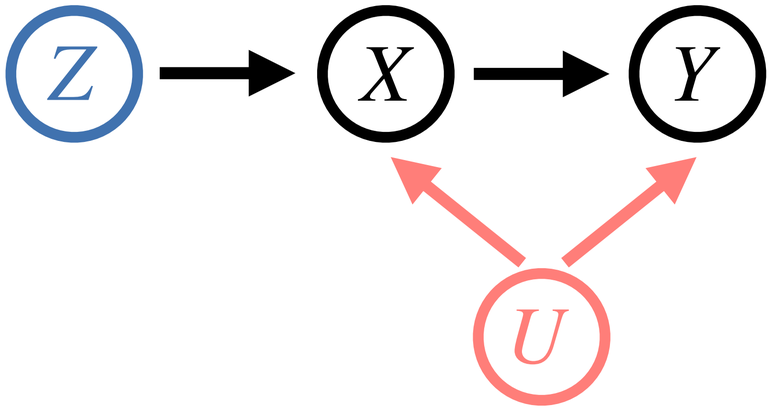

X
Y
U


Z
X
Y
U

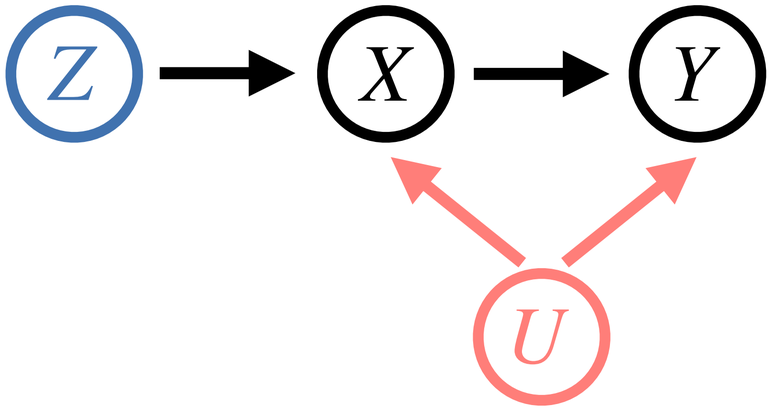


***Key Idea**: We can condition on instrument Z to counter the effect of confounder U on X.*


Z
X
Y
U

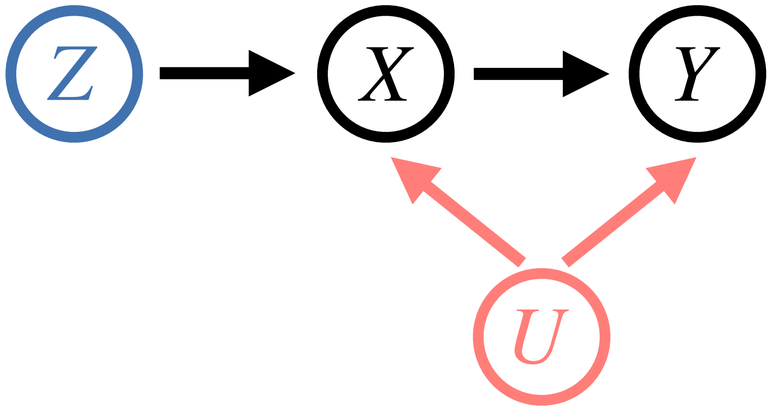


$$X = g(Z, U)$$



$$X = g(Z, U)$$

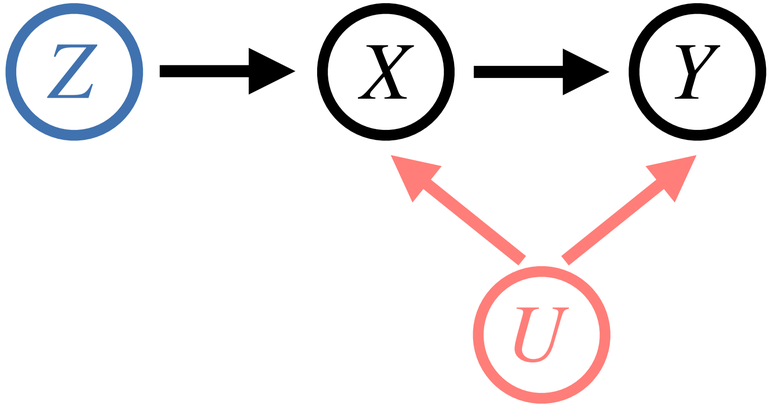
$$Y = h(X) + U$$



$$X = g(Z, U)$$

$$Y = h(X) + U$$

$$\mathbb{E}[U] = 0$$

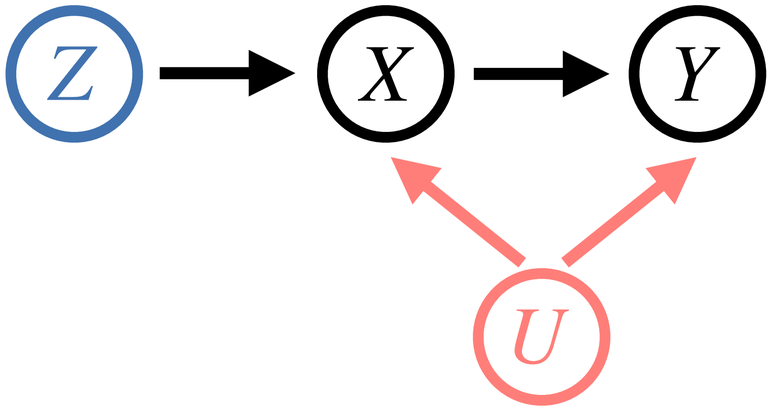


$$X = g(Z, U)$$

$$Y = h(X) + U$$

$$\mathbb{E}[U] = 0$$

$$0 = \mathbb{E}[U] = \mathbb{E}[U \mid z] = \mathbb{E}[Y - h(X) \mid z]$$



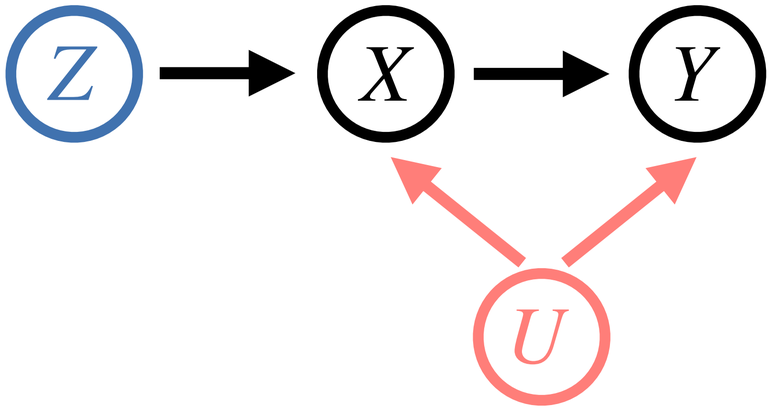
$$X = g(Z, U)$$

$$Y = h(X) + U$$

$$\mathbb{E}[U] = 0$$

$$0 = \mathbb{E}[U] = \mathbb{E}[U \mid z] = \mathbb{E}[Y - h(X) \mid z]$$

$$\Rightarrow \mathbb{E}[Y \mid z] = \mathbb{E}[h(X) \mid z], \forall z$$



$$X = g(Z, U)$$

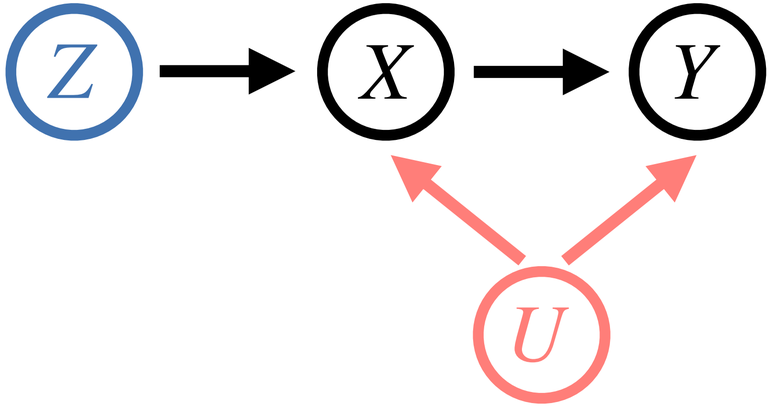
$$Y = h(X) + U$$

$$\mathbb{E}[U] = 0$$

$$0 = \mathbb{E}[U] = \mathbb{E}[U \mid z] = \mathbb{E}[Y - h(X) \mid z]$$

$$\Rightarrow \mathbb{E}[Y \mid z] = \mathbb{E}[h(X) \mid z], \forall z$$

$$\Rightarrow \min_h \mathbb{E}_z[(\mathbb{E}[Y \mid z] - \mathbb{E}[h(X) \mid z])^2]$$



$$X = g(Z, U)$$

$$Y = h(X) + U$$

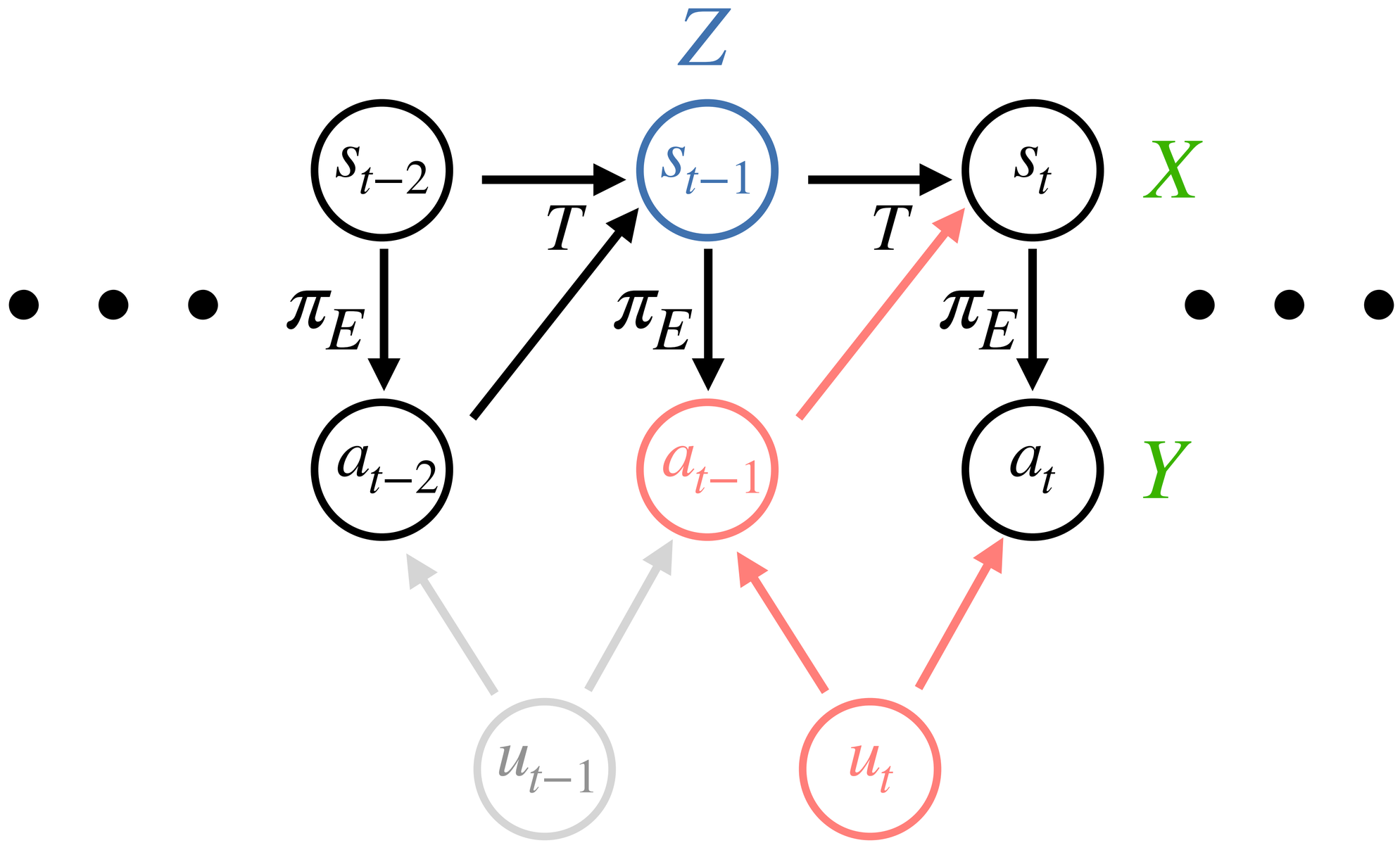
$$\mathbb{E}[U] = 0$$

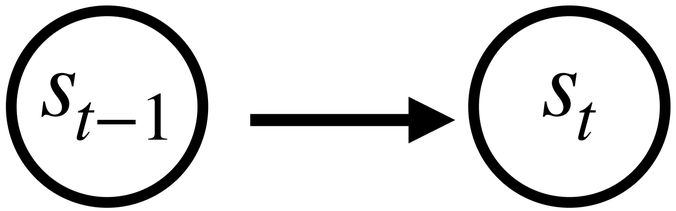
$$0 = \mathbb{E}[U] = \mathbb{E}[U \mid z] = \mathbb{E}[Y - h(X) \mid z]$$

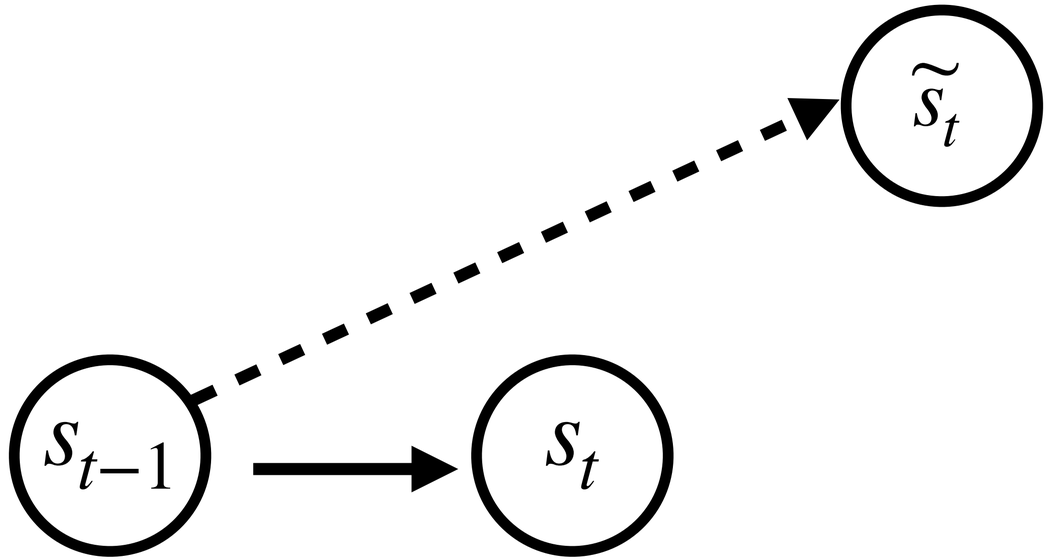
$$\Rightarrow \mathbb{E}[Y \mid z] = \mathbb{E}[h(X) \mid z], \forall z$$

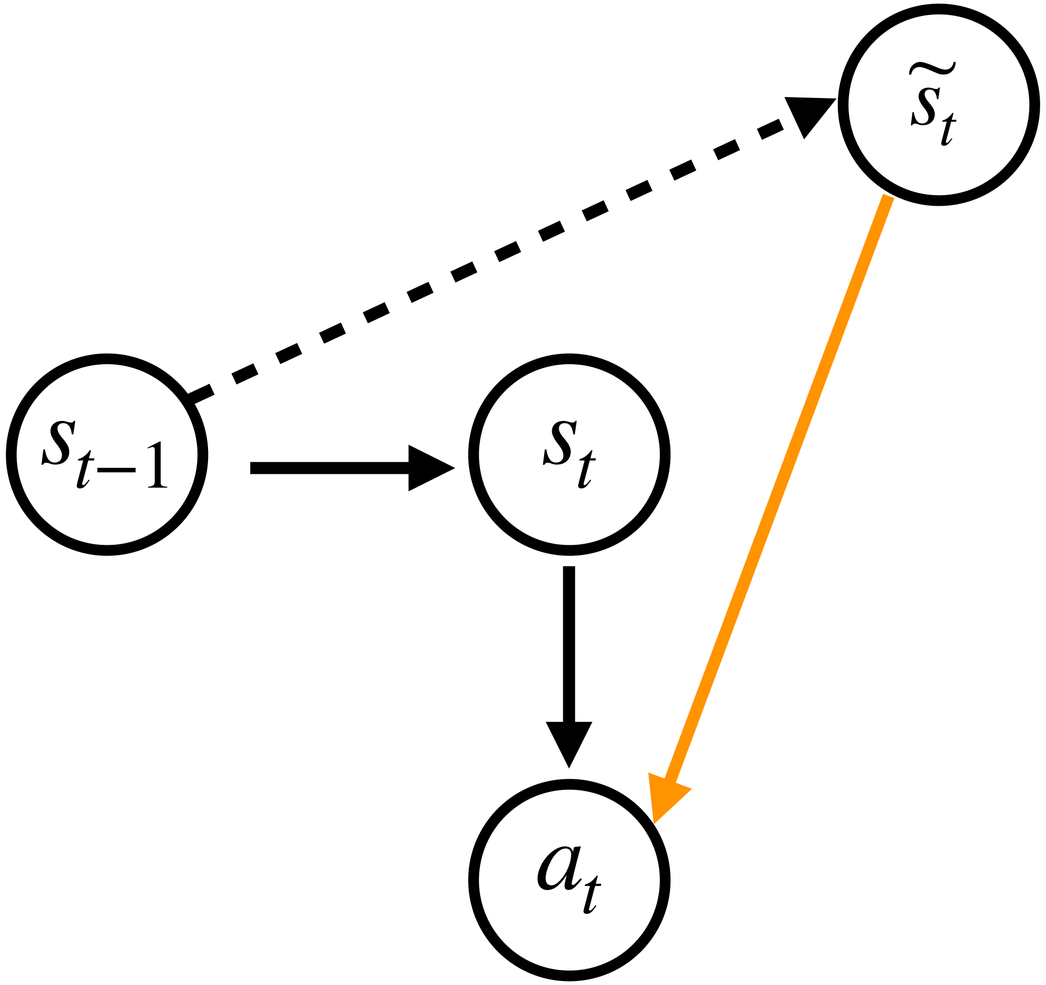
$$\Rightarrow \min_h \mathbb{E}_z[(\mathbb{E}[Y \mid z] - \mathbb{E}[h(X) \mid z])^2]$$

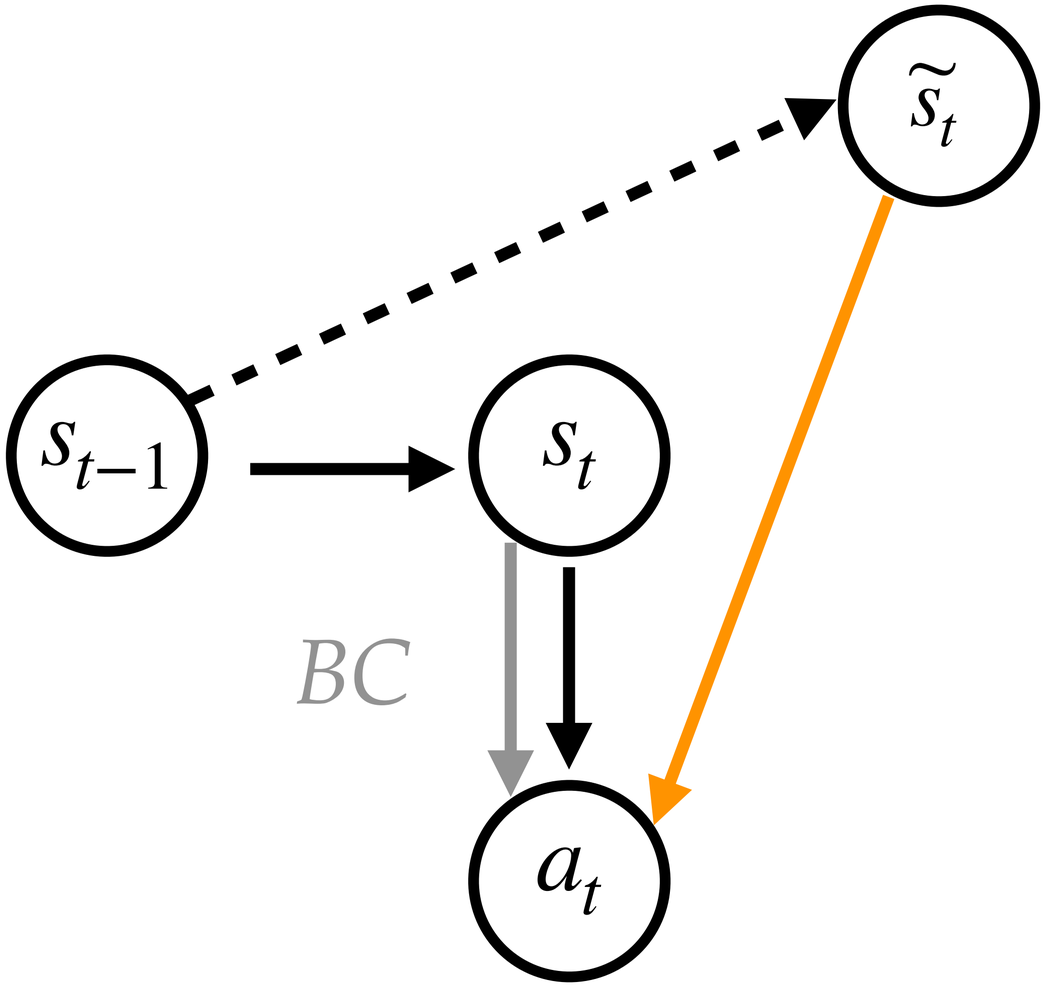
$$\Leftrightarrow \min_h \max_f \mathbb{E}_z[2(Y - h(X))f(Z) - f^2(Z)]$$

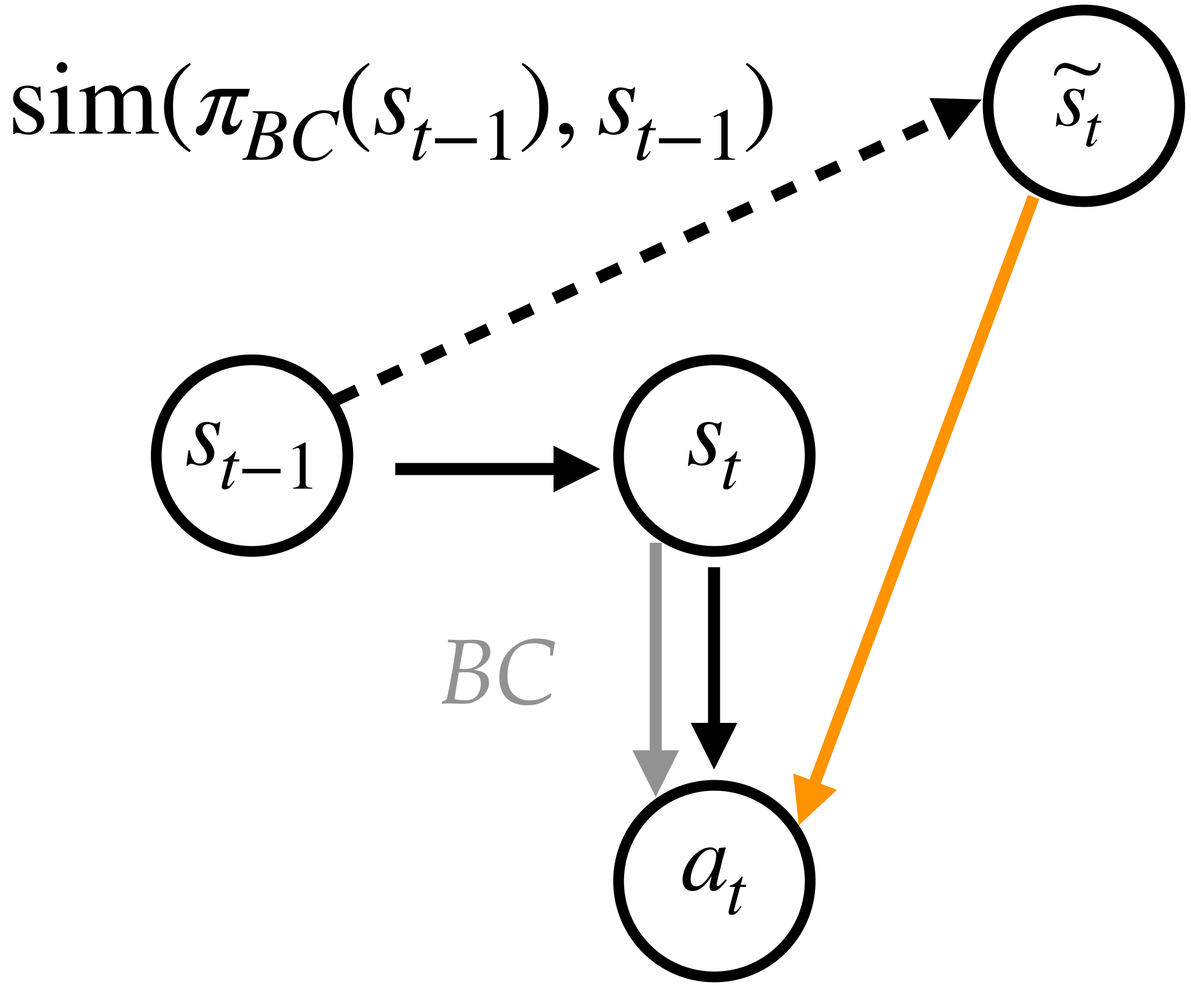

Z
s_{t-2}
s_{t-1}
s_t
X
T
T
π_E
π_E
π_E
a_{t-2}
a_{t-1}
a_t
Y
u_{t-1}
u_t

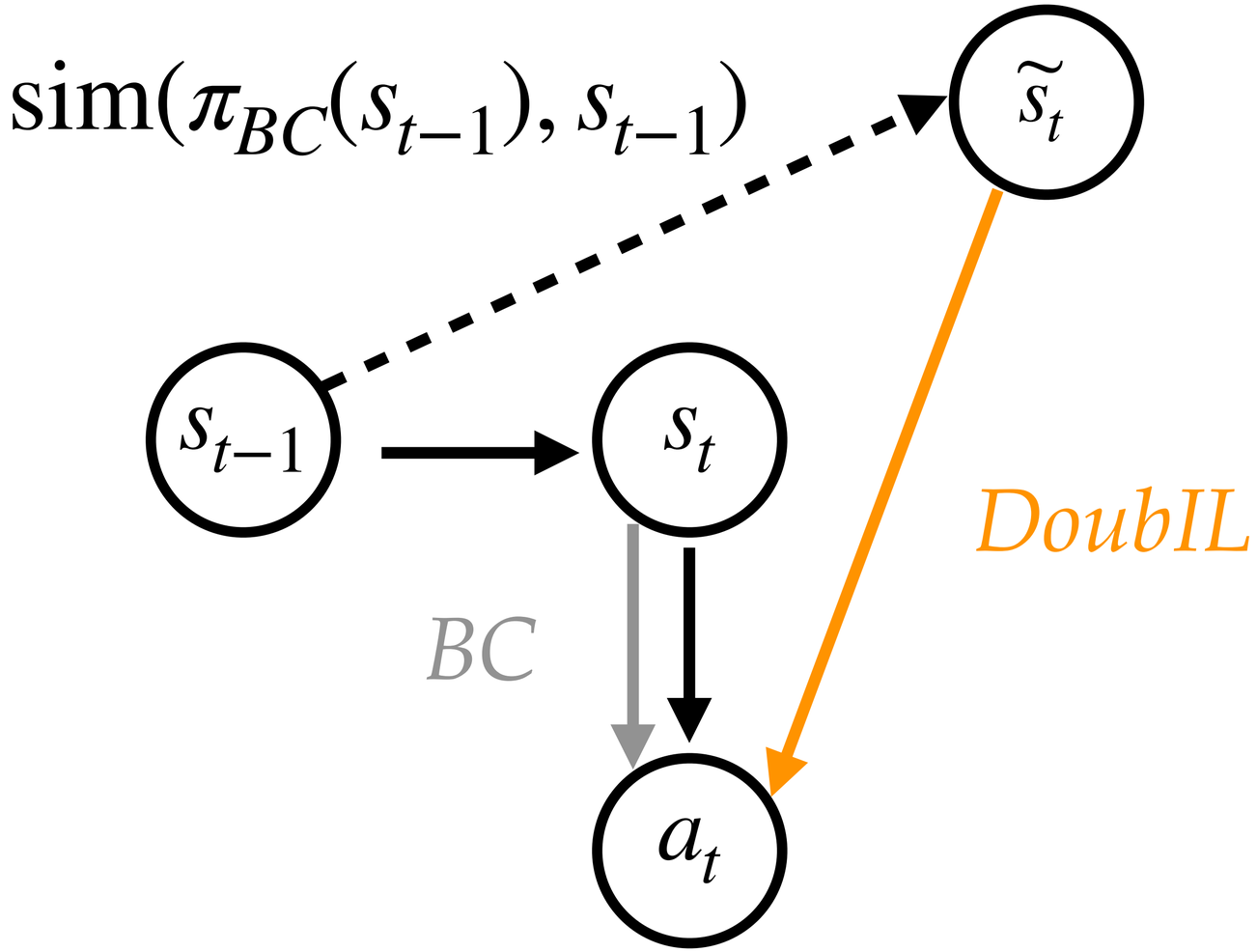

s_{t-1}
s_t

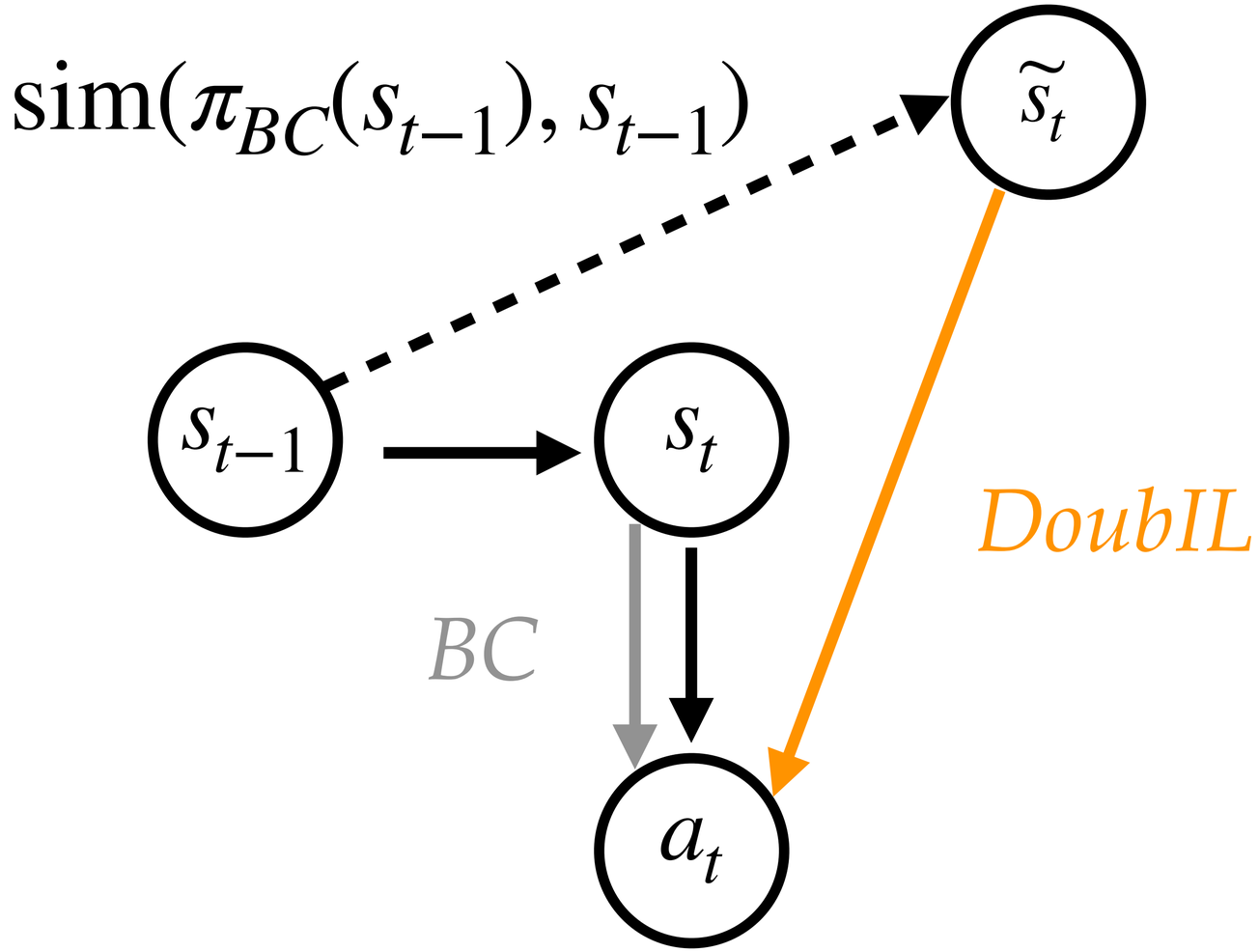

$\widetilde{s}_t$
s_{t-1}
s_t


$\tilde{s}_t$
s_{t-1}
s_t
a_t


$\tilde{s}_t$
s_{t-1}
s_t
BC
a_t


$\mathrm{sim}(\pi_{BC}(s_{t-1}), s_{t-1})$
$\tilde{s}_t$
s_{t-1}
s_t
BC
a_t


$\mathrm{sim}(\pi_{BC}(s_{t-1}), s_{t-1})$
$\tilde{s}_t$
s_{t-1}
s_t
a_t
BC
DoubIL



$$J(\pi_E) - J(\pi) \leq c(\sqrt{\epsilon} + \sqrt{\delta})\kappa(\Pi)T^2$$

$$\min_{\pi} \max_{f} \mathbb{E}[2(a_t - \pi(s_t))f(s_{t-1}) - f(s_{t-1})^2]$$

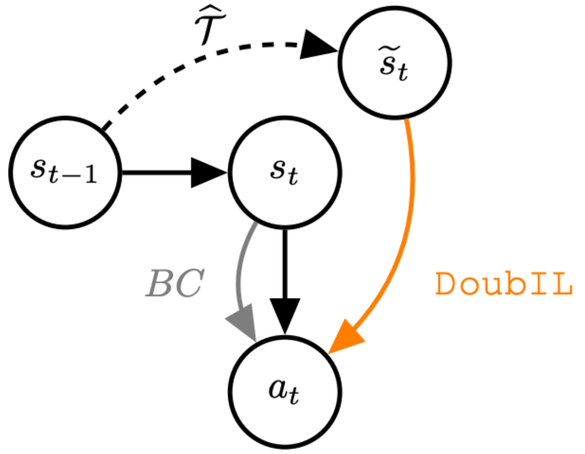
$$\min_{\pi} \max_{f} \mathbb{E}[2(a_t - \pi(s_t))f(s_{t-1}) - f(s_{t-1})^2]$$

$$J(\pi_E) - J(\pi) \leq c\sqrt{\epsilon}\kappa(\Pi)T^2$$

Instrumental Variable Imitation Learning

generative modeling

DoubIL



$$J(\pi_E) - J(\pi) \leq c(\sqrt{\epsilon} + \sqrt{\delta})\kappa(\Pi)T^2$$

game-theoretic

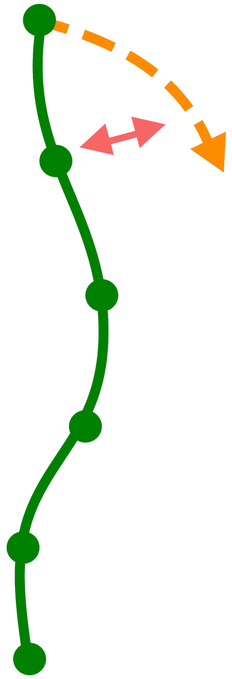
ResiduIL

$$\min_{\pi} \max_{f} \mathbb{E}[2(a_t - \pi(s_t))f(s_{t-1}) - f(s_{t-1})^2]$$

$$J(\pi_E) - J(\pi) \leq c\sqrt{\epsilon}\kappa(\Pi)T^2$$

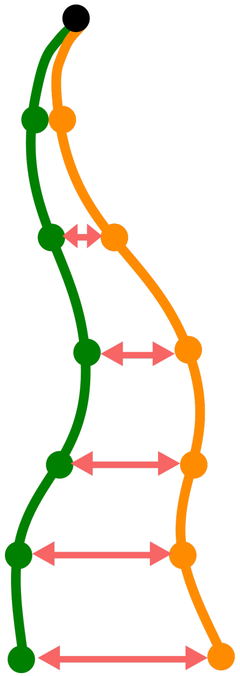
$\pi_E \overset{f}{\longleftrightarrow} \pi$

Offline

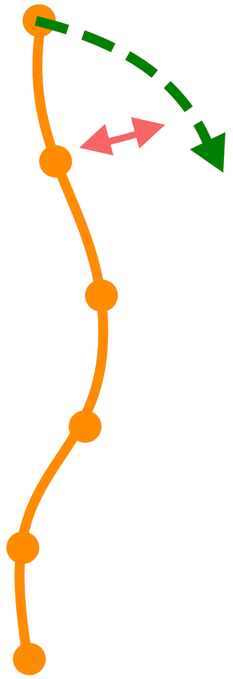


Inconsistent,
IVR Consistent

Online



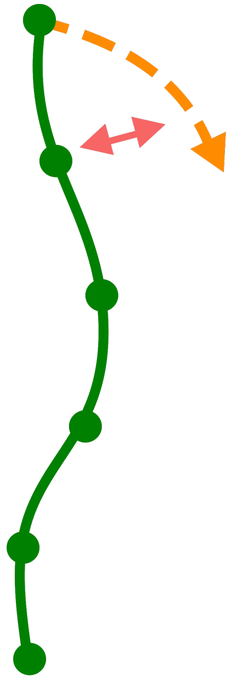
Interactive



Consistent

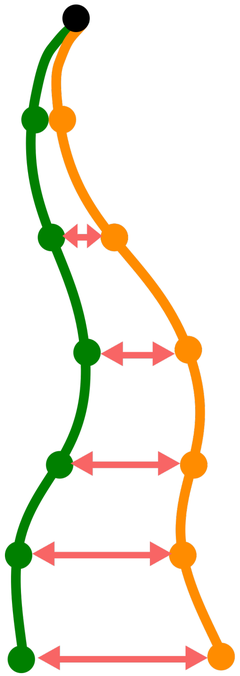
$\pi_E \overset{f}{\leftrightarrow} \pi$

Offline



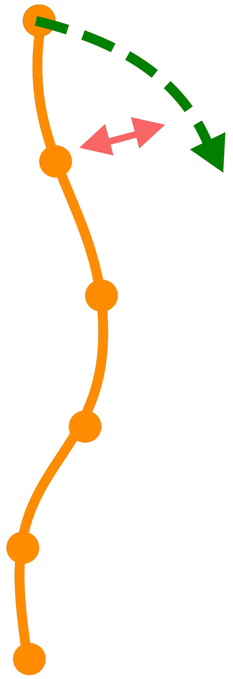
Inconsistent,
IVR Consistent

Online

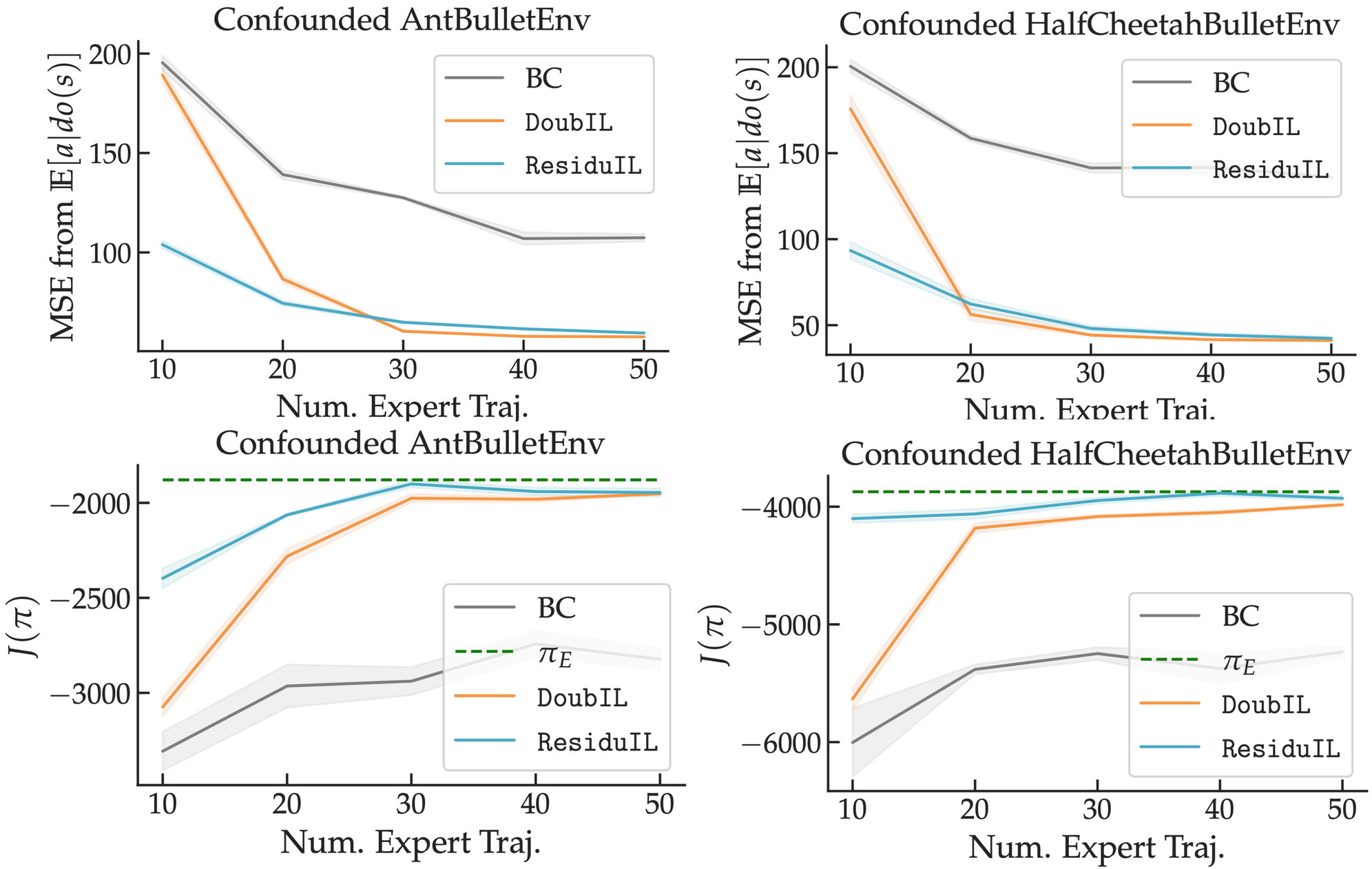


Inconsistent,
Hybrid?

Interactive

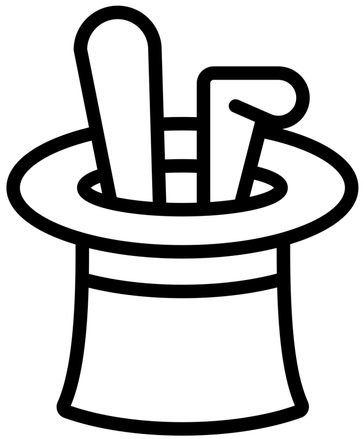


Consistent


Confounded AntBulletEnv
MSE from $\mathbb{E}[a|do(s)]$
200
150
100
10
20
30
40
50
Num. Expert Traj.
BC
DoubIL
ResiduIL
Confounded HalfCheetahBulletEnv
MSE from $\mathbb{E}[a|do(s)]$
200
150
100
50
10
20
30
40
50
Num. Expert Traj.
BC
DoubIL
ResiduIL
Confounded AntBulletEnv
$J(\pi)$
−2000
−2500
−3000
10
20
30
40
50
Num. Expert Traj.
BC
π_E
DoubIL
ResiduIL
Confounded HalfCheetahBulletEnv
$J(\pi)$
−4000
−5000
−6000
10
20
30
40
50
Num. Expert Traj.
BC
π_E
DoubIL
ResiduIL

	Offline	Online	Interactive
Covariate Shift	✘	✔	✔
Hidden Context	✘	✔ w/ History	✔ w/ History
TCN	✔ w/ IVR	✔ w/ IVR	✔

Interventions happen via interaction with the environment in sequential decision making.



Thanks!

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