# System Dynamics for Complex Adaptive Systems

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# Outline

- Complex Adaptive Systems
- Modeling paradigms
  – System Dynamics
    - Population Growth
    - Epidemiology
  – Agent-based Modeling
  – Strengths/weaknesses of each
- Embedded (Hybrid) Models

# Complex Adaptive Systems

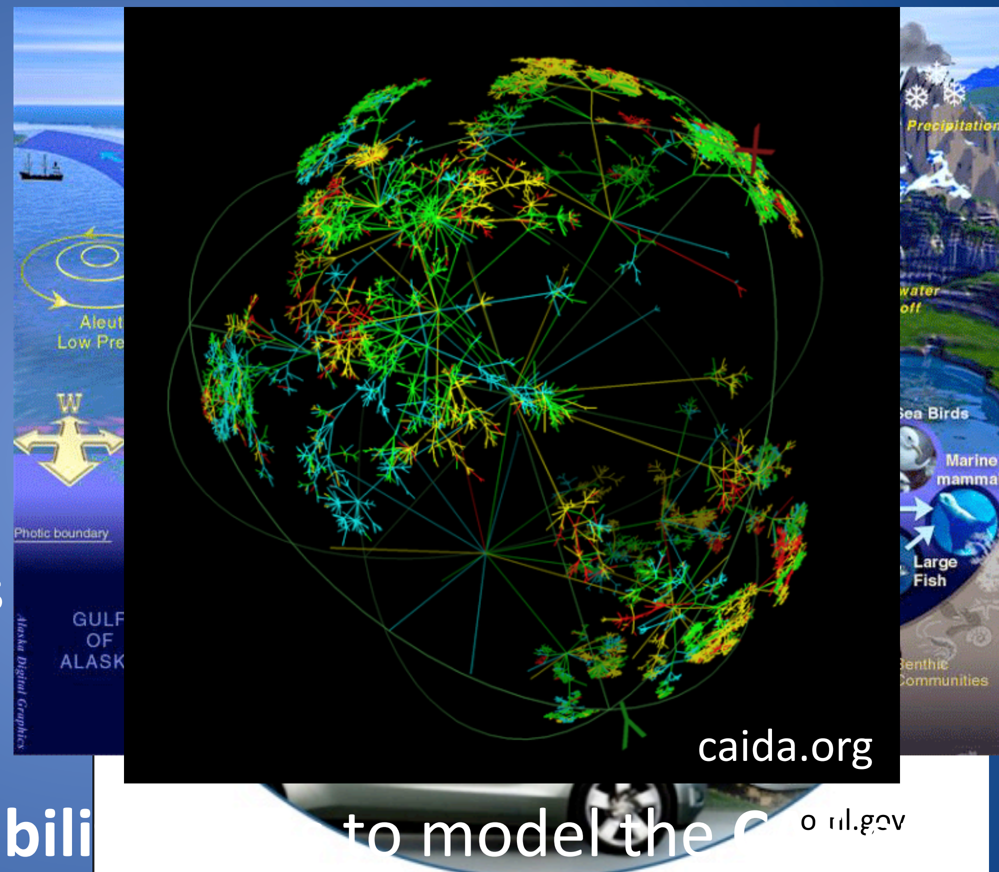
*"The **whole** is not only **more than** but **very different** than the **sum of its parts."***
**(**Anderson, Phil W. 1972. "More is Different." *Science* 177: 393-96)

- ***<u>Dynamic</u>*** network of many agents (e.g. cells, species, individuals, firms, nations) acting in parallel, constantly acting and reacting to what the other agents are doing
- ***<u>Control</u>*** of CAS is dispersed, decentralized
- ***<u>Emergent</u>*** behaviors (e.g. equilibria, patterns) arise from competition and cooperation among agents

**System behavior is unpredictable and the result of decisions made every moment by many individual agents**

# Complex Adaptive Systems

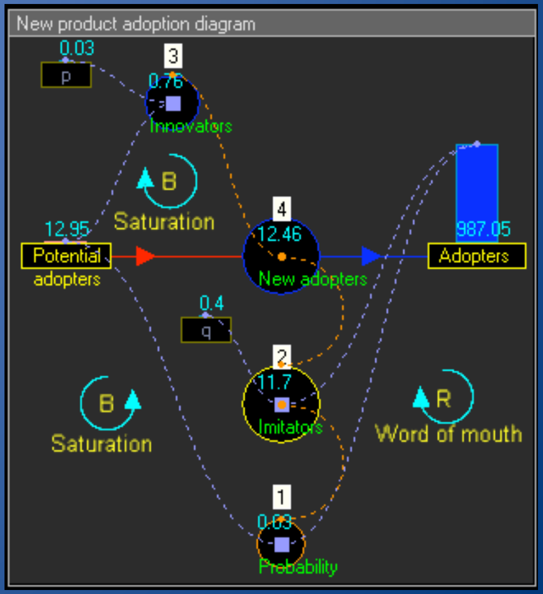
- **Examples:**
  - Energy grids
  - Ecosystems
  - Social diffusion
  - Disease dynamics
  - Politics
  - Supply chain networks
  - Etc.



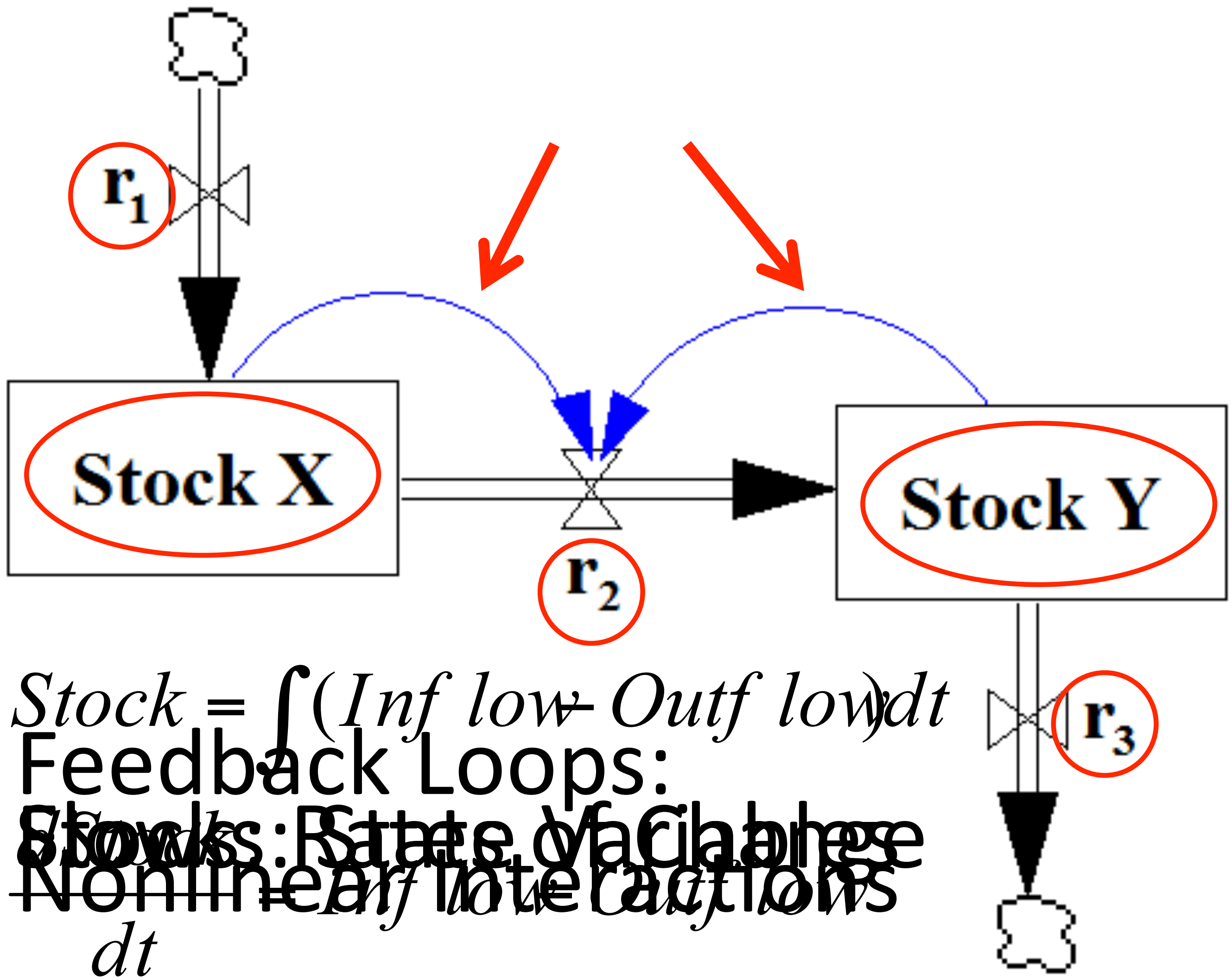

**Computational Sustainabili** to model the C our world in the hopes of guiding them toward **long-term, sustainable outcomes**

# Modeling CAS

- Two basic approaches:
  - Top-down: System Dynamics
    - ODEs, Stock and Flow Diagrams
  - Bottom-up: Agent-based Modeling
    - Cellular Automata, Intelligent Agents



# System Dynamics




$$Stock = \int (Inflow - Outflow)\,dt$$

Feedback Loops:

Stocks: State Variables

Flows: Rates of Change

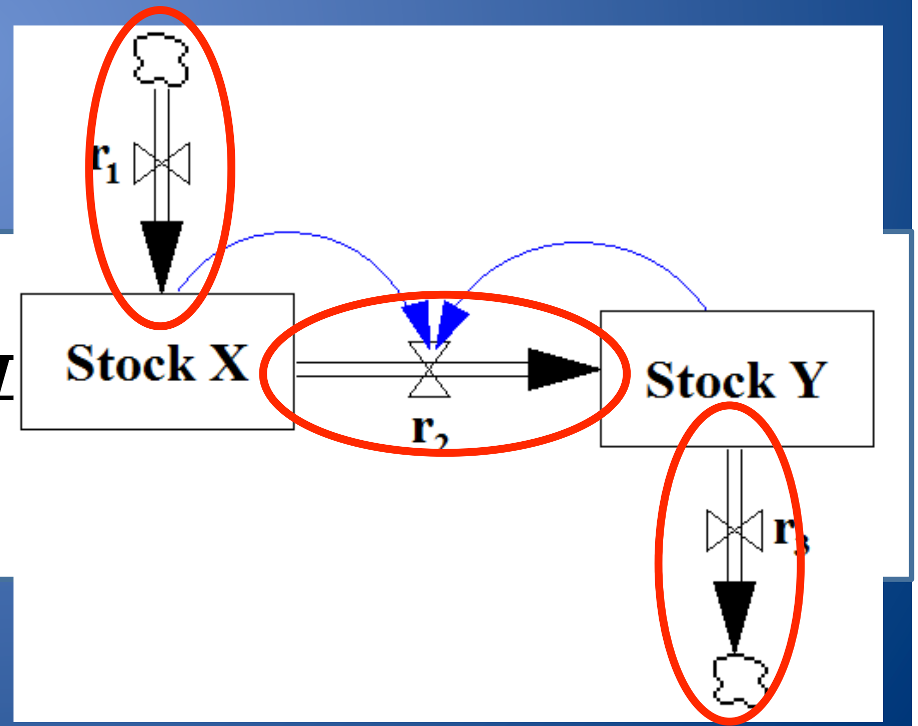
Nonlinear Interactions

$$\frac{dStock}{dt} = Inflow - Outflow$$

# System Dynamics

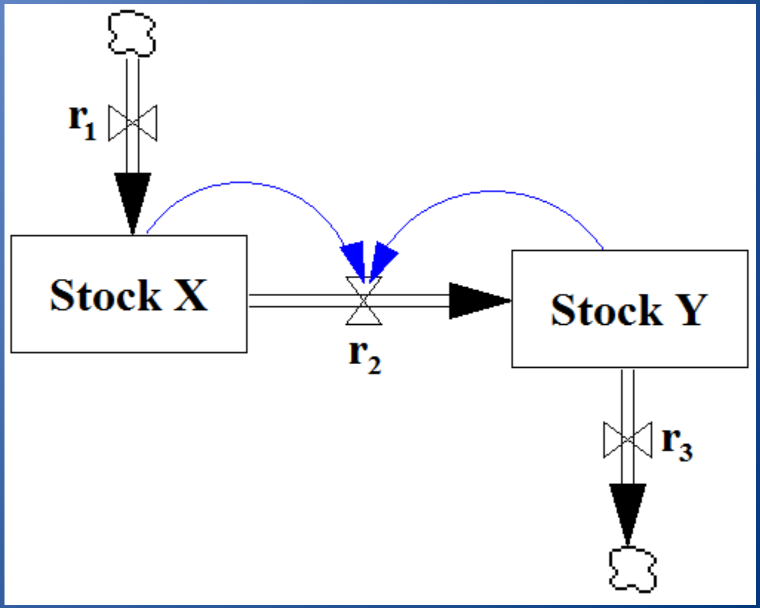
- Flows can be of three types:
  - Generative ($F_{gen}$)
  - Stock-to-Stock ($F_{in}$, $F_{out}$)
  - Destructive ($F_{des}$)

$$\frac{dStock}{dt} = (F_{gen} + F$$



# System Dynamics

- Discrete systems:
  - Stocks: homogenous groups of well-mixed agents
  - Flows: movement of agents between groups
  - Feedback Loops: nonlinear interactions and effects
    - **$r_2 = f(X,Y)$**



# System Dynamics

- Example 1: Population Growth

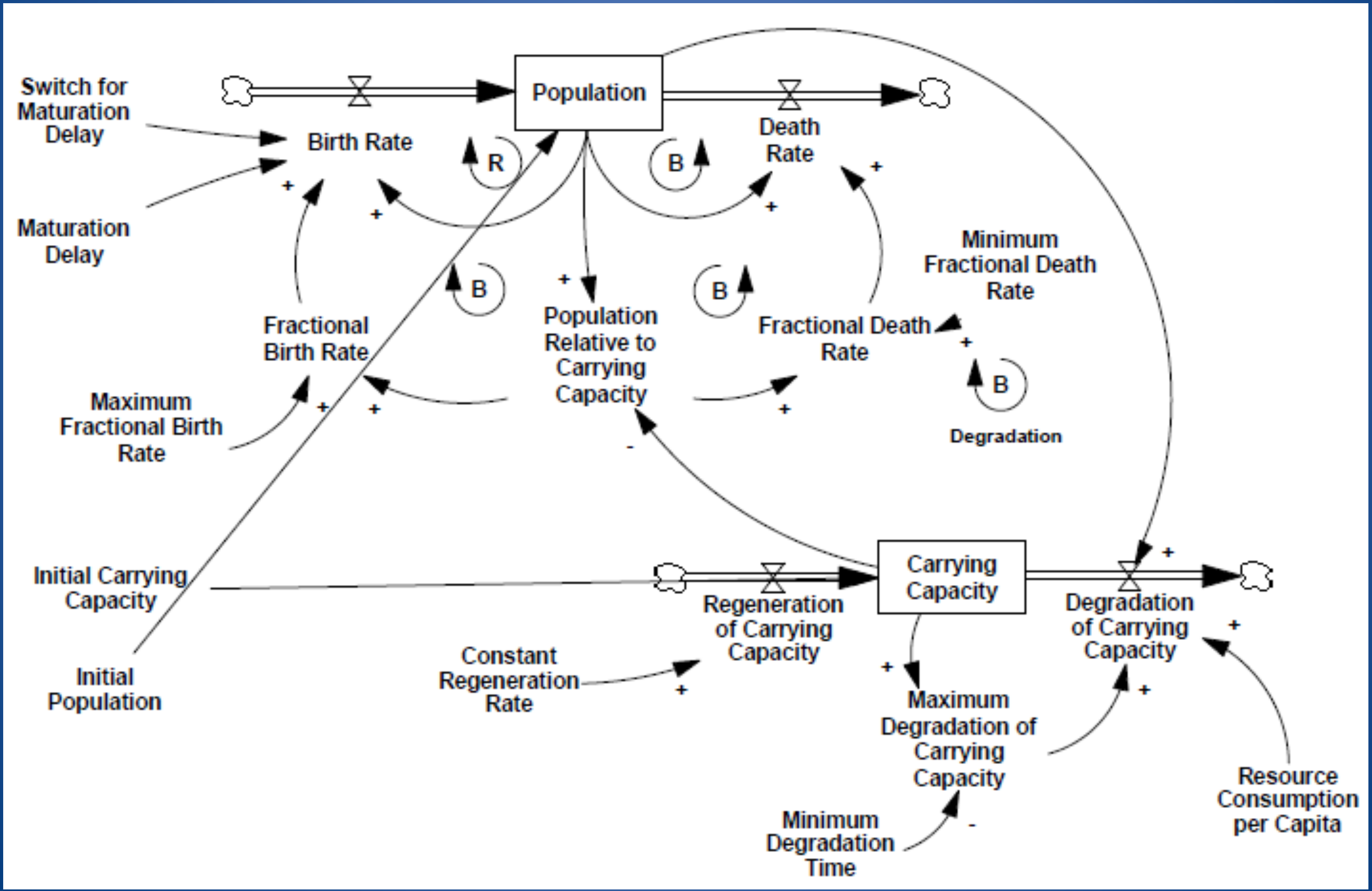
$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$$

- *P* – population size at time *t*
- *r* – growth rate
- *K* – habitat carrying capacity

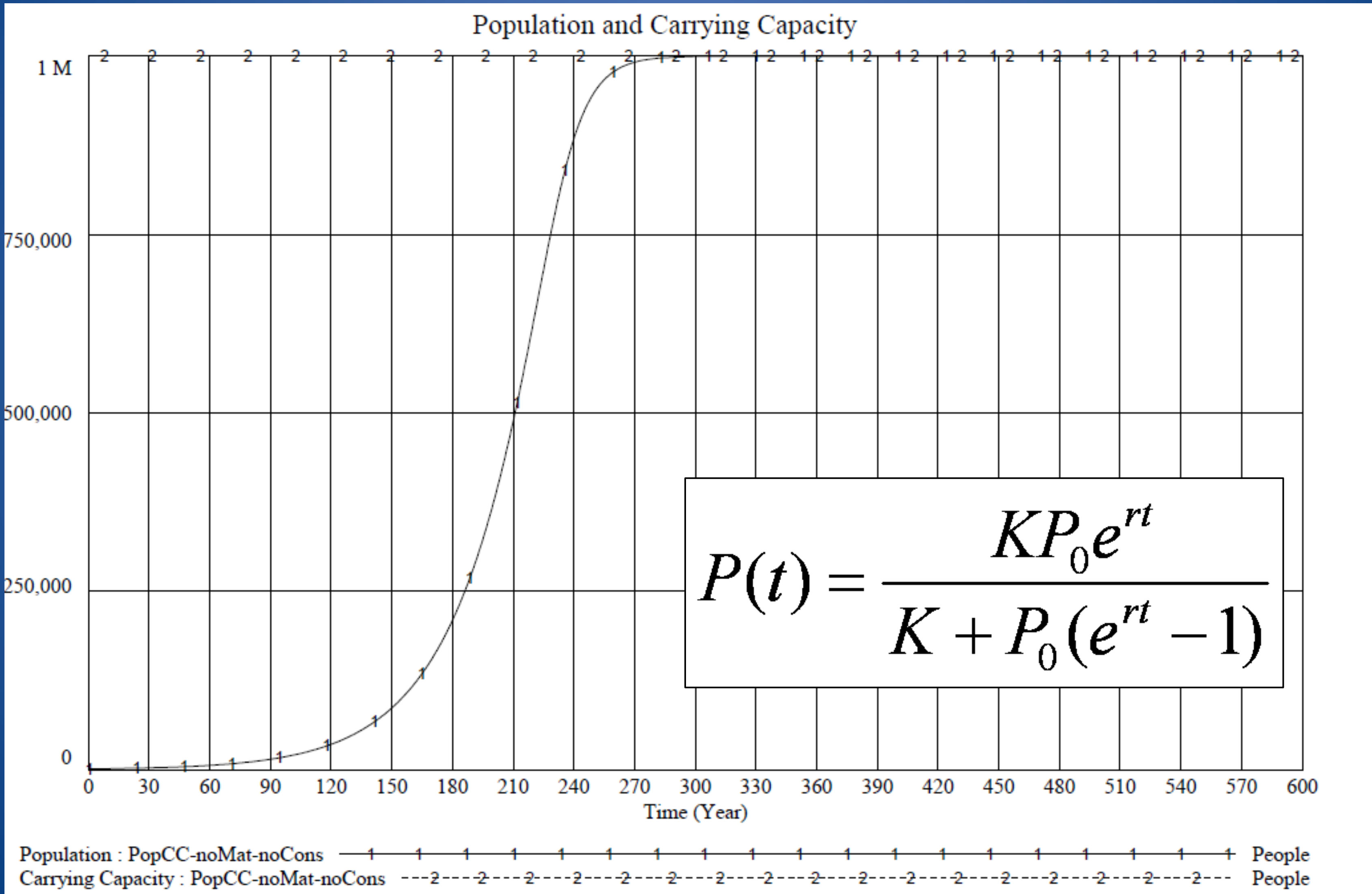
  – Exponential growth when population is small

  – Exponential decay when population above *K*

# Population Growth

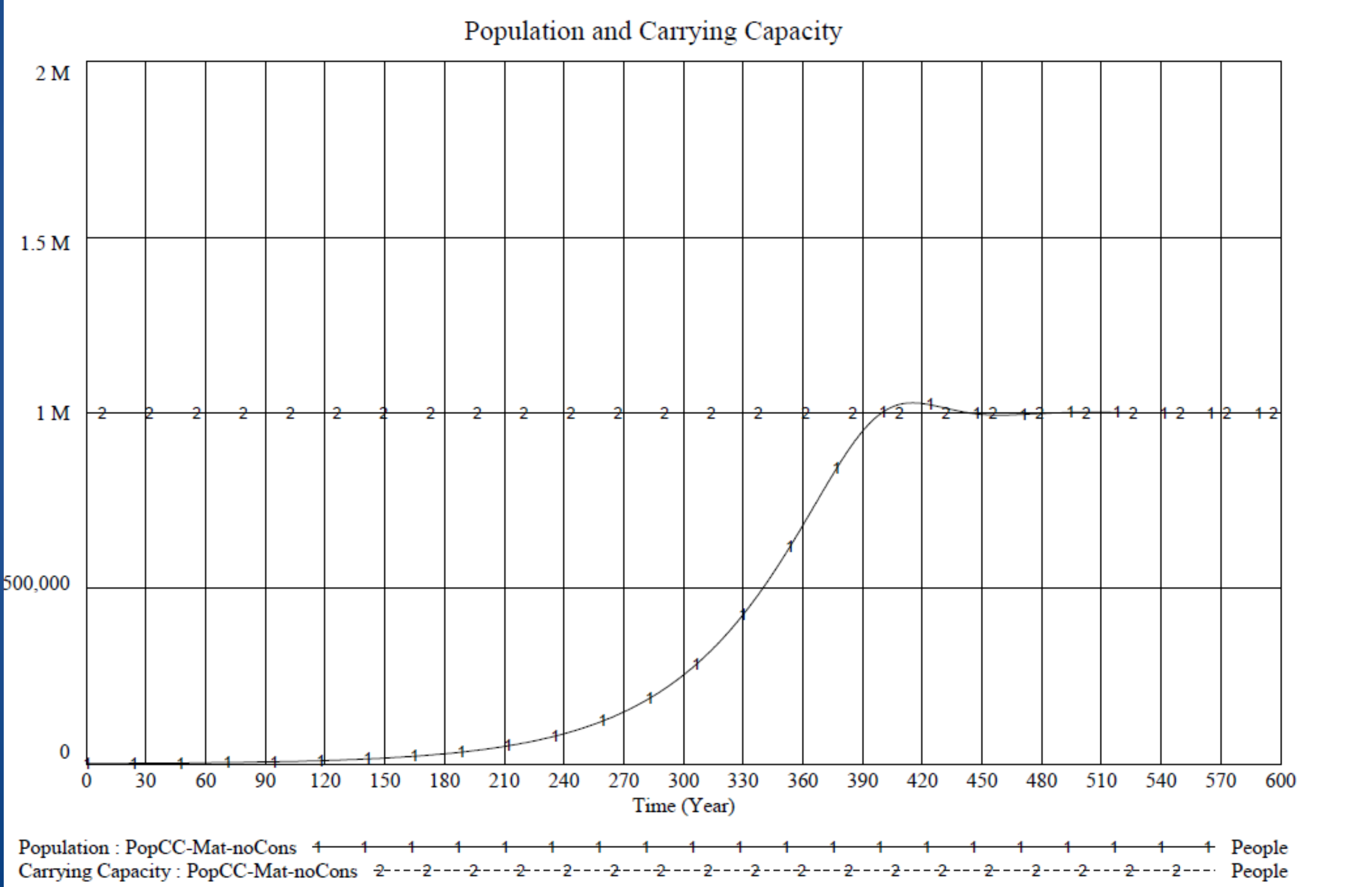


# Population Growth

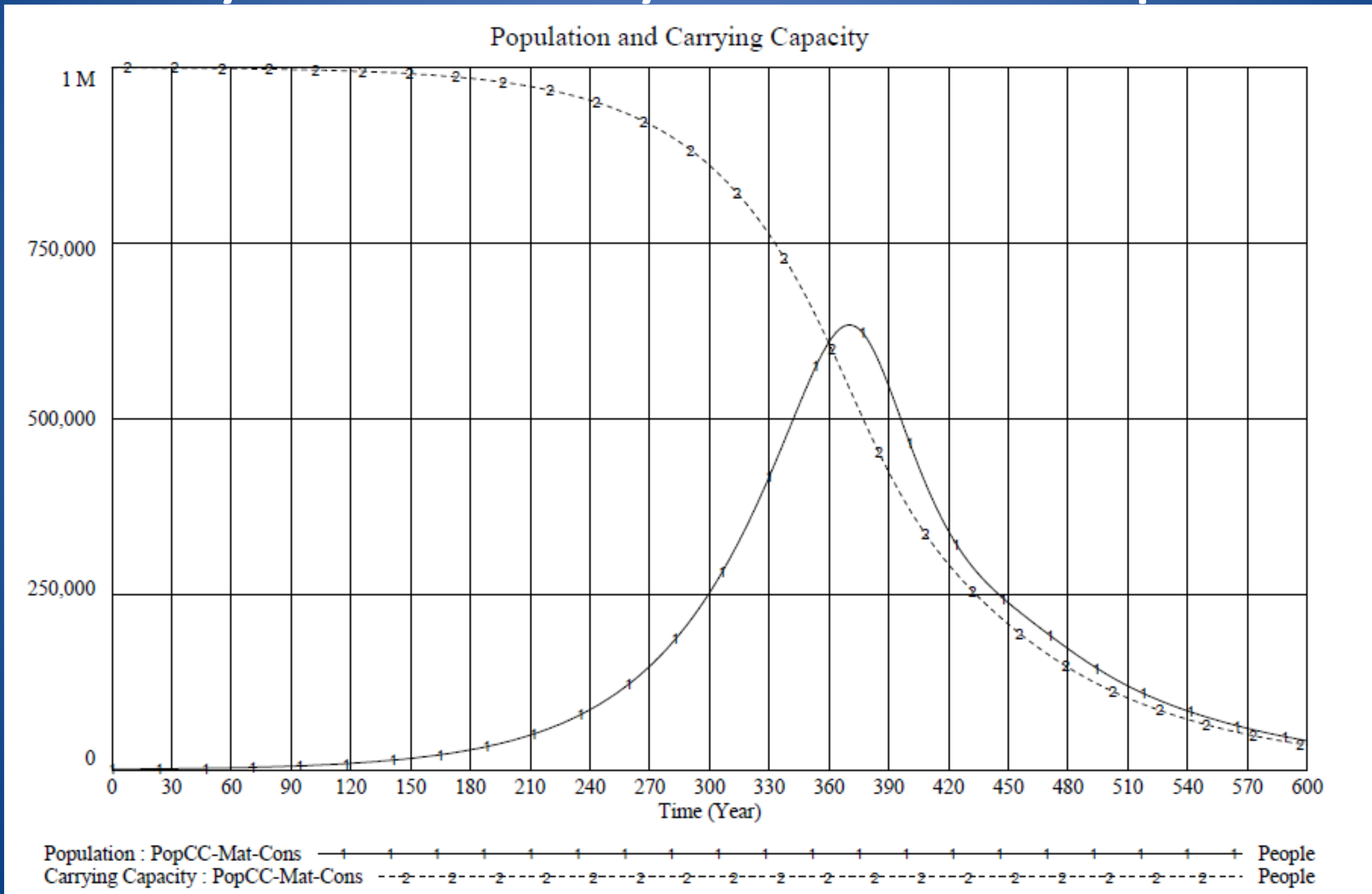



$$P(t) = \frac{K P_0 e^{rt}}{K + P_0 (e^{rt} - 1)}$$

# Population Growth: Delayed Maturity



# Population Growth: Delayed Maturity and Consumption



# System Dynamics

- Example 2: Infectious Disease

$$\frac{dS}{dt} = -\beta IS$$

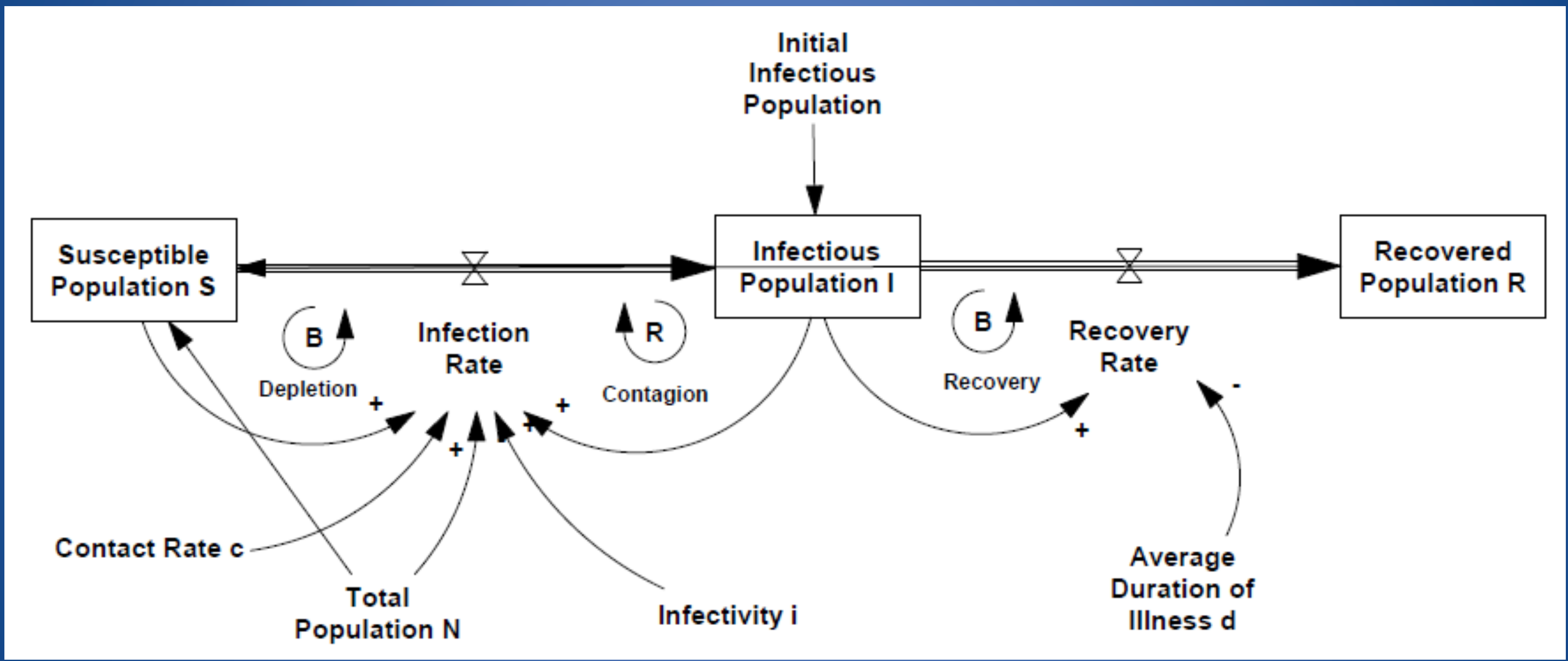
$$\frac{dI}{dt} = \beta SI - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$

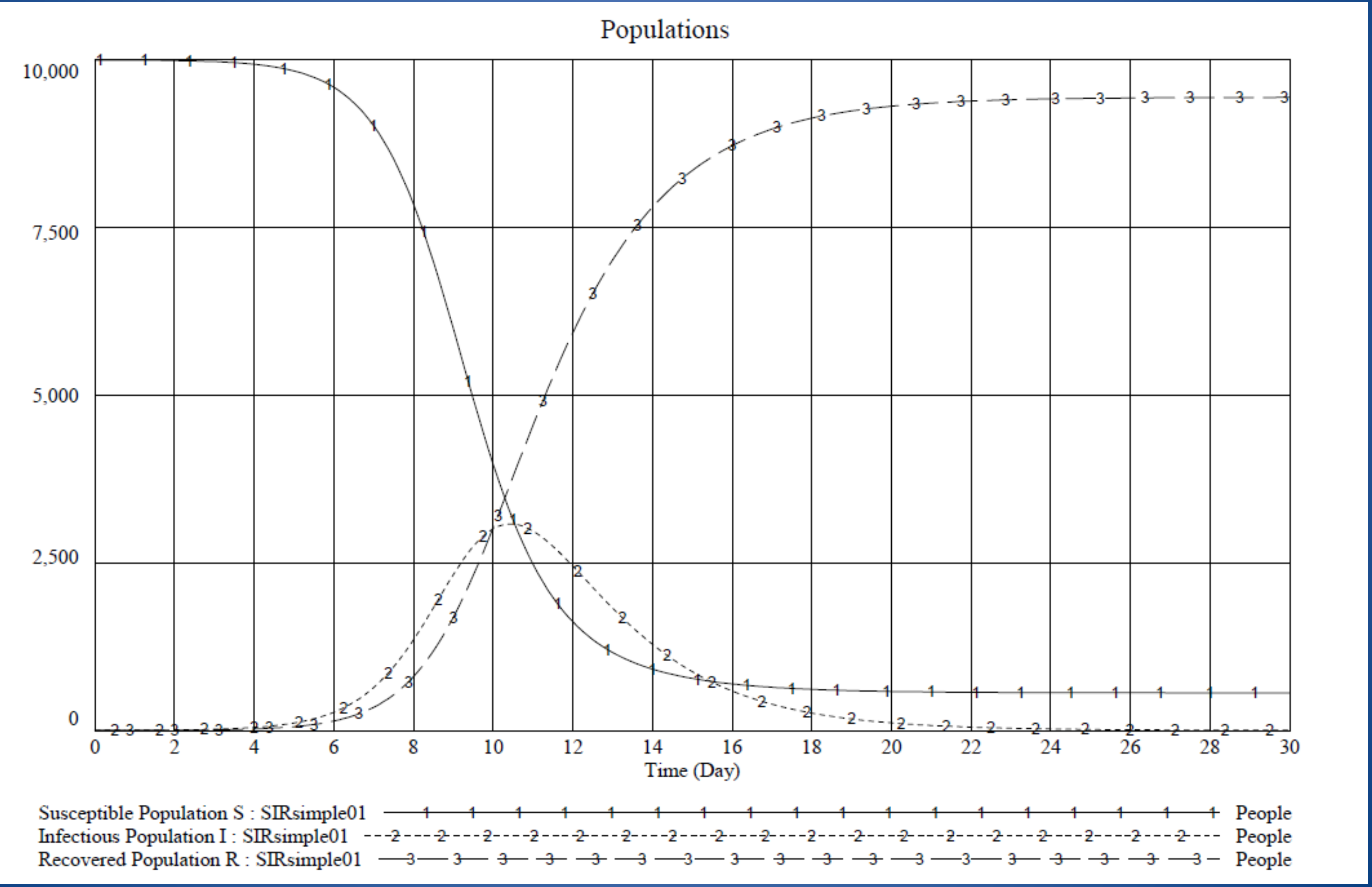
- S – Susceptible population
- I – Infectious population
- R – Recovered population

- β – Transmission rate
- γ – Recovery rate

# Infectious Disease



# Infectious Disease



# System Dynamics

- Simulation
  - For most systems of ODEs, analytical solutions do not exist
  - Continuous stock (e.g. money in savings): use numerical methods to approximate solution
  - Discrete stock (e.g. people): use stochastic simulation methods

# Stochastic Simulation of ODEs

- Assumption: Future state of system depends only on present state, independent of history

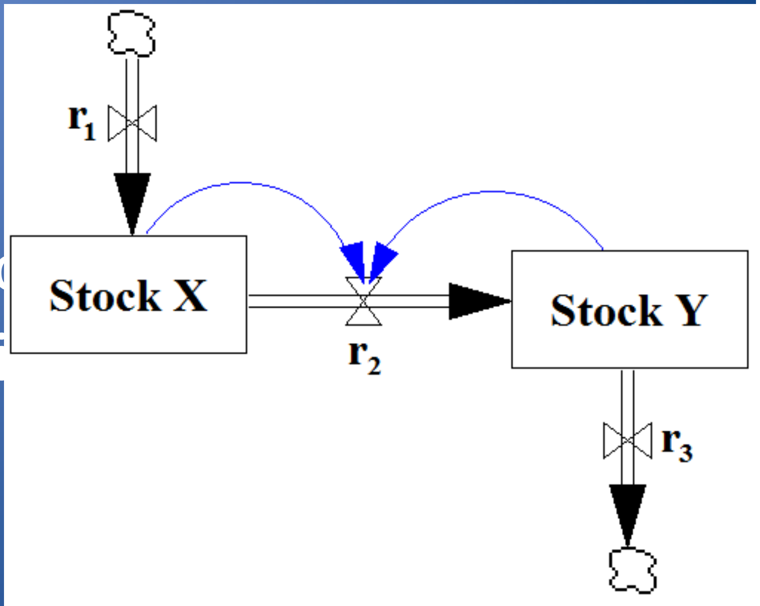
→Continuous Time Markov Chain!

  – Time between events Exponentially distributed

  – Event occurrences in [$t$, $t+\Delta t$) Poisson distributed

# Stochastic Simulation of ODEs

- ODEs as CTMCs
  - Flows are interpreted as transition probabilities per unit time
  - Events:
    - $\{X \rightarrow X+1\} \sim \textbf{Pois}(r_1 \Delta t)$
    - $\{(X,Y) \rightarrow (X-1,Y+1)\} \sim \textbf{Pois}(r_2 \Delta t)$
    - $\{Y \rightarrow Y-1\} \sim \textbf{Pois}(r_3 \Delta t)$
  - For $\Delta t$ small, probability of mo per time step is $o(\Delta t^2)$, negligi



# Stochastic Simulation of ODEs

**Algorithm 1** $\tau$-leap Method

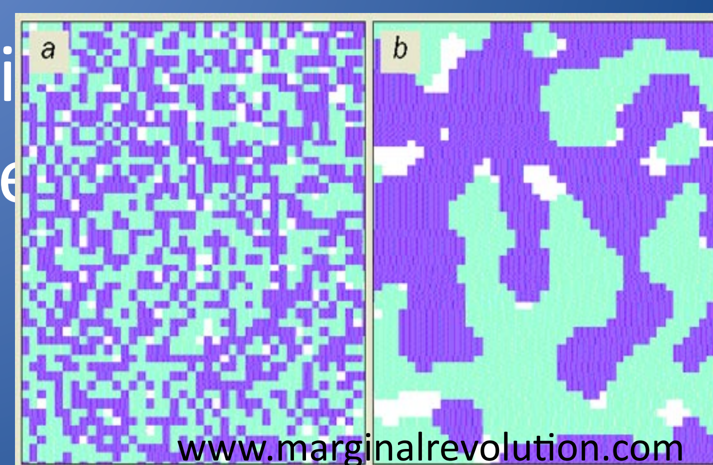
**while** $Time < MaxTime$ **do**
  **for** all event types i **do**
    $\Delta E_i \leftarrow Poisson(r_i \Delta t)$
  **end for**
  Update size of each stock based on which transition events occur.
  Randomly select $\Delta E_i$ agents uniformly from the appropriate stock and transition according to event $E_i$.
  $Time \leftarrow Time + \Delta t$
**end while**

# System Dynamics

- Strengths:
  - Easy model construction and validation with available data
  - Simulation methods computationally efficient
- Weaknesses:
  - Assumes homogenous and well-mixed population
  - Captures only average behavior
  - Assumes mathematical equations capture all feedback structure in system
  - Assumes macro-level behavior is independent of micro-level behavior
  - Difficult to model certain interventions (actions by outsiders) that influence flows in the model

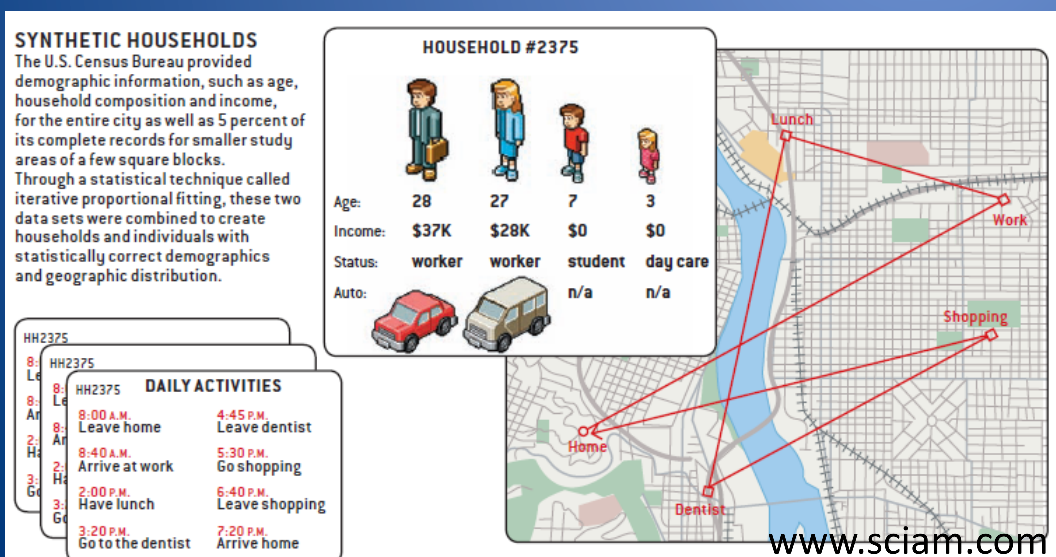
# Agent-Based Modeling

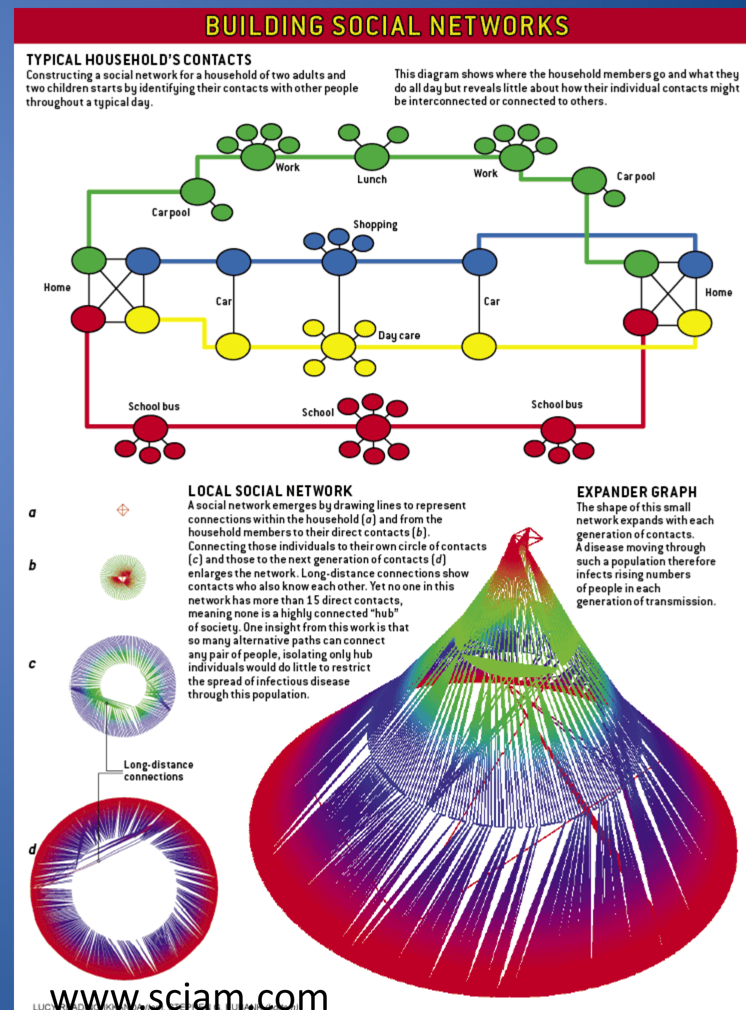
- System modeled as population of heterogeneous agents with evolving state space (e.g. Schelling Segregation Model)
- Agent interactions can cause complex emergent behavior to arise
- Object-oriented programmi
representing interacting age



# Agent-Based Modeling

- Example: EpiSims
  - Highly detailed
  - Virtual laboratory



# Agent-Based Modeling

- Strengths:
  - Allows sophisticated interactions between agents with heterogeneous state space (e.g. contact network)
  - Yields greater and more intuitive information that can be used by researchers and policymakers
  - More "lifelike" than system dynamics models
- Weaknesses:
  - larger state space means poor computational efficiency
  - Model construction is difficult: hard to link observed behavior to local interactions, capture all critical feedback loops
  - Model calibration, validation, and sensitivity analysis require large amounts of data and time

# Embedded (Hybrid) Models

- A complete agent-based model need not be fitted, but individual-level granularity in the model is maintained and heterogeneity in agents can be exploited
- Allows for simulation of novel, complex intervention strategies at the level of agents that might otherwise be difficult or impossible to express succinctly in system dynamics terminology