Training and optimization

Stochastic gradient descent

- Gradient on single example = unbiased sample of true gradient
- Idea: at each iteration sample single example $x^{(t)}$

$$\mathbf{w}^{(t+1)} \leftarrow \mathbf{w}^{(t)} - \underbrace{\lambda}_{\text{step size}} \nabla_{\mathbf{w}} L(h_{\mathbf{w}}(x^{(t)}), y^{(t)})$$

- Con: variance in estimate of gradient → slow convergence, jumping near optimum

Minibatch stochastic gradient descent

- Compute gradient on a small batch of examples
- Same mean (=true gradient), but variance inversely proportional to minibatch size

$$\mathbf{w}^{(t+1)} \leftarrow \mathbf{w}^{(t)} - \lambda \frac{1}{|B^{(t)}|} \sum_{(x,y) \in B^{(t)}} \nabla_{\mathbf{w}} L(h_{\mathbf{w}}(x), y)$$

Momentum

- *Average* multiple gradient steps
- Use *exponential averaging*

$$\mathbf{p}^{(t+1)} \leftarrow (1-\mu)\mathbf{p}^{(t)} - \mu \frac{1}{|B^{(t)}|} \sum_{(x,y)\in B^{(t)}} \nabla_{\mathbf{w}} L(h_{\mathbf{w}}(x), y)$$

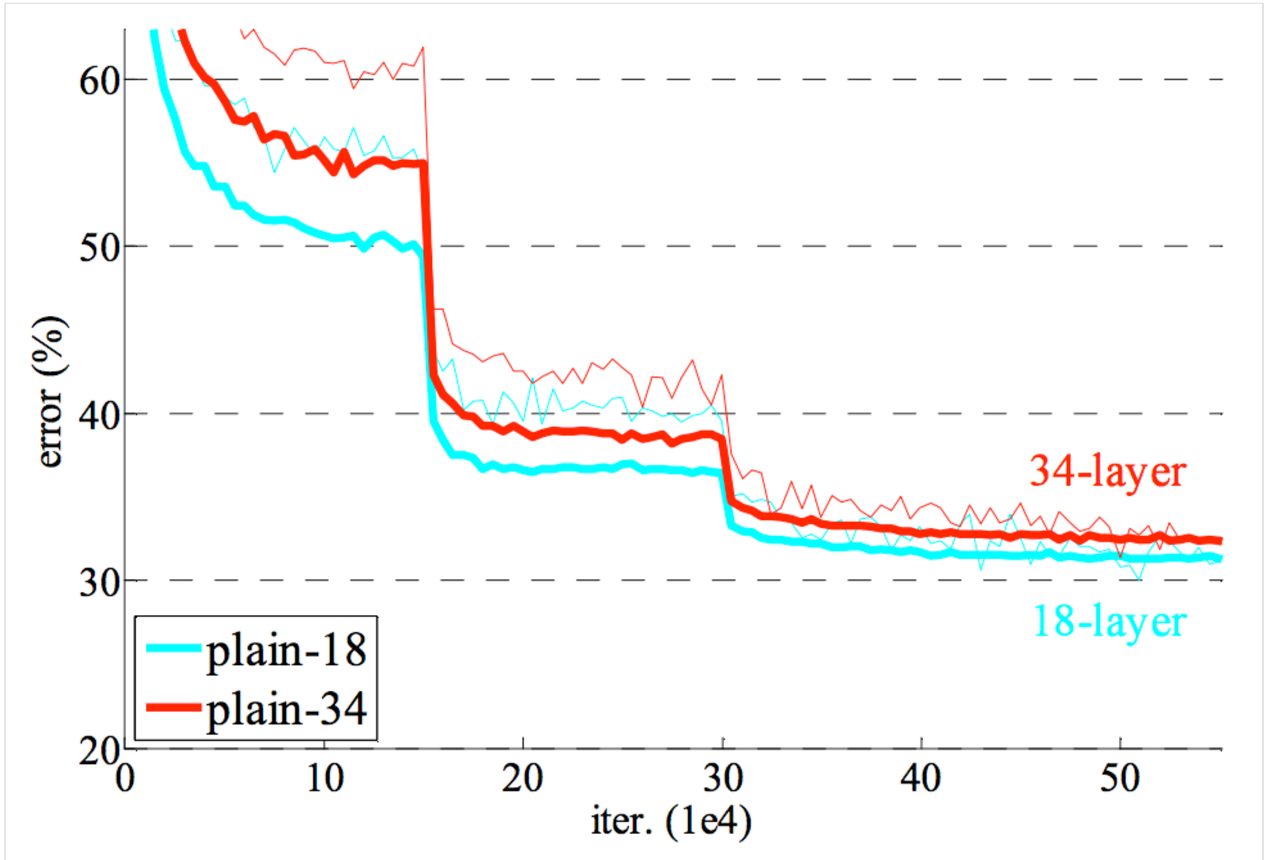
$$\mathbf{w}^{(t+1)} \leftarrow \mathbf{w}^{(t)} - \lambda \mathbf{p}^{(t+1)}$$

Weight decay

- Add $-a\mathbf{w}^{(t)}$ to the gradient
- Prevents $\mathbf{w}^{(t)}$ from growing to infinity
- Equivalent to L2 regularization of weights

Learning rate decay

- Large step size / learning rate
  - Faster convergence initially
  - Bouncing around at the end because of noisy gradients
- Learning rate must be decreased over time
- Usually done in steps

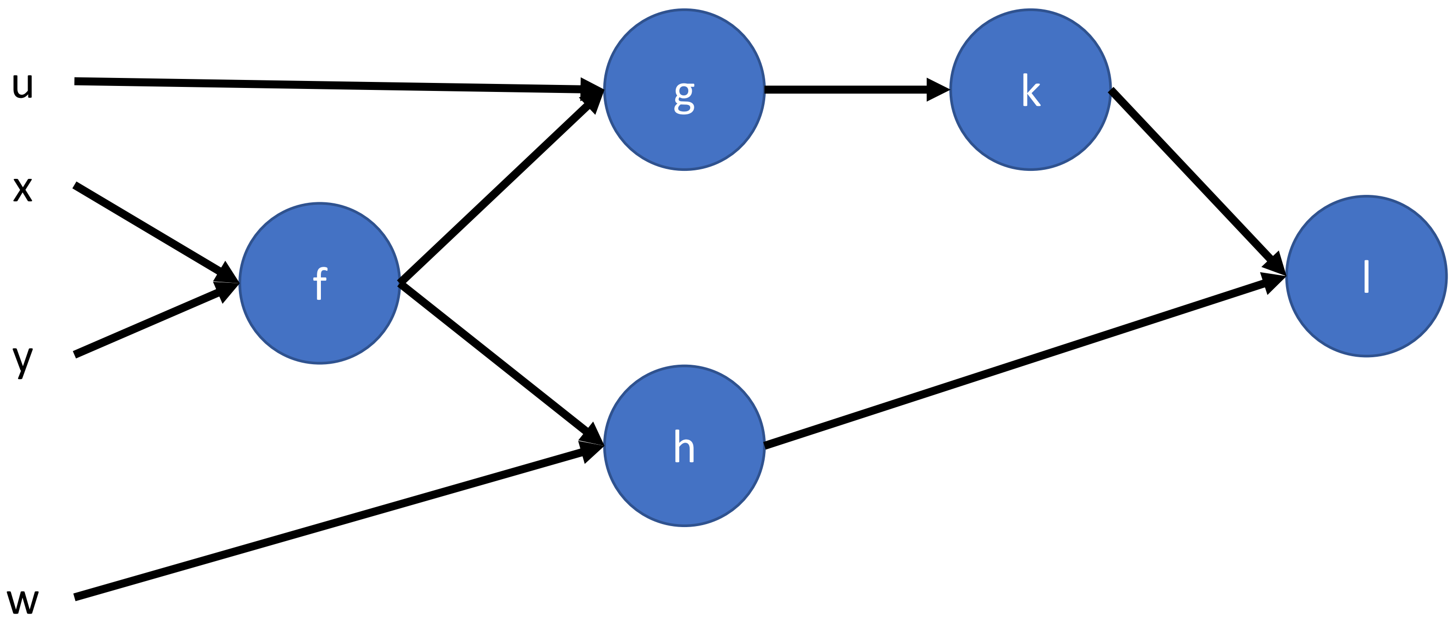


Convolutional network training

- Initialize network
- Sample *minibatch* of images
- Forward pass to compute loss
- Backpropagate loss to compute gradient
- Combine gradient with momentum and weight decay
- Take step according to current learning rate

Beyond sequences: computation graphs

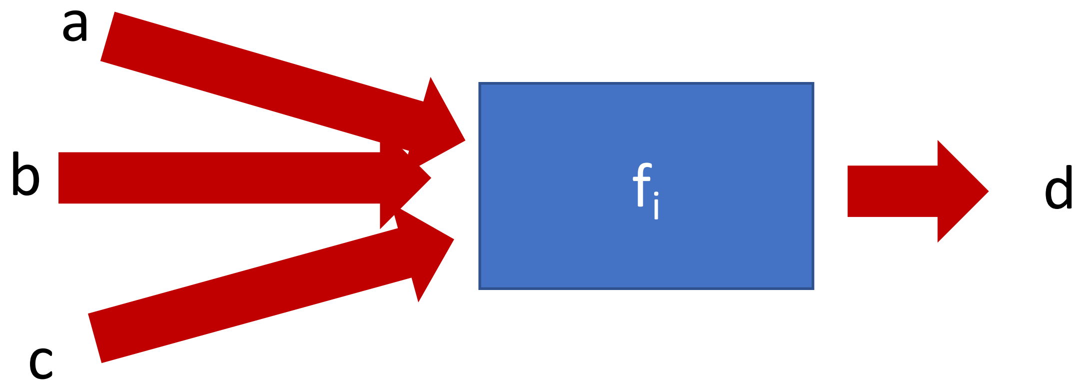
- Multi-layer perceptrons and first convolutional networks were *sequences of functions*
- In general, can have *arbitrary DAGs* of functions



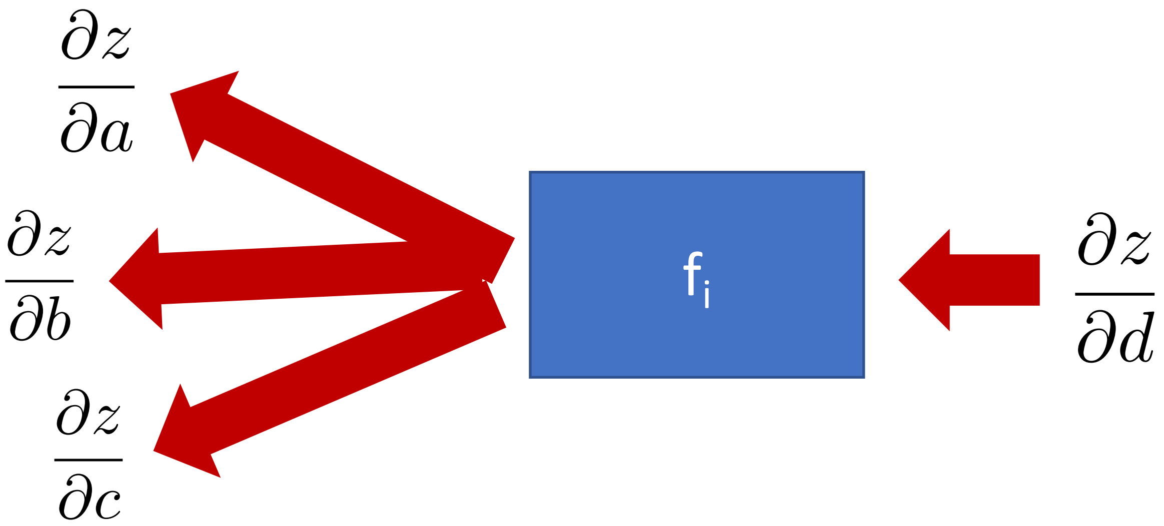
Computation graphs

- Each node implements two methods
  - A “forward”
    - Computes output given input
  - A “backward”
    - Computes derivative of z w.r.t input, given derivative of z w.r.t output

Computation graphs

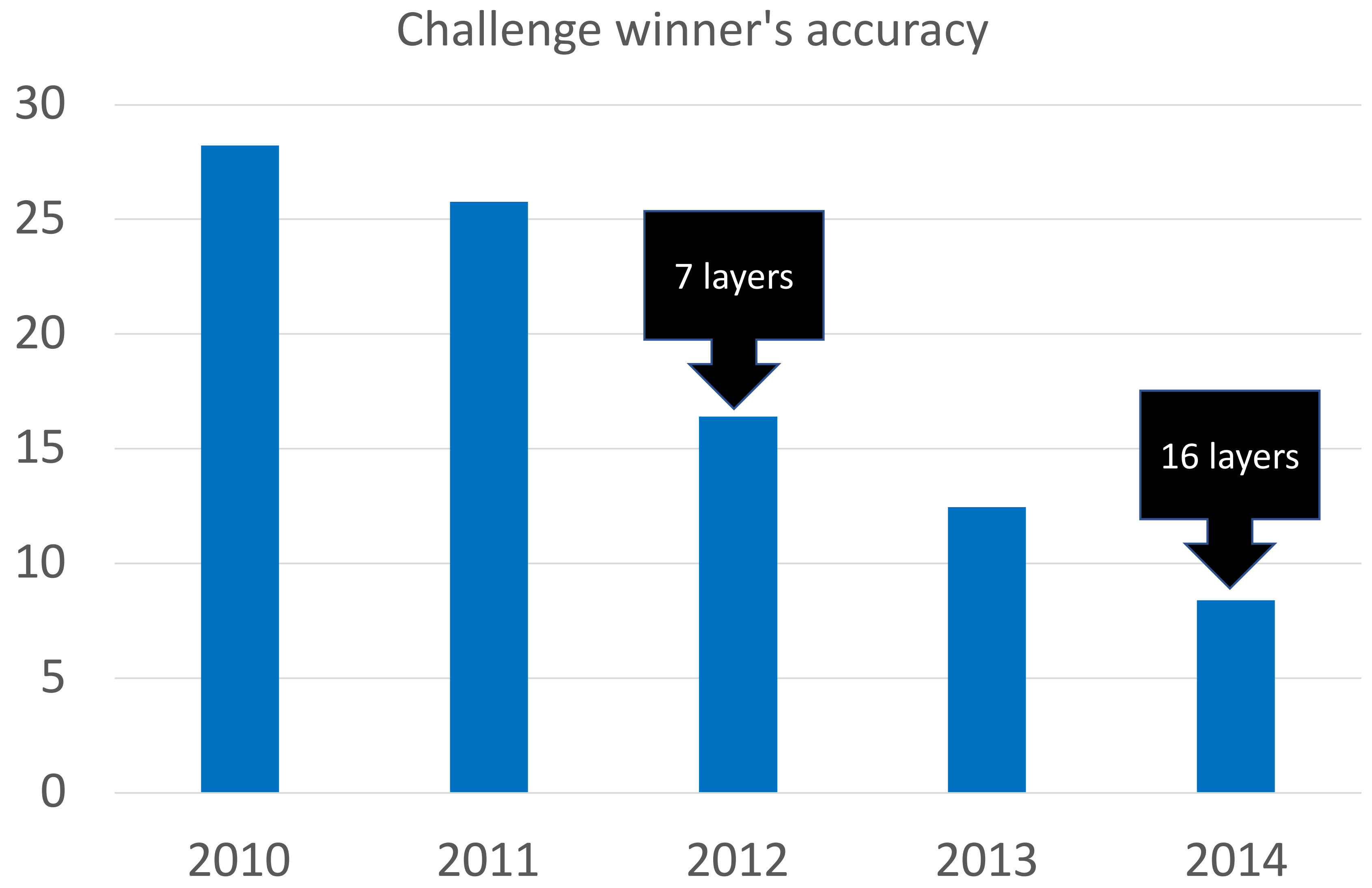


Computation graphs

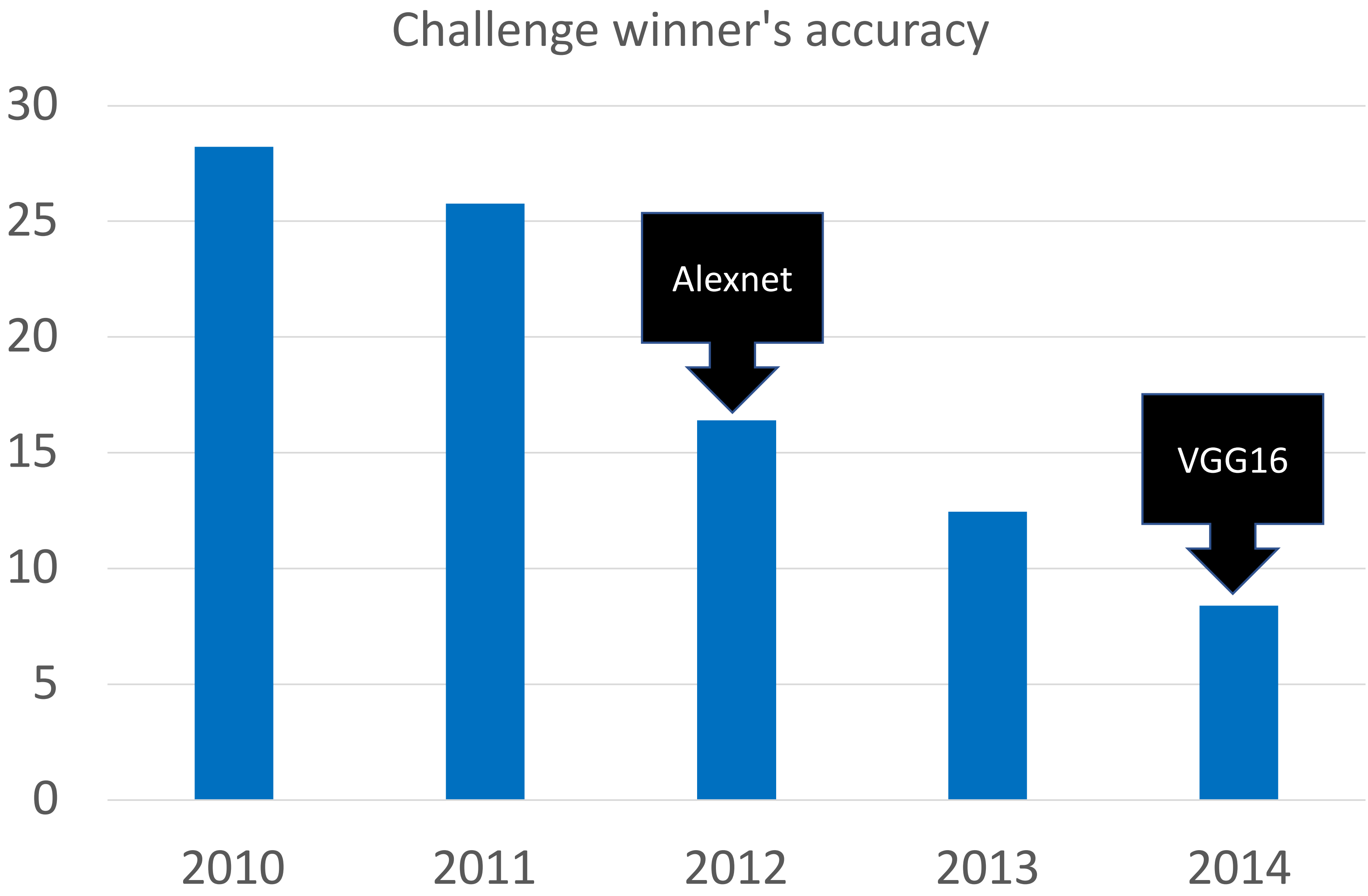


Exploring convnet architectures

Deeper is better



Deeper is better



The VGG pattern

- Every convolution is 3x3, padded by 1
- Every convolution followed by ReLU
- ConvNet is divided into “stages”
  - Layers within a stage: no subsampling
  - Subsampling by 2 at the end of each stage
- Layers within stage have same number of channels
- Every subsampling → double the number of channels

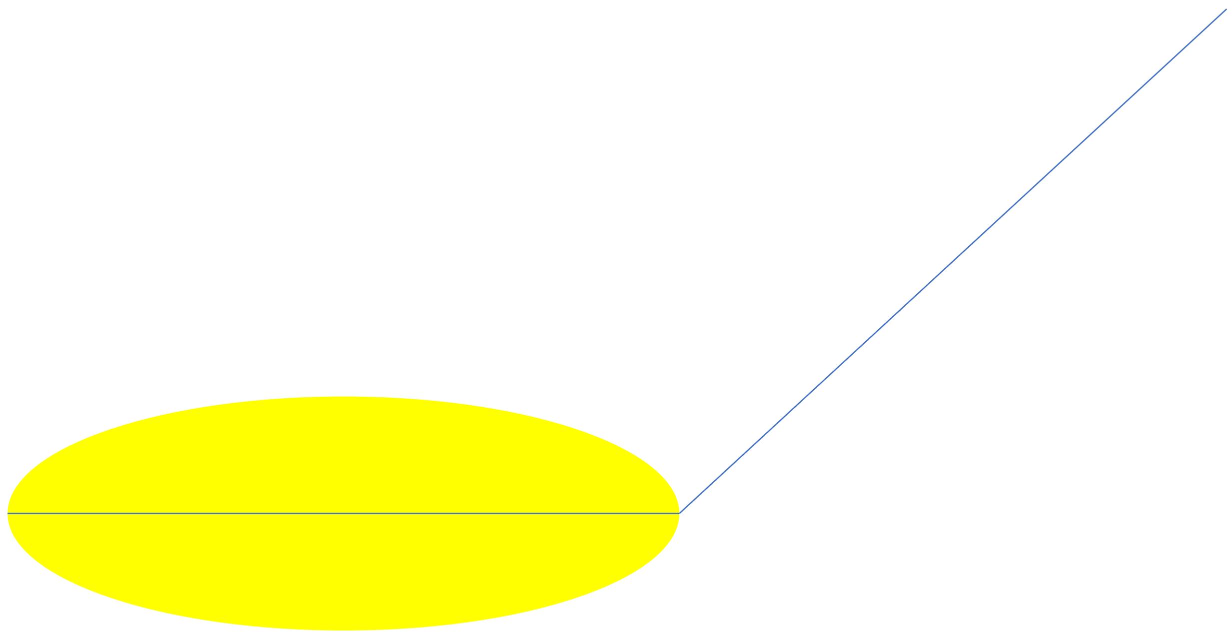
Challenges in training: exploding / vanishing gradients

- Vanishing / exploding gradients

$$\frac{\partial z}{\partial z_i} = \frac{\partial z}{\partial z_{n-1}} \frac{\partial z_{n-1}}{\partial z_{n-2}} \cdots \frac{\partial z_{i+1}}{\partial z_i}$$

  - If each term is (much) greater than 1 → *explosion of gradients*
  - If each term is (much) less than 1 → *vanishing gradients*

Challenges in training: dependence on init



Solutions

- Careful init
- Batch normalization
- Residual connections

Careful initialization

- Key idea: want variance to remain approx. constant
  - Variance increases in backward pass => exploding gradient
  - Variance decreases in backward pass => vanishing gradient
- "MSRA initialization"
  - weights = Gaussian with 0 mean and variance = 2/(k*k*d)

Delving Deep into Rectifiers: Surpassing Human-Level Performance on ImageNet Classification. K. He, X. Zhang, S. Ren, J. Sun

Batch normalization

- Key idea: normalize so that each layer output has zero mean and unit variance
  - Compute mean and variance for each channel
  - Aggregate over batch
  - Subtract mean, divide by std
- Need to reconcile train and test
  - No ”batches” during test
  - After training, compute means and variances on train set and store

Batch Normalization: Accelerating Deep Network Training by Reducing Internal Covariate Shift. S. Ioffe, C. Szegedy. In *ICML,* 2015.

Residual connections

- In general, gradients tend to vanish
- Key idea: allow gradients to flow unimpeded

$$z_{i+1} = f_{i+1}(z_i, w_{i+1})$$

$$\frac{\partial z_{i+1}}{\partial z_i} = \frac{\partial f_{i+1}(z_i, w_{i+1})}{\partial z_i}$$

$$\frac{\partial z}{\partial z_i} = \frac{\partial z}{\partial z_{n-1}} \frac{\partial z_{n-1}}{\partial z_{n-2}} \cdots \frac{\partial z_{i+1}}{\partial z_i}$$

Residual connections

- In general, gradients tend to vanish
- Key idea: allow gradients to flow unimpeded

$$z_{i+1} = g_{i+1}(z_i, w_{i+1}) + z_i$$

$$\frac{\partial z_{i+1}}{\partial z_i} = \frac{\partial g_{i+1}(z_i, w_{i+1})}{\partial z_i} + I$$

$$\frac{\partial z}{\partial z_i} = \frac{\partial z}{\partial z_{n-1}} \frac{\partial z_{n-1}}{\partial z_{n-2}} \cdots \frac{\partial z_{i+1}}{\partial z_i}$$

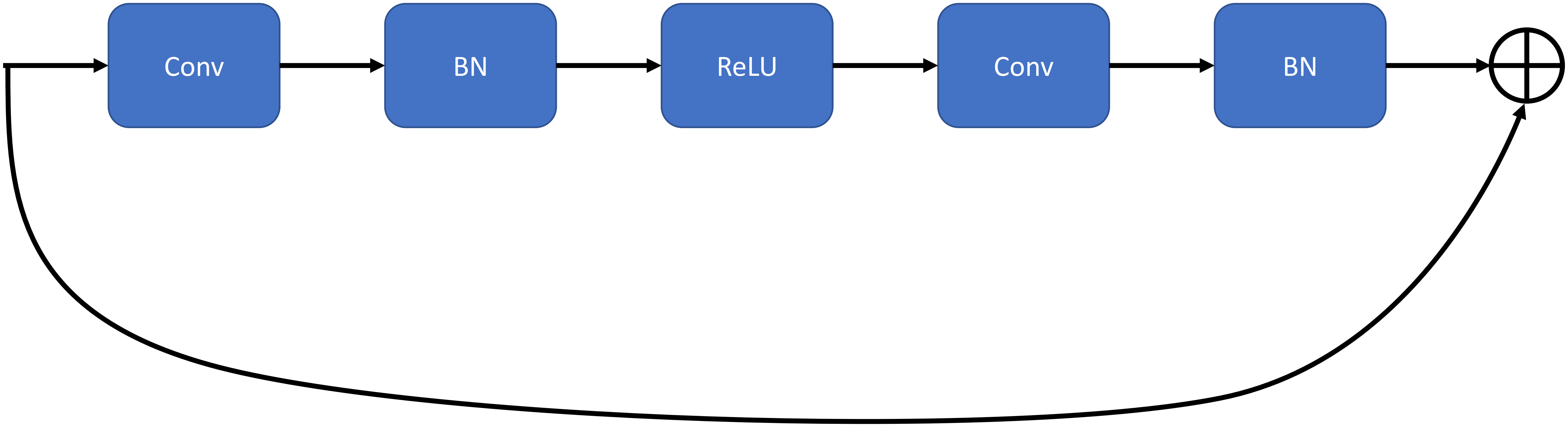
Residual connections

- Assumes all z_i have the same size
- True within a stage
- Across stages?
  - Doubling of feature channels
  - Subsampling
- Increase channels by 1x1 convolution
- Decrease spatial resolution by subsampling

$$z_{i+1} = g_{i+1}(z_i, w_{i+1}) + \text{subsample}(W z_i)$$

A residual block

- Instead of single layers, have residual connections over block



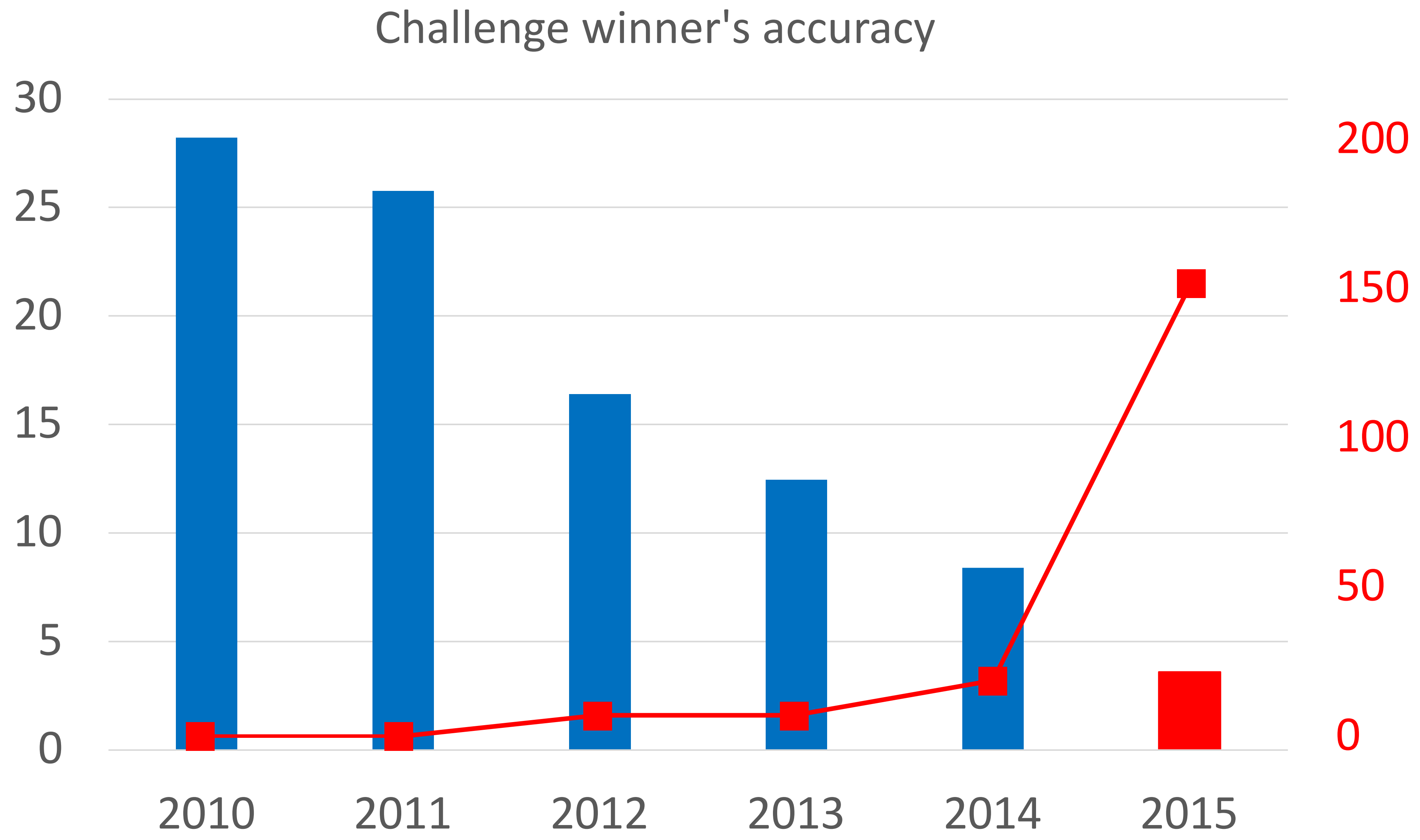
Bottleneck blocks

- Problem: When channels increases, 3x3 convolutions introduce many parameters
  - $3 \times 3 \times c^2$
- Key idea: use 1x1 to project to lower dimensionality, do convolution, then come back
  - $c \times d \ + \ 3 \times 3 \times d^2 \ + \ d \times c$

The ResNet pattern

- Decrease resolution substantially in first layer
  - Reduces memory consumption due to intermediate outputs
- Divide into stages
  - maintain resolution, channels in each stage
  - halve resolution, double channels between stages
- Divide each stage into residual blocks
- At the end, compute average value of each channel to feed linear classifier

Putting it all together - Residual networks



Computational complexity

Analyzing computational complexity

- What is the computational complexity of a single convolutional layer?
  - $h \times w \times c$ input and output
  - $k \times k$ kernel
- Space:
  - Input/output: hwc
  - Filters: k^2c^2
- Time (Flops): hwk^2c^2

Reducing computational complexity

- …while maintaining accuracy?
- Multiple ways:
  - Make architecture *a priori* cheaper
  - Make *weights* and *operations* cheaper
  - Make inference adaptive

Cheaper convolutional blocks

- Standard convolution:
  - Each filter operates on all channels
  - Single $k \times k$ filter operating on c channels producing one output channel: $k^2 c$ parameters, cost
  - c such filters: $k^2 c^2$ parameters, cost
- *Depthwise separable* convolution
  - Each filter operates on a single channel
  - c filters operating on c channels: $k^2 c$ parameters, cost
  - But each channel is independently processed
  - Add a 1x1 convolution at the end with cost c^2 : $k^2 c + c^2$ parameters

Cheaper convolutional blocks

- Depthwise separable convolutions are specific instance of more general idea: *grouped convolutions*
- Grouped convolutions in original AlexNet network
- Grouped convolution:
  - Divide input channels into g groups
  - Apply convolutional layers on each group independently
  - Concatenate

Grouped and depth-wise convolutions

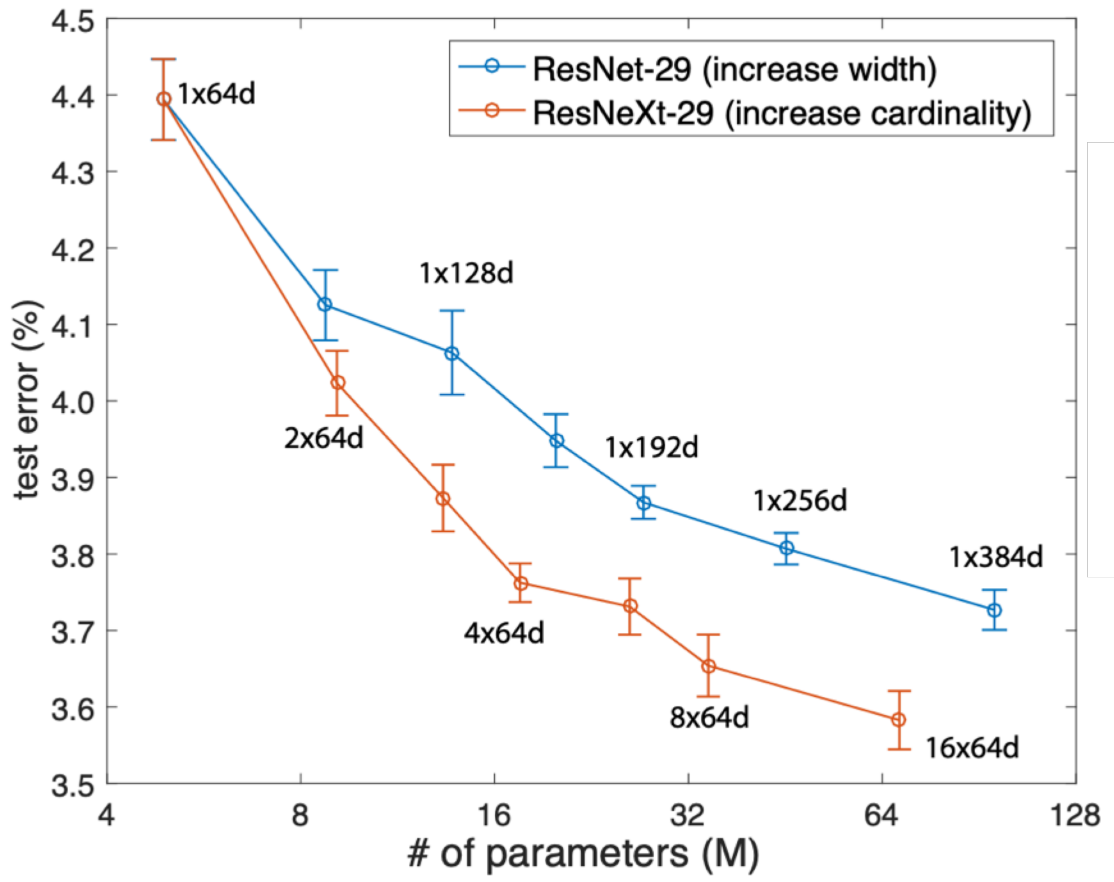


Table 4. Depthwise Separable vs Full Convolution MobileNet

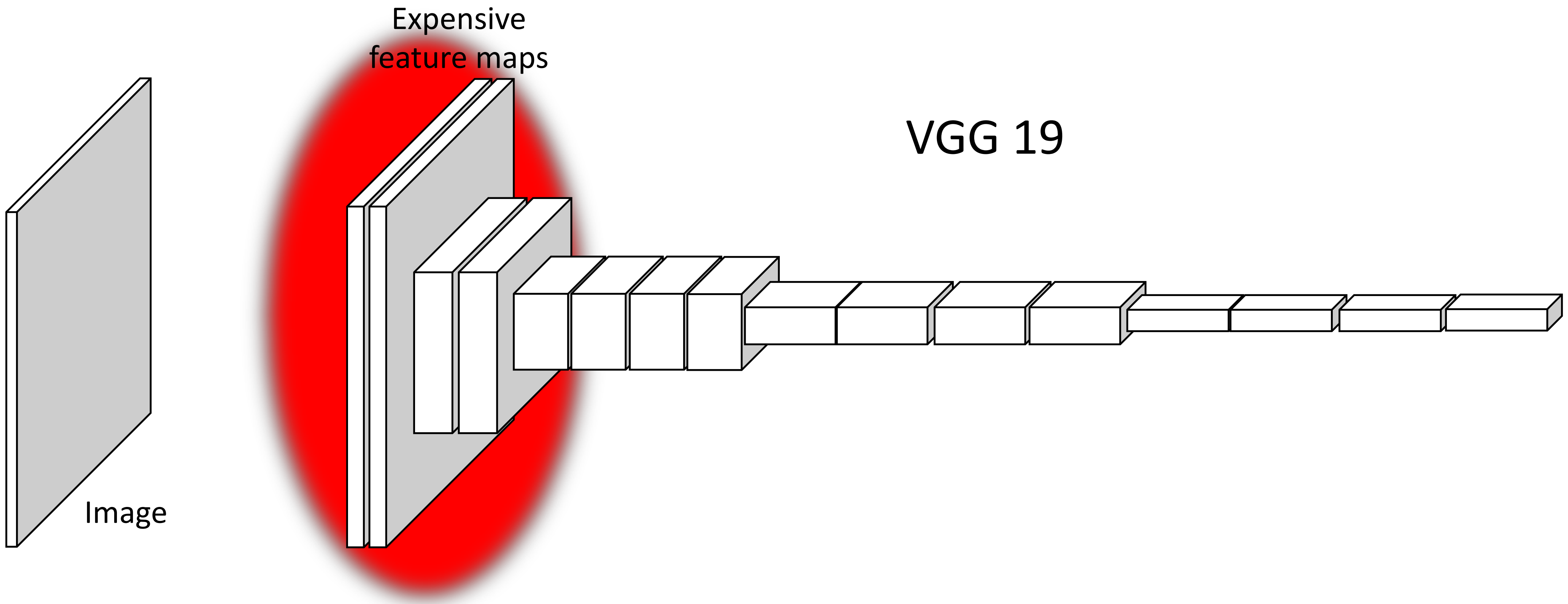
Model	ImageNet Accuracy	Million Mult-Adds	Million Parameters
Conv MobileNet	71.7%	4866	29.3
MobileNet	70.6%	569	4.2

Xie, Saining, et al. "Aggregated residual transformations for deep neural networks." *Proceedings of the IEEE conference on computer vision and pattern recognition*. 2017.

Howard, Andrew G., et al. "Mobilenets: Efficient convolutional neural networks for mobile vision applications." *arXiv preprint arXiv:1704.04861* (2017).

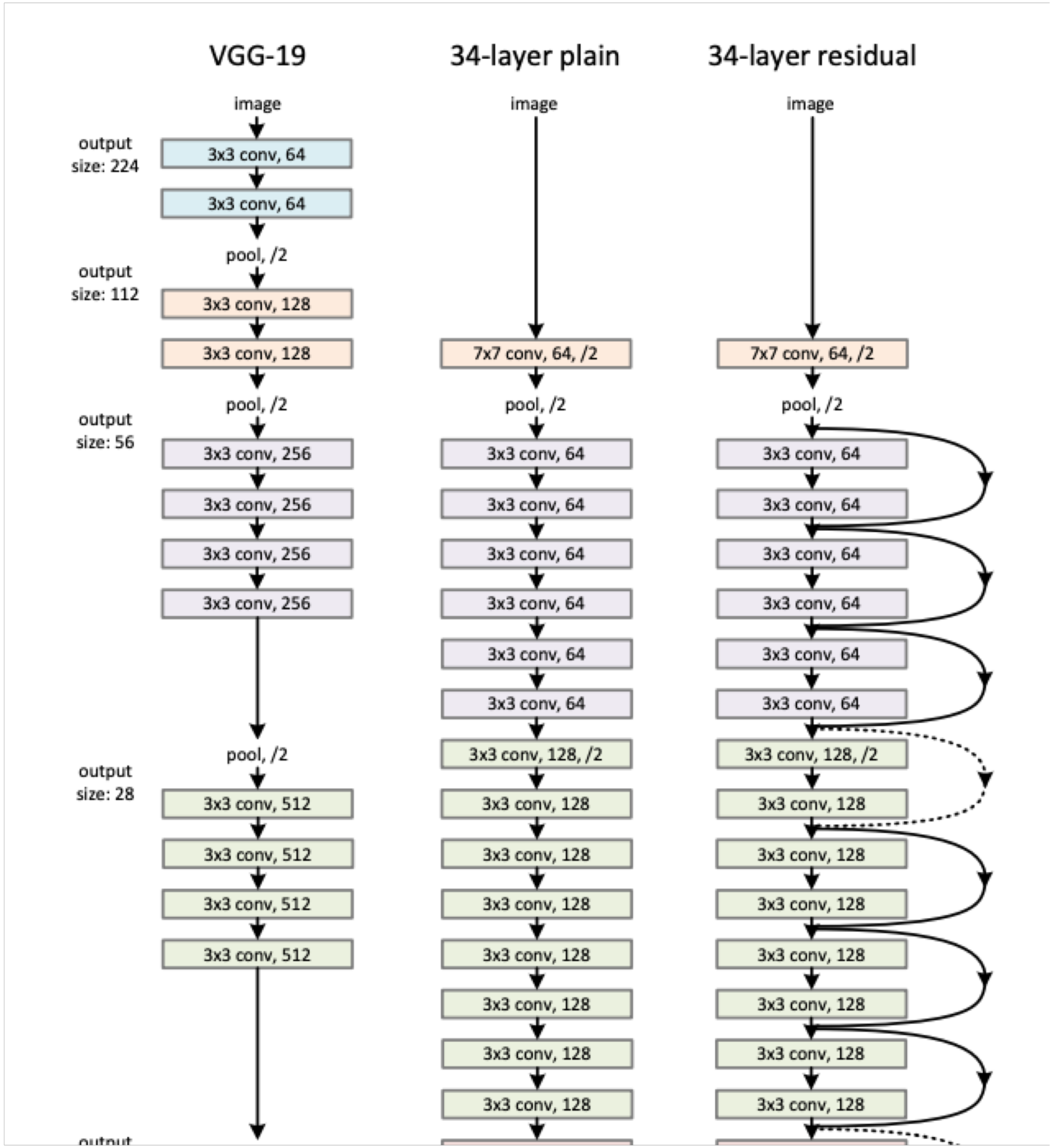
Other architectural changes

- Biggest memory consumption: large feature maps



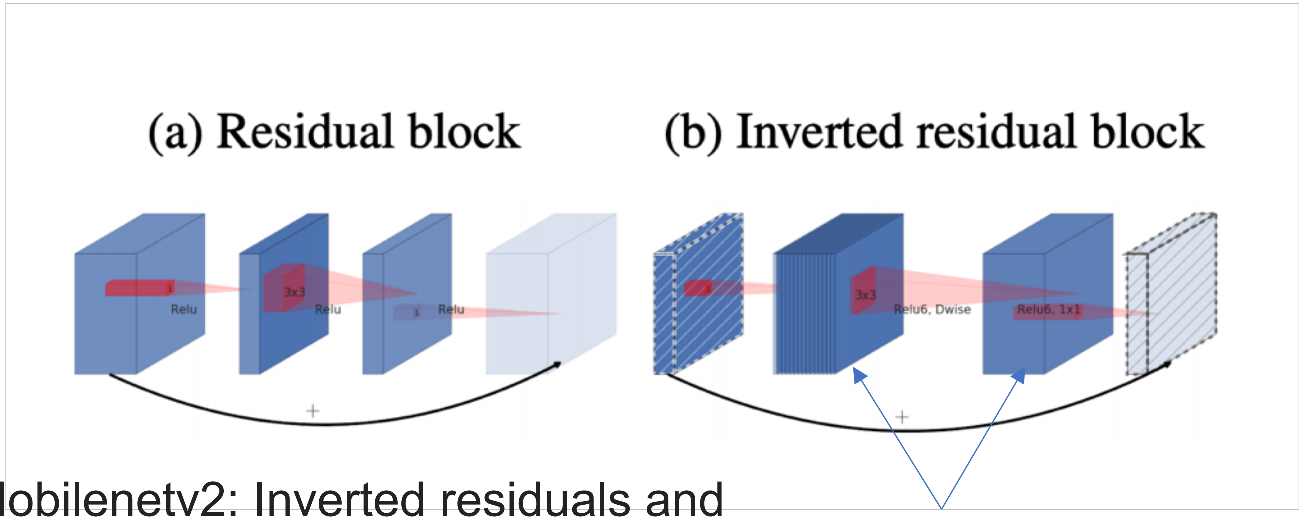
Other architectural changes

- Biggest memory consumption: large feature maps
- Simple solution: reduce resolution early



Other architectural changes

- Biggest memory consumption: large feature maps
- Simple solution (ResNet):
  - Reduce resolution drastically (/4) early
- More sophisticated changes: Inverted residuals (MobileNet v2)



Dispose of these

Sandler, Mark, et al. "Mobilenetv2: Inverted residuals and linear bottlenecks." *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*. 2018.

Other kinds of connections

- DenseNets
  - Replace addition of residuals with concatenation
  - Alternative to solving vanishing gradient problem
  - Should *increase* number of parameters, but *decreases* them
  - Better re-use of features

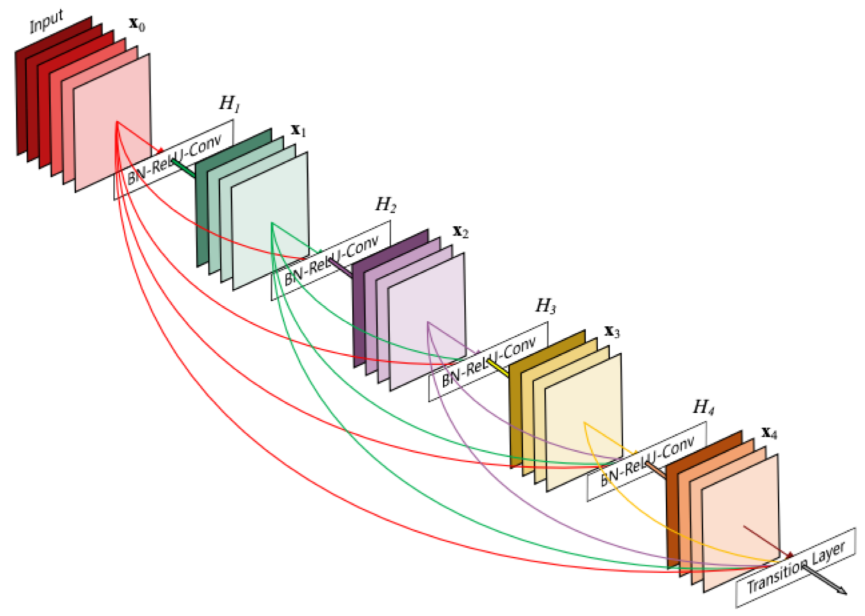
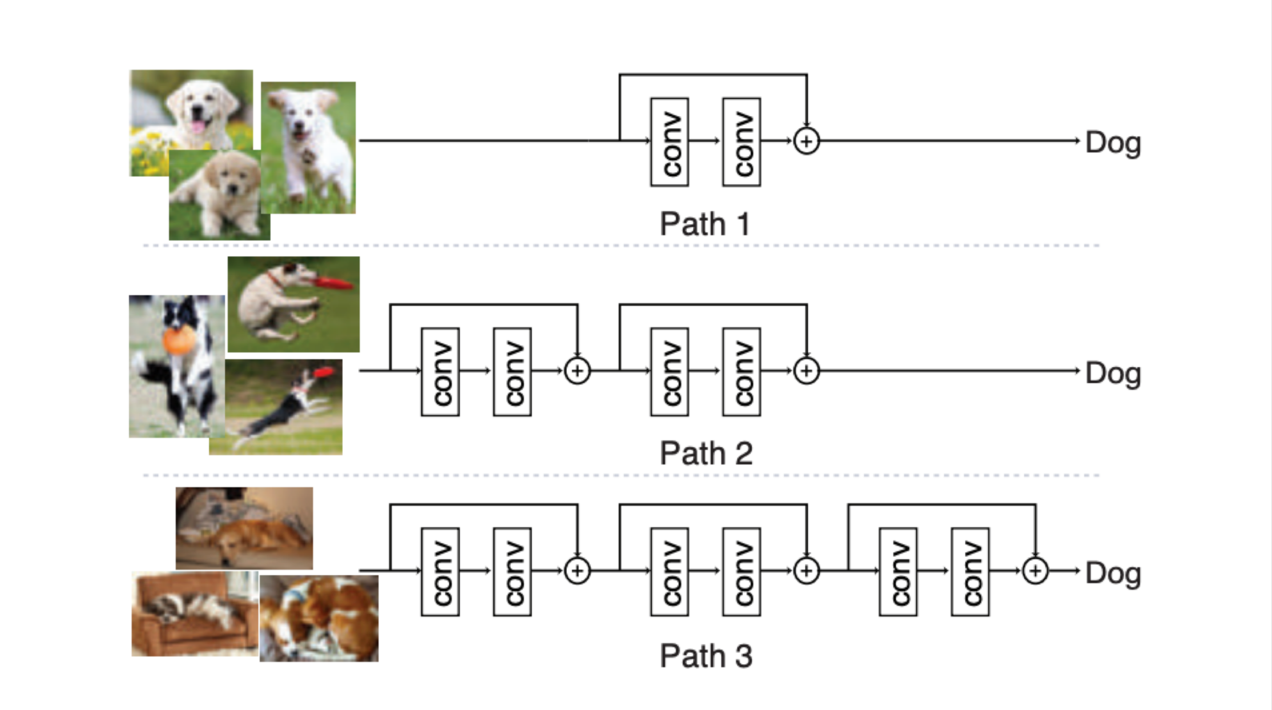


Figure 1: A 5-layer dense block with a growth rate of $k = 4$. Each layer takes all preceding feature-maps as input.

Adaptive inference

- Some examples are harder than others
- Should be able to use different amounts of computation for different examples
- Version 1: skip some residuals



Veit, Andreas, and Serge Belongie. "Convolutional networks with adaptive inference graphs." *Proceedings of the European Conference on Computer Vision (ECCV)*. 2018.

Wu, Zuxuan, et al. "Blockdrop: Dynamic inference paths in residual networks." *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*. 2018.

Adaptive inference

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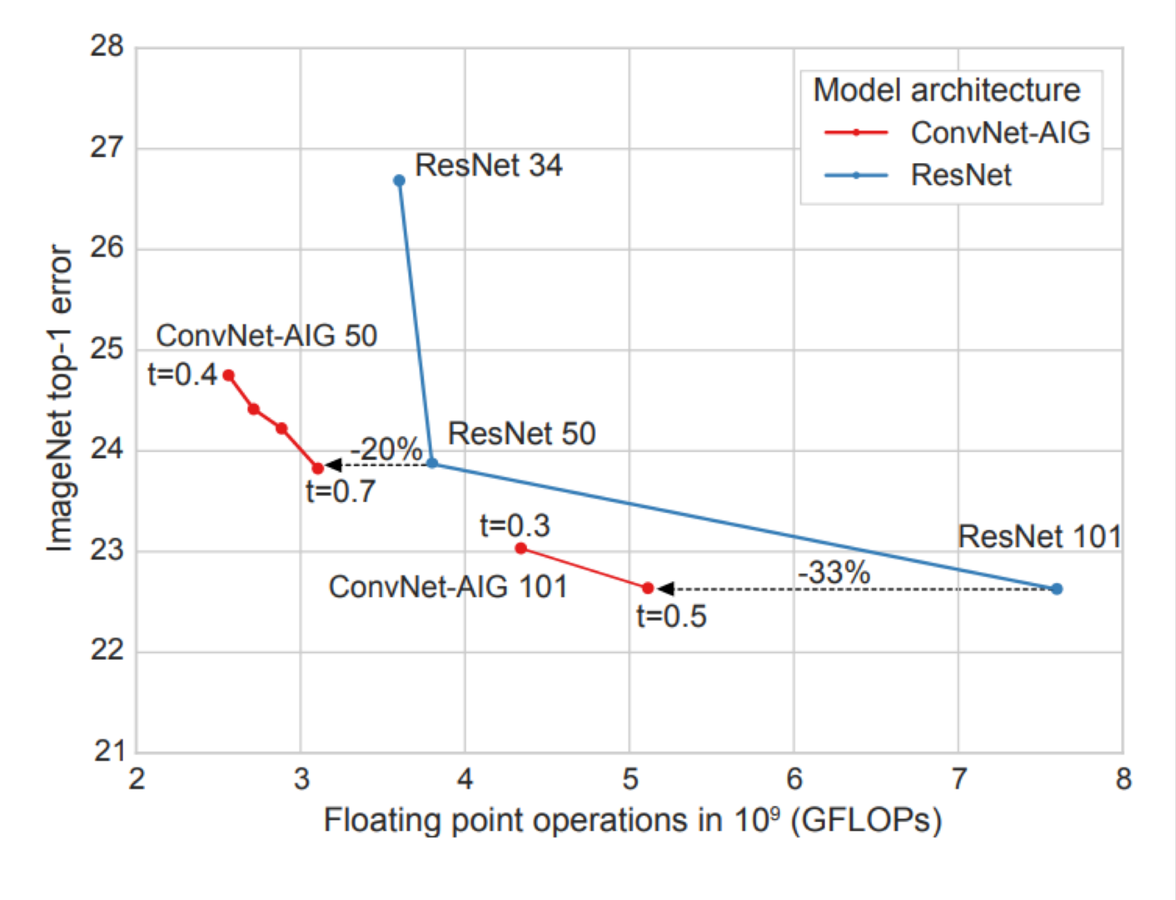

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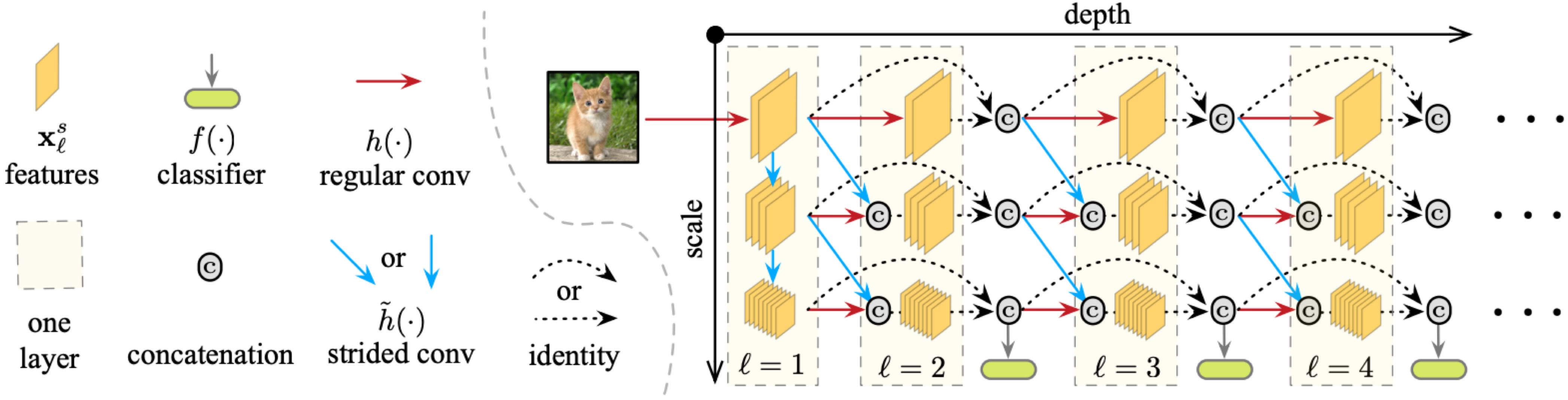
Wu, Zuxuan, et al. "Blockdrop: Dynamic inference paths in residual networks." *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*. 2018.

Adaptive inference

- Some examples are harder than others
- Should be able to use different amounts of computation for different examples
- Version 2: reduce resolution at different rates



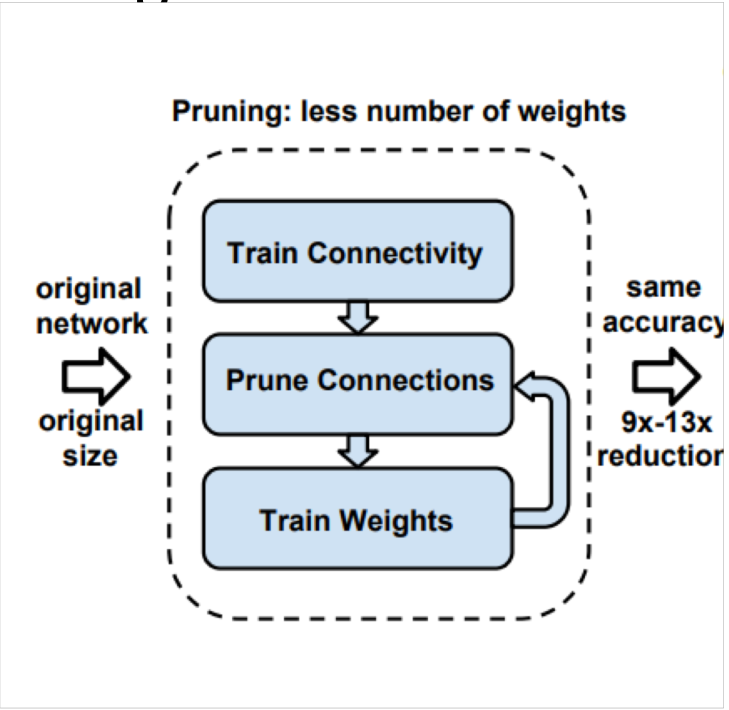
Huang, Gao, et al. "Multi-scale dense networks for resource efficient image classification." *ICLR 2018*

Compressing model weights

- All of model storage: filters
- Flops also scale with non-zero entries in filters (in principle)
- Compress filters
  - Sparsify them
  - Represent them with fewer bits

Pruning network connections

- Simple approach: prune weights that are below a threshold
- Retrain rest of the weights
- Repeat



Han, Song, Huizi Mao, and William J. Dally. "Deep compression: Compressing deep neural networks with pruning, trained quantization and huffman coding." *ICLR, 2016*

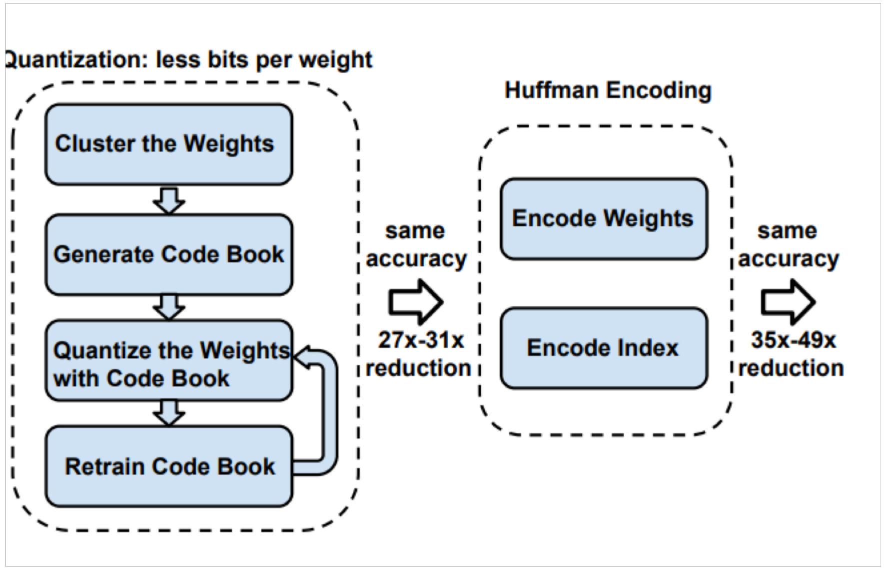
Pruning network connections

- Simple approach: prune weights that are below a threshold
- Retrain rest of the weights
- Repeat
- Sophisticated alternative
  - Train with *regularizer* that penalizes expensive connections
  - Prune
  - If model within budget, expand and retrain

Gordon, Ariel, et al. "Morphnet: Fast & simple resource-constrained structure learning of deep networks." *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*. 2018.

Filter quantization

- Two questions:
  - How do we quantize?
  - Quantization → discrete values. How do we optimize?
- Example 1: *cluster*
  - Weights → indices into dictionary
  - Update dictionary elements as parameters.



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Filter quantization

- Two questions:
  - How do we quantize?
  - Quantization → discrete values. How do we optimize?
- Example 2: *binarize/ternarize*
  - Weights → binary/ternary, + real-valued scale
  - Parameter updates happen in real space

Zhu, Chenzhuo, et al. "Trained ternary quantization." *ICLR, 2017*.

Rastegari, Mohammad, et al. "Xnor-net: Imagenet classification using binary convolutional neural networks." *European Conference on Computer Vision*. Springer, Cham, 2016.