## Multilayer perceptrons

- Key idea: build complex functions by composing simple functions



## Multilayer perceptrons

- Key idea: build complex functions by composing simple functions
- Caveat: simple functions must include non-linearities
- $W(U(V x))=(W U V) x$


## Reducing capacity



Reducing capacity

## Idea 1: local connectivity

- Inputs and outputs are feature maps
- Pixels only related to nearby pixels



## Idea 2: Translation invariance

- Pixels only related to nearby pixels



## Local connectivity + translation invariance = convolution

| 5.4 | 0.1 | 3.6 |
| :---: | :---: | :---: |
| 1.8 | 2.3 | 4.5 |
| 1.1 | 3.4 | 7.2 |



## Local connectivity + translation invariance = convolution

| 5.4 | 0.1 | 3.6 |
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## Local connectivity + translation invariance $=$ convolution

| 5.4 | 0.1 | 3.6 |
| :---: | :---: | :---: |
| 1.8 | 2.3 | 4.5 |
| 1.1 | 3.4 | 7.2 |



Feature map


## Convolution as a primitive



## Invariance to distortions



## Invariance to distortions



Invariance to distortions: Pooling


## Invariance to distortions: Subsampling



## Convolution subsampling convolution



## Convolution subsampling convolution

- Convolution in earlier steps detects more local patterns less resilient to distortion
- Convolution in later steps detects more global patterns more resilient to distortion
- Subsampling allows capture of larger, more invariant patterns


## Convolutional networks



## Convolutional networks



## Convolutional networks



## Vagaries of optimization

- Non-convex
- Local optima
- Sensitivity to initialization
- Vanishing / exploding gradients

$$
\frac{\partial z}{\partial z_{i}}=\frac{\partial z}{\partial z_{n-1}} \frac{\partial z_{n-1}}{\partial z_{n-2}} \ldots \frac{\partial z_{i+1}}{\partial z_{i}}
$$

- If each term is (much) greater than $1 \rightarrow$ explosion of gradients
- If each term is (much) less than $1 \rightarrow$ vanishing gradients


## Vanishing and exploding gradients

$$
\begin{gathered}
\frac{\partial \mathbf{z}}{\partial \mathbf{z}_{i}}=\frac{\partial \mathbf{z}}{\partial \mathbf{z}_{n-1}} \frac{\partial \mathbf{z}_{n-1}}{\partial \mathbf{z}_{n-2}} \cdots \frac{\partial \mathbf{z}_{i+1}}{\partial \mathbf{z}_{i}} \\
\frac{\partial L}{\partial \mathbf{z}_{i}}=\frac{\partial L}{\partial \mathbf{z}} \frac{\partial \mathbf{z}}{\partial \mathbf{z}_{i}} \\
\lambda_{\min }\left(\frac{\partial \mathbf{z}}{\partial \mathbf{z}_{i}}\right) \frac{\partial L}{\partial \mathbf{z}} \leq \frac{\partial L}{\partial \mathbf{z}} \frac{\partial \mathbf{z}}{\partial \mathbf{z}_{i}} \leq \lambda_{\max }\left(\frac{\partial \mathbf{z}}{\partial \mathbf{z}_{i}}\right) \frac{\partial L}{\partial \mathbf{z}} \\
\lambda_{\max }(U V) \leq \lambda_{\max }(U) \lambda_{\max }(V) \\
\lambda_{\min }(U V) \geq \lambda_{\min }(U) \lambda_{\min }(V) \\
\lambda_{\max }\left(A^{n}\right)=\lambda_{\max }(A)^{n} \\
\lambda_{\min }\left(A^{n}\right)=\lambda_{\min }(A)^{n}
\end{gathered}
$$

## Sigmoids cause vanishing gradients



## Convolution subsampling convolution



## Rectified Linear Unit (ReLU)

- max (x,0)
- Also called half-wave rectification (signal processing)


## Image Classification

## ImageNet

## - 1000 categories <br> - ~1000 instances per category



Olga Russakovsky*, Jia Deng*, Hao Su, Jonathan Krause, Sanjeev Satheesh, Sean Ma, Zhiheng Huang, Andrej Karpathy, Aditya Khosla, Michael Bernstein, Alexander C. Berg and Li Fei-Fei. (* = equal contribution) ImageNet Large Scale Visual Recognition Challenge. International Journal of Computer Vision, 2015.

## Challenge winner's accuracy



Transfer learning

## Transfer learning with convolutional networks



## Transfer learning with convolutional networks

| Dataset | Non-Convnet <br> Method | Non-Convnet <br> perf | Pretrained <br> convnet + <br> classifier | Improvement |
| :--- | :--- | :--- | :--- | :--- |
| Caltech 101 | MKL | 84.3 | 87.7 | +3.4 |
| VOC 2007 | SIFT+FK | 61.7 | 79.7 | +18 |
| CUB 200 | SIFT+FK | 18.8 | 61.0 | +42.2 |
| Aircraft | SIFT+FK | 61.0 | 45.0 | -16 |
| Cars | SIFT+FK | 59.2 | 36.5 | -22.7 |

## Why transfer learning?

- Availability of training data
- Computational cost
- Ability to pre-compute feature vectors and use for multiple tasks
- Con: NO end-to-end learning

Finetuning


## Finetuning



## Finetuning

| Dataset | Non- <br> Convnet <br> Method | Non- <br> Convnet <br> perf | Pretrained <br> convnet + <br> classifier | Finetuned <br> convnet | Improvem <br> ent |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Caltech <br> 101 | MKL | 84.3 | 87.7 | 88.4 | +4.1 |
| VOC 2007 | SIFT+FK | 61.7 | 79.7 | 82.4 | +20.7 |
| CUB 200 | SIFT+FK | 18.8 | 61.0 | 70.4 | +51.6 |
| Aircraft | SIFT+FK | 61.0 | 45.0 | 74.1 | +13.1 |
| Cars | SIFT+FK | 59.2 | 36.5 | 79.8 | +20.6 |

