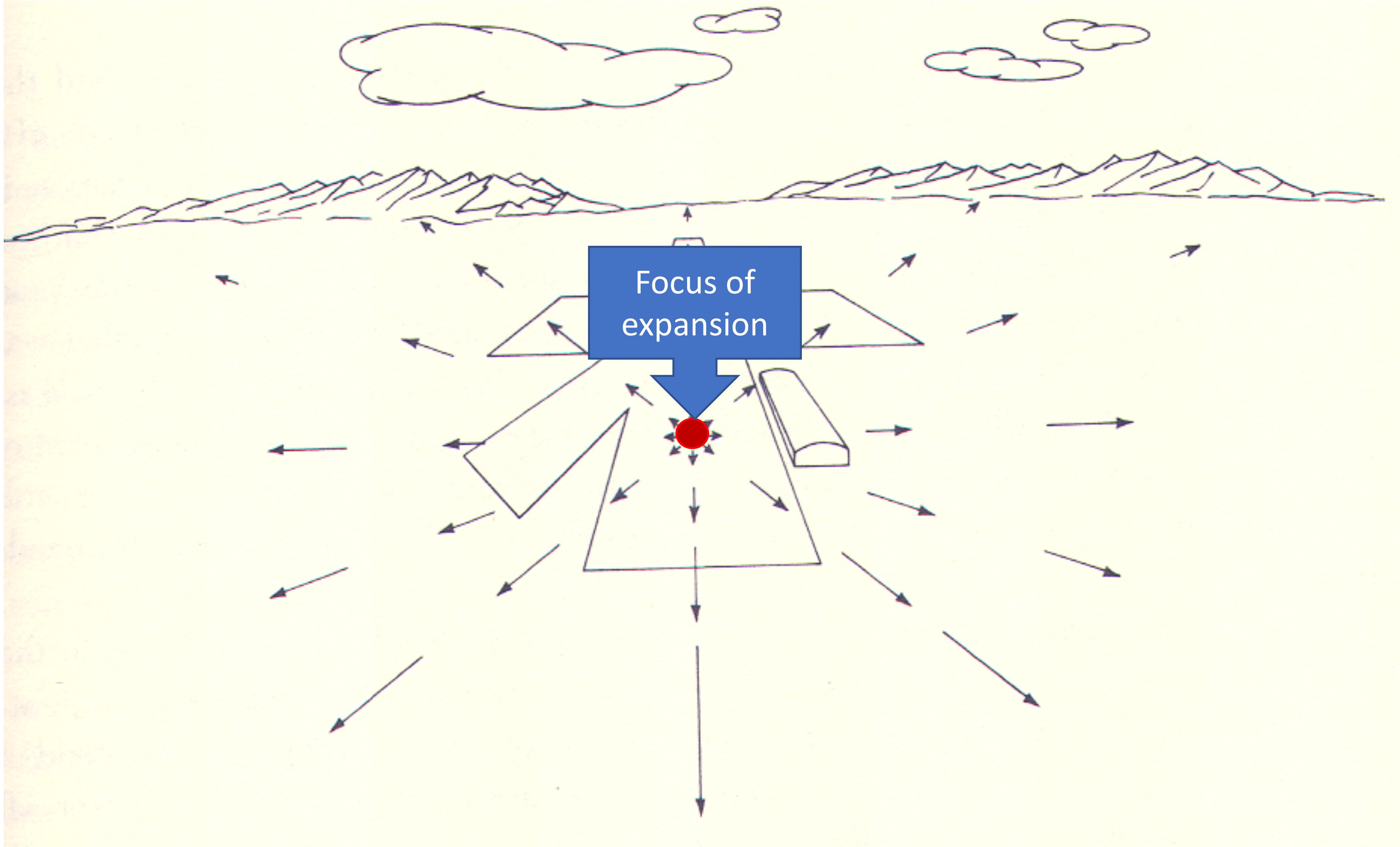
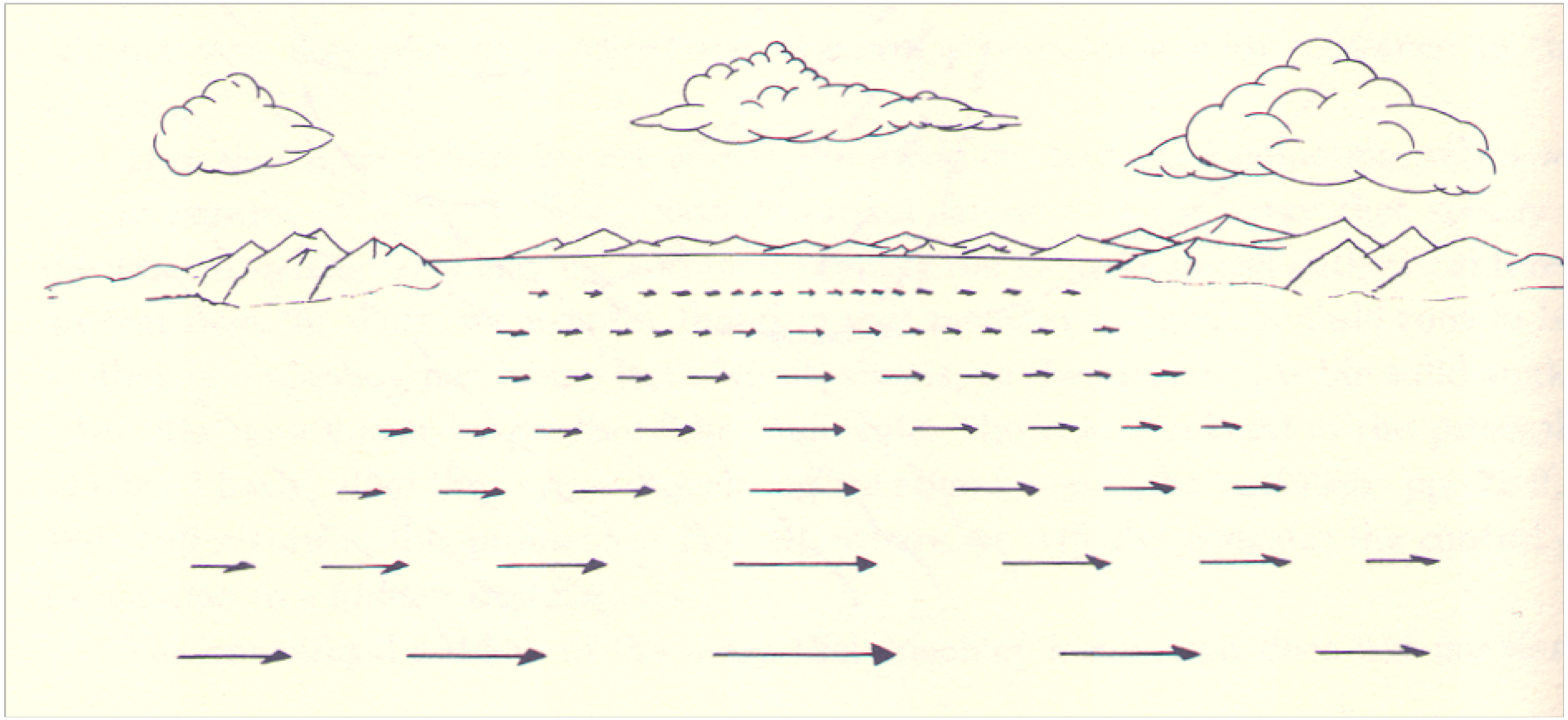
Optical flow



J. J. Gibson



Optical flow due to camera motion

- Consider camera translating and rotating

$$\mathbf{P} = (X, Y, Z)^T$$

$$x = \frac{X}{Z} \qquad y = \frac{Y}{Z}$$

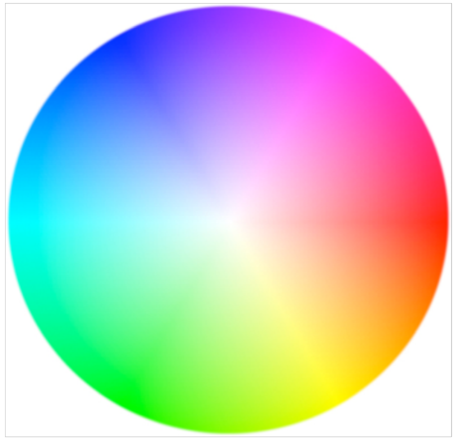
$$\dot{\mathbf{P}} = -\mathbf{t} - \omega \times \mathbf{P}$$

Optical flow due to camera motion

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \frac{1}{Z}\begin{bmatrix} -1 & 0 & x \\ 0 & -1 & y \end{bmatrix}\begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} + \begin{bmatrix} xy & -(1+x^2) & y \\ 1+y^2 & -xy & -x \end{bmatrix}\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

Optical flow for moving scenes

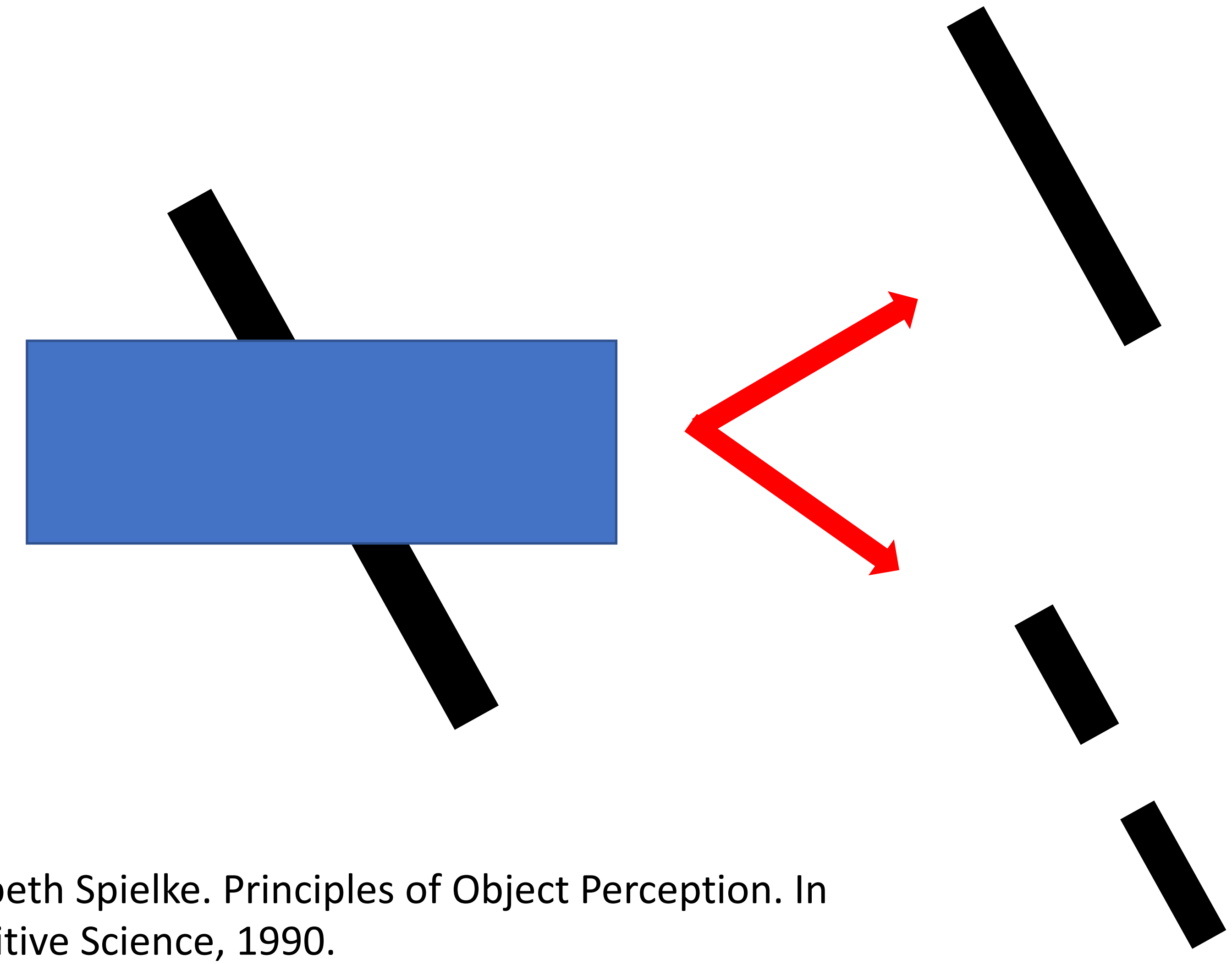
Optical flow for moving scenes



Optical flow for moving scenes

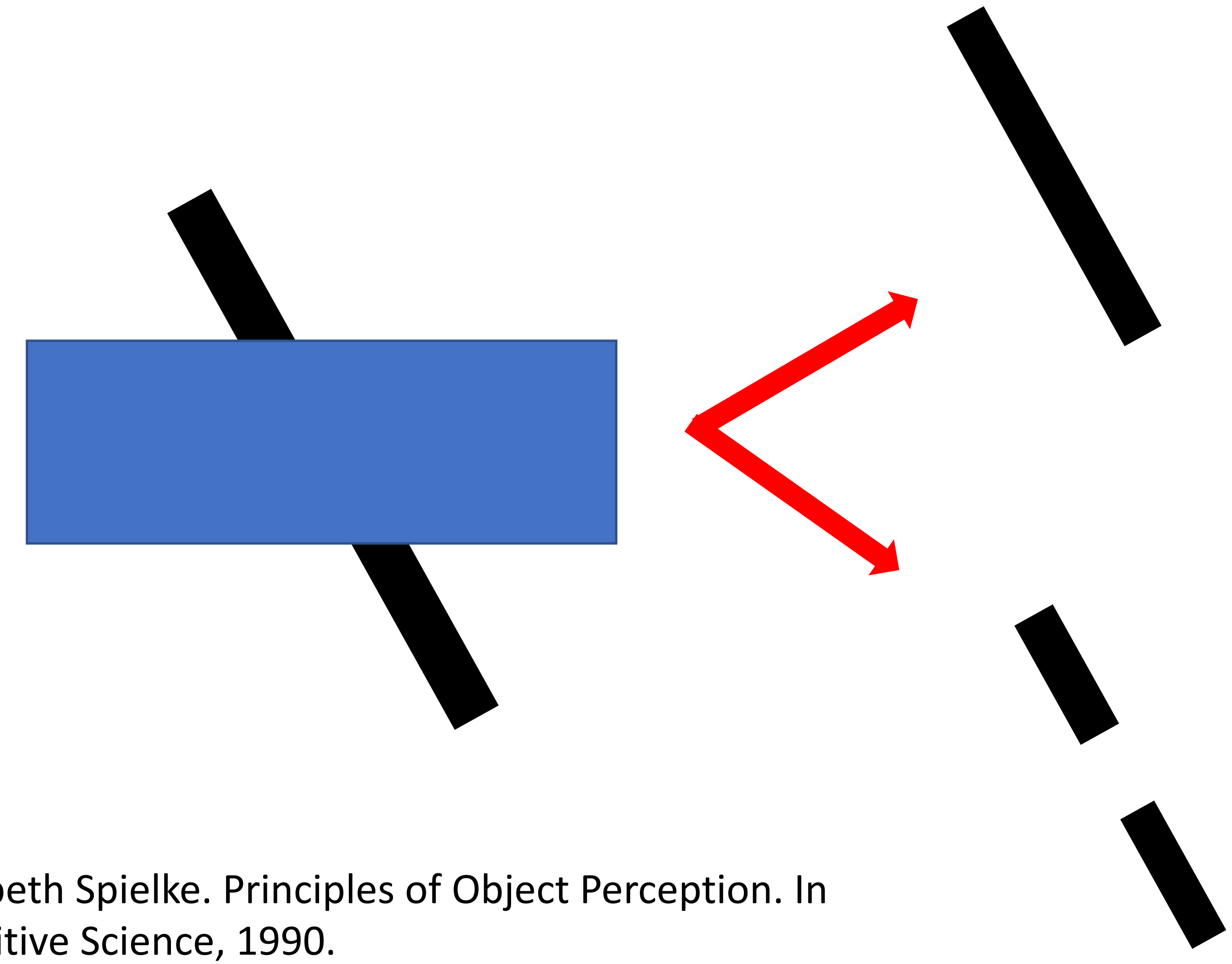
- Optical flow helps *grouping*
- *Gestalt principle of common fate*
  - *Things that move together belong together*

Motion segmentation in humans



Elizabeth Spielke. Principles of Object Perception. In Cognitive Science, 1990.

Motion segmentation in humans



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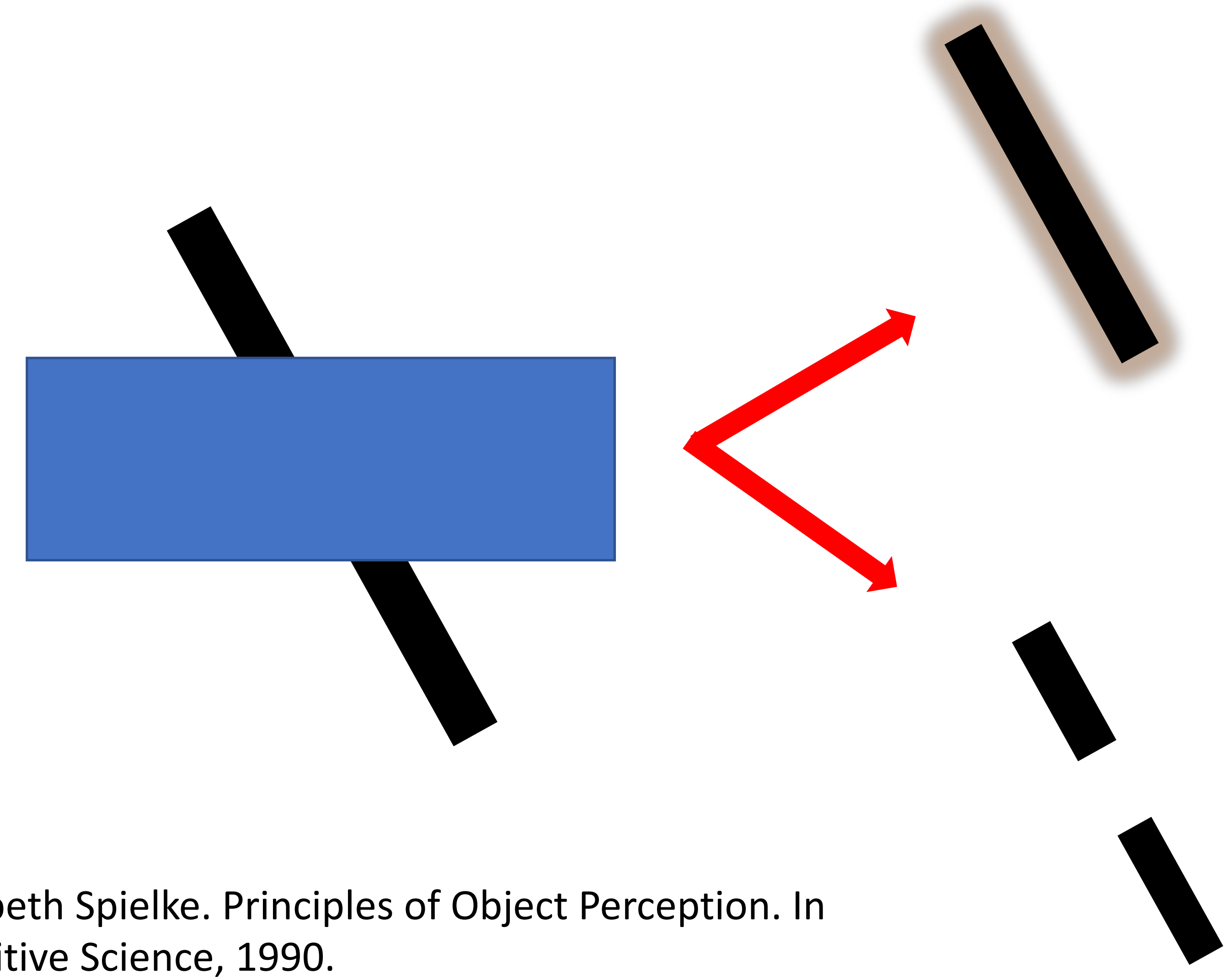
Motion segmentation in humans



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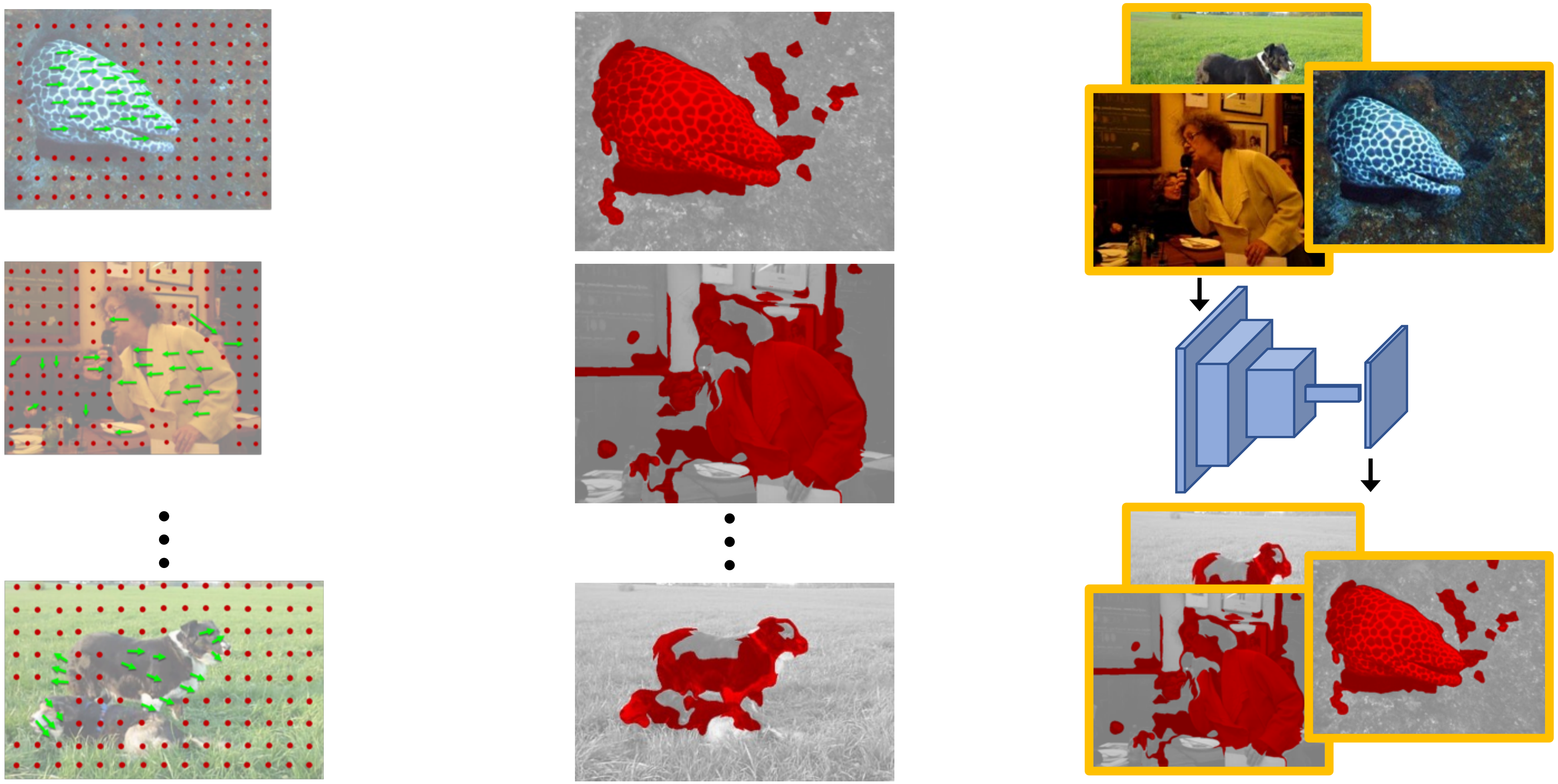


1. Collect videos

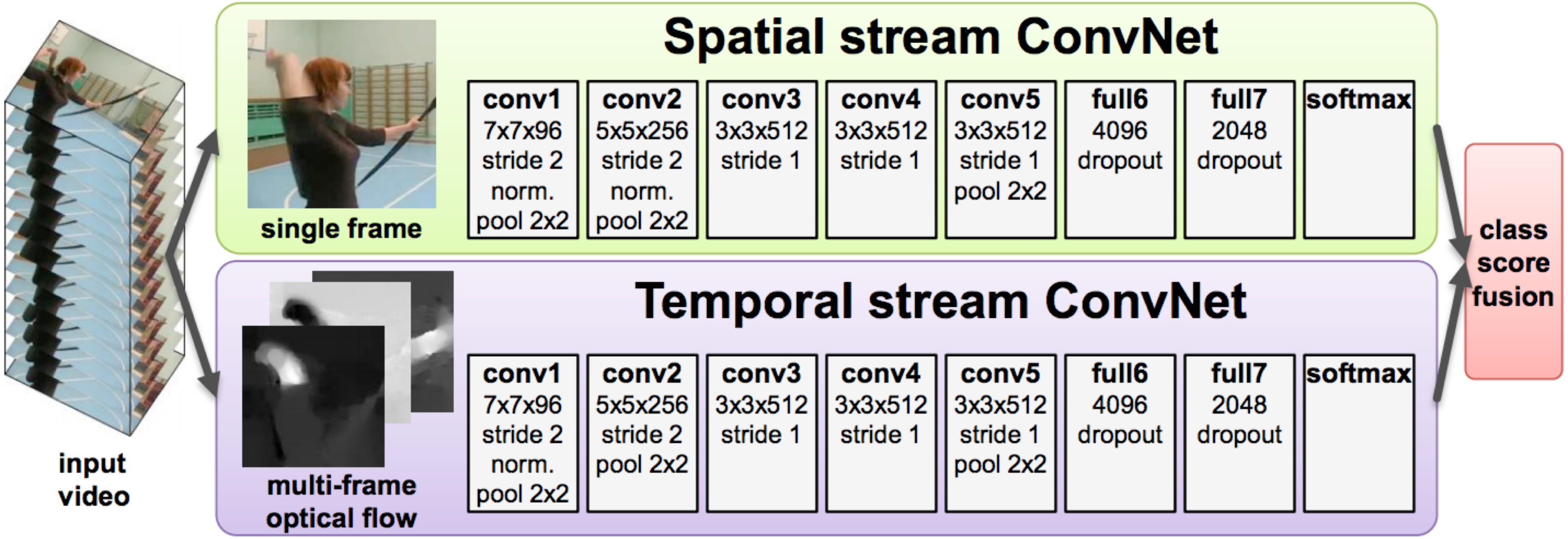
2. Segment using motion

3. Train ConvNet

Learning Features by Watching Objects Move. D. Pathak, R. Girshick, P. Dollar, T. Darrell, B. Hariharan. *CVPR,* 2017.

Optical flow for moving scenes

- Motion is cue for recognition
  - Gestures, actions, …



Two-Stream Convolutional Networks for Action Recognition in Videos. Simonyan and Zisserman. In *NIPS 2014.*

Optical flow for moving scenes

- Motion is cue for recognition
  - Gestures, actions, …

Model	Accuracy
Without optical flow	73.0%
With optical flow	**88.0%**

Two-Stream Convolutional Networks for Action Recognition in Videos. Simonyan and Zisserman. In *NIPS 2014.*

Estimating optical flow

- Yet another correspondence problem!
- But:
  - Bad: scene can move
  - Good: changes are usually very small (often sub-pixel)

Optical flow constraint equation

- Image intensity *continuous* function of x, y, t
- In time dt, pixel (x,y,t) moves to (x + u dt, y + v dt, t + dt)

$$\min_{u,v}(I(x + u\Delta t, y + v\Delta t, t + \Delta t) - I(x, y, t))^2$$

$$\equiv \min_{u,v}(I(x, y, t) + I_x u\Delta t + I_y v\Delta t + I_t\Delta t - I(x, y, t))^2$$

$$\equiv \min_{u,v}(I_x u\Delta t + I_y v\Delta t + I_t\Delta t)^2$$

$$I_x u + I_y v + I_t = 0$$

- Optical flow constraint equation: One equation, two variables

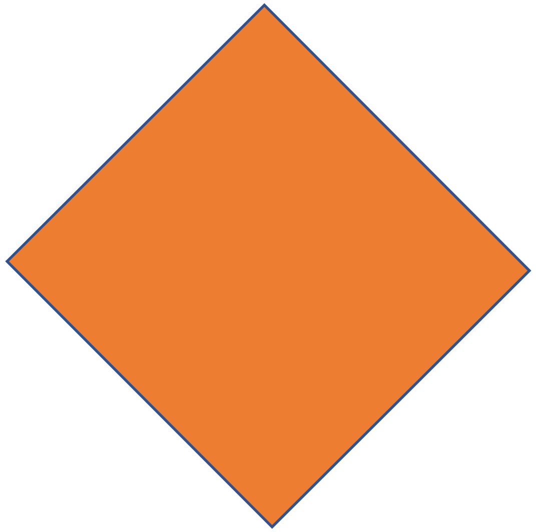
Lucas-Kanade

- Assume all pixels in patch have the same flow
- When will this produce a unique solution?

$$\begin{pmatrix} \nabla I(x_1, y_1)^T \\ \vdots \\ \nabla I(x_n, y_n)^T \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = - \begin{pmatrix} I_t(x_1, y_1) \\ \vdots \\ I_t(x_n, y_n) \end{pmatrix}$$

Aperture problem

Aperture problem



Lucas-Kanade

$$\begin{pmatrix} \nabla I(x_1, y_1)^T \\ \vdots \\ \nabla I(x_n, y_n)^T \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = - \begin{pmatrix} I_t(x_1, y_1) \\ \vdots \\ I_t(x_n, y_n) \end{pmatrix}$$

- Equation of the form Ax = b
- Solve using Normal equations: $x = (A^T A)^{-1} A^T b$
- Need $A^T A$ to be invertible - corners!

Lucas-Kanade

- What if we consider the whole image as one patch?
  - Constant optical flow for the entire image?
- Better: what if we consider flow as a *parametric function* of pixel location?
  - e.g. affine $\begin{bmatrix} u \\ v \end{bmatrix} = A\mathbf{x} + b$
  - More generally: $\mathbf{x}' = W(\mathbf{x}; \mathbf{p})$
    - W is some 2D → 2D parametric warp function
    - **p** is a parameter vector
  - “Motion models”

Lucas-Kanade

$$\min_{\mathbf{p}} \sum_{\mathbf{x}} (I(W(\mathbf{x};\mathbf{p})) - T(\mathbf{x}))^2$$

- T is the previous frame, also called *template*
- I is the current frame
- Goal is to find **p**

Baker, Simon, and Iain Matthews. "Lucas-kanade 20 years on: A unifying framework." *International journal of computer vision* 56.3 (2004): 221-255.

Lucas-Kanade

- Iterative process
- Assume that we have a current iterate $\boldsymbol{p}$ and we want to find the next iterate $\boldsymbol{p} + \Delta\boldsymbol{p}$
- Find $\Delta\boldsymbol{p}$ by optimizing $\min_{\Delta\mathbf{p}} \sum_{\mathbf{x}} (I(W(\mathbf{x};\mathbf{p}+\Delta\mathbf{p})) - T(\mathbf{x}))^2$
- Hard because I and W are both non-linear
- Assume $\Delta\boldsymbol{p}$ is small and linearize:
  - Linearize W: $W(\mathbf{x};\mathbf{p}+\Delta\mathbf{p}) \approx W(\mathbf{x};\mathbf{p}) + \frac{\partial W}{\partial \mathbf{p}}\Delta\mathbf{p}$
  - Linearize I: $I(W(\mathbf{x};\mathbf{p}+\Delta\mathbf{p})) \approx I(W(\mathbf{x};\mathbf{p})) + \nabla I \frac{\partial W}{\partial \mathbf{p}}\Delta\mathbf{p}$

Baker, Simon, and Iain Matthews. "Lucas-kanade 20 years on: A unifying framework." *International journal of computer vision* 56.3 (2004): 221-255.

Lucas Kanade

- Iterative process
- At each step, find $\Delta \boldsymbol{p}$ that optimizes

$$\min_{\Delta \mathbf{p}} (I(W(\mathbf{x};\mathbf{p})) + \nabla I \frac{\partial W}{\partial \mathbf{p}} \Delta \mathbf{p} - T(\mathbf{x}))^2$$

  - Warped image
  - Gradient of warped image
  - Jacobian of warp function
  - Template
- Quadratic in $\Delta \boldsymbol{p}$, solve exactly

Lucas-Kanade

- Solve by iterating on parameters
- Equivalent to Newton iteration + linearization
- Can we remove the parametric assumption?

Baker, Simon, and Iain Matthews. "Lucas-kanade 20 years on: A unifying framework." *International journal of computer vision* 56.3 (2004): 221-255.

Horn-Schunk

$$E(\mathbf{u},\mathbf{v}) = E_{data}(\mathbf{u},\mathbf{v}) + E_{smoothness}(\mathbf{u},\mathbf{v})$$

$$E(\mathbf{u},\mathbf{v}) = \int\int (I(x+u(x,y)\Delta t, y+v(x,y)\Delta t, t+\Delta t) - I(x,y,t))^2$$ ← Data

$$+ \alpha(\|\nabla u\|^2 + \|\nabla v\|^2)dxdy$$ ← Smoothness

Horn-Schunk

$$E(\mathbf{u},\mathbf{v}) = E_{data}(\mathbf{u},\mathbf{v}) + E_{smoothness}(\mathbf{u},\mathbf{v})$$

$$E(\mathbf{u},\mathbf{v}) = \int\int (I_x u + I_y v + I_t)^2 + \alpha(\|\nabla u\|^2 + \|\nabla v\|^2)dxdy$$

Data

Smoothness

Variational minimization

- u and v are *functions*
- Euler-lagrange equations
  - Similar to “gradient=0”

$$\min_q \int L(t, q(t), \dot{q}(dt))dt$$

$$\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} = 0$$

Variational minimization

$$\min_q \int L(t, q(t), \dot{q}(dt))dt$$

$$\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} = 0$$

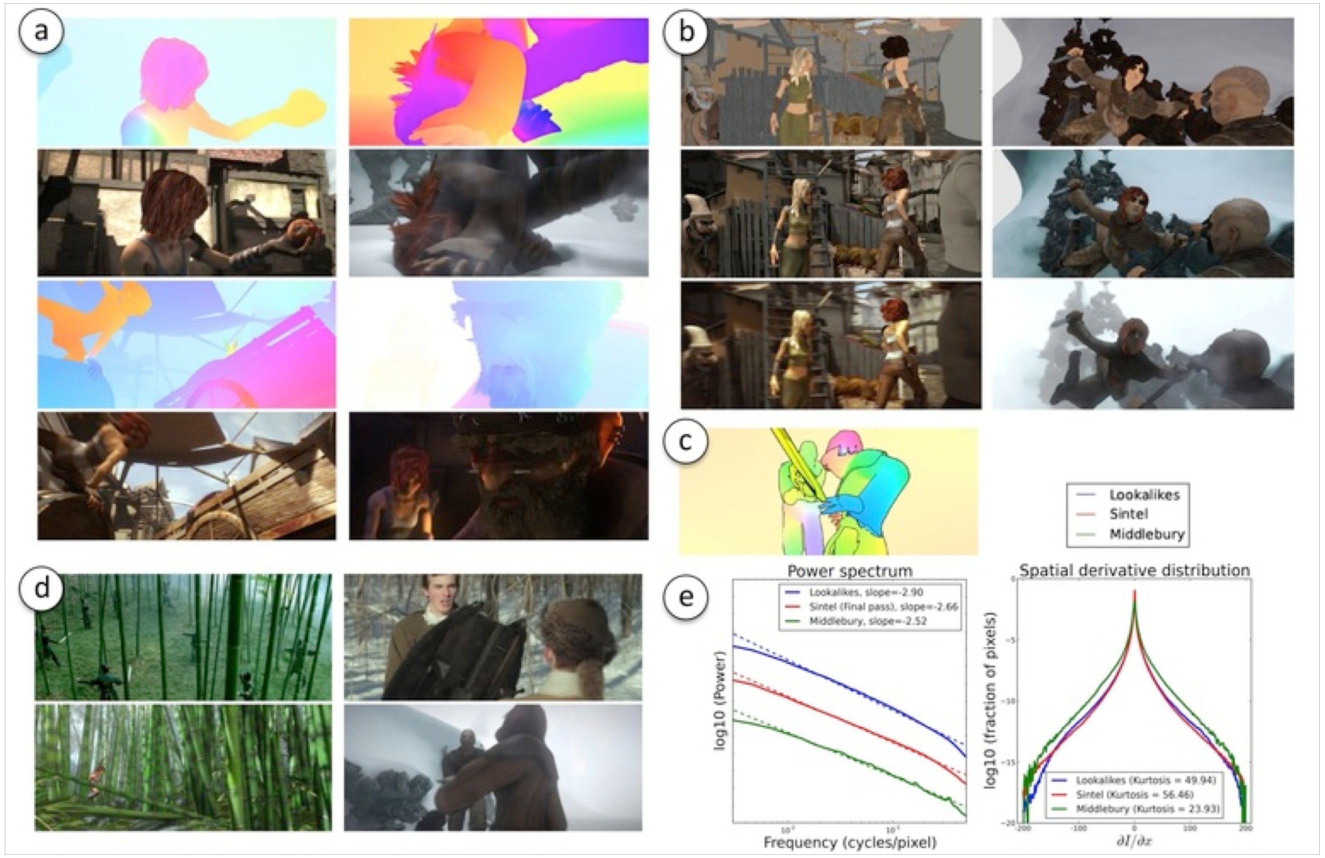
$$\min_{u,v} \int\int f(x, y, u, v, u_x, u_y, v_x, v_y)dxdy$$

$$\frac{\partial f}{\partial u} - \frac{d}{dx}\frac{\partial f}{\partial u_x} - \frac{d}{dy}\frac{\partial f}{\partial u_y} = 0$$

$$\frac{\partial f}{\partial v} - \frac{d}{dx}\frac{\partial f}{\partial v_x} - \frac{d}{dy}\frac{\partial f}{\partial v_y} = 0$$

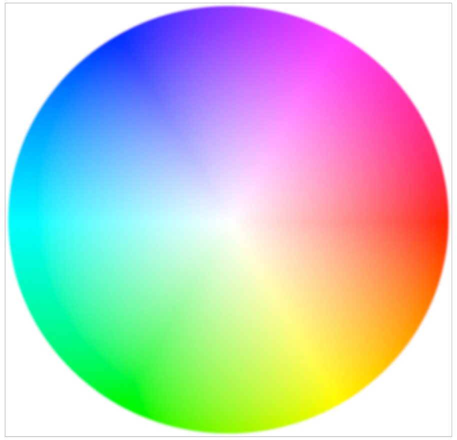
MPI-Sintel

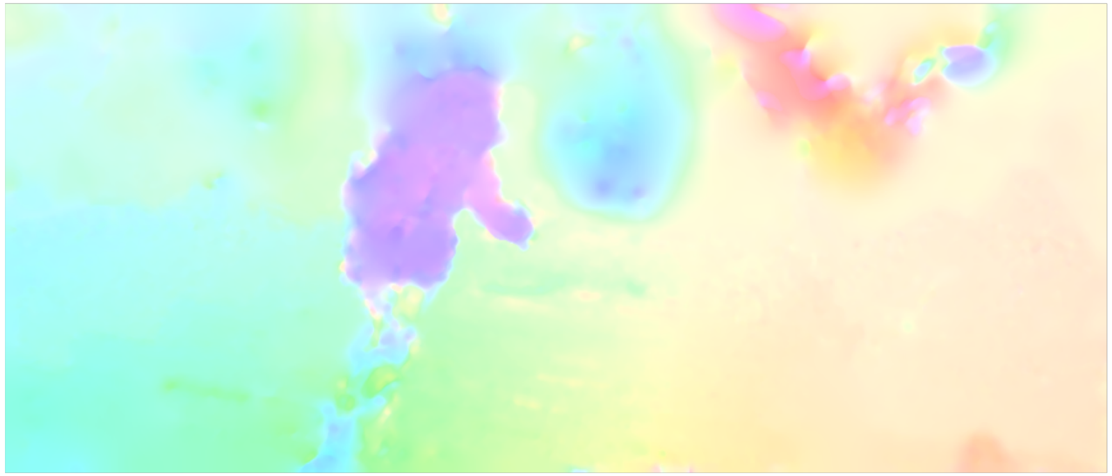
- Open-source animated movie "Sintel"
- "Naturalistic" video
- Ground truth optical flow
- Large motions
- Complex scenes



Butler, D. J. and Wulff, J. and Stanley, G. B. and Black, M. J. A naturalistic open source movie for optical flow evaluation. *ECCV,* 2012

MPI-Sintel results



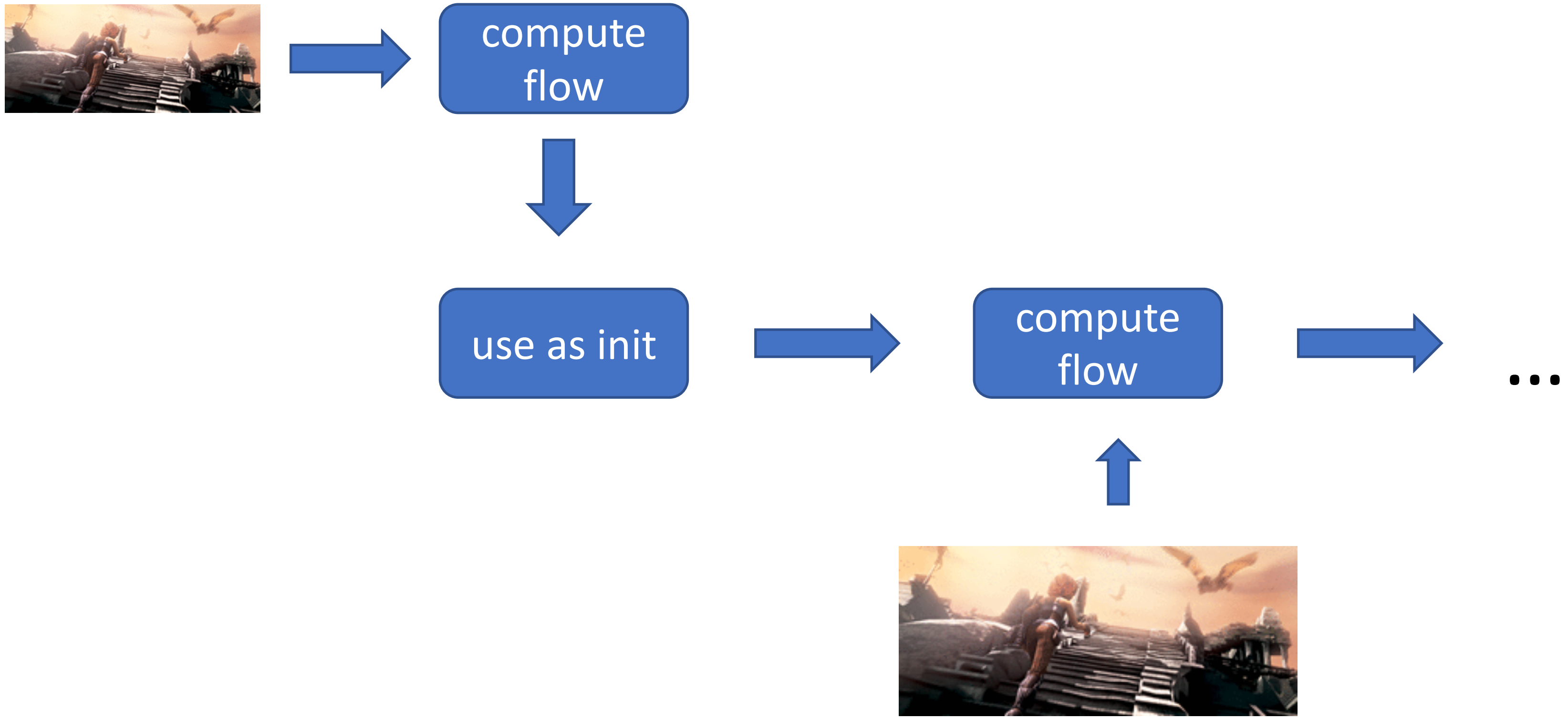


Optical flow with large displacements

- Optical flow constraint equation assumes differential optical flow
- “Large displacement”?
- Key idea: reducing resolution reduces displacement
- Reduce resolution, then upsample?
    - will lose fine details

Optical flow with large displacements

- Key idea 2: Use upsampled flow as *initialization*
- *Changes to initialization will be infinitesimal*



Brox, Thomas, et al. "High accuracy optical flow estimation based on a theory for warping." *Computer Vision-ECCV 2004* (2004)

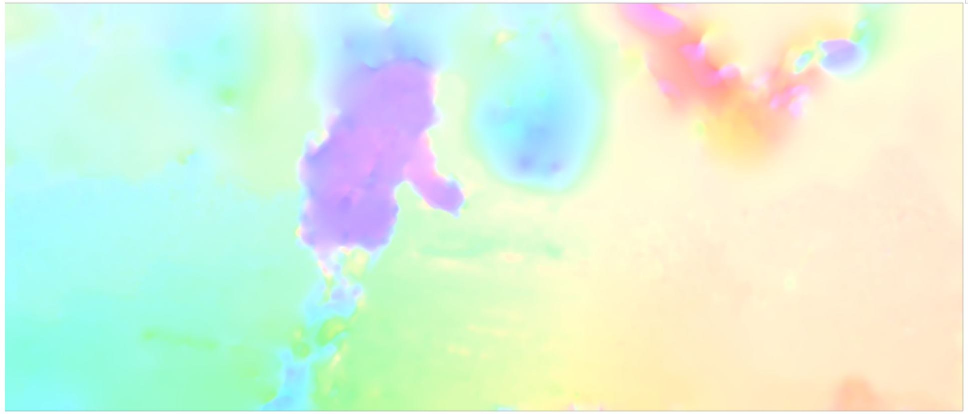
Optical flow for large displacements

- Possible issue: large appearance change => incorrect matching based on color alone
- Use descriptor matching (e.g. SIFT) on sparse points
- Use smoothness to propagate to all pixels:
  - Flow is weighted average of nearest neighbors: $F(p) = \frac{\sum_q k(p,q)F(q)}{\sum_q k(p,q)}$
  - *Kernel k* dependent on relative position + edges
- Use this as initialization for optimization

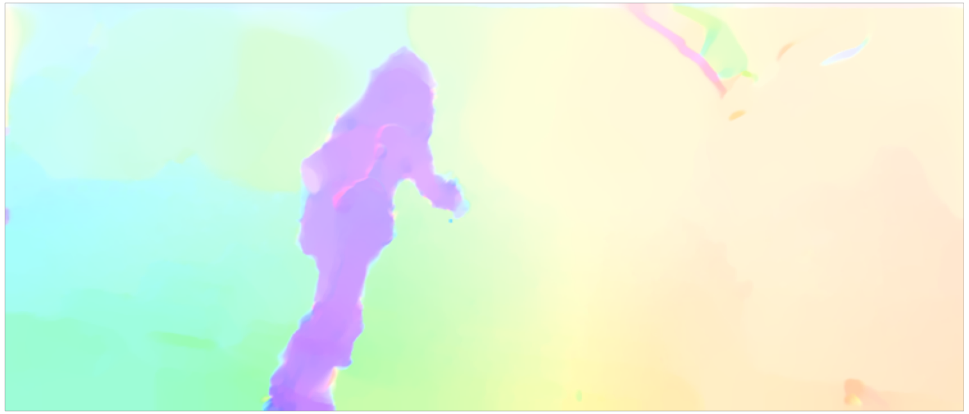
Brox, Thomas, Christoph Bregler, and Jitendra Malik. "Large displacement optical flow." *CVPR,* 2009.
Revaud, Jerome, et al. "Epicflow: Edge-preserving interpolation of correspondences for optical flow." CVPR. 2015.

LDOF and EpicFlow

Video



Basic Horn-Schunk (Error = 2.069)



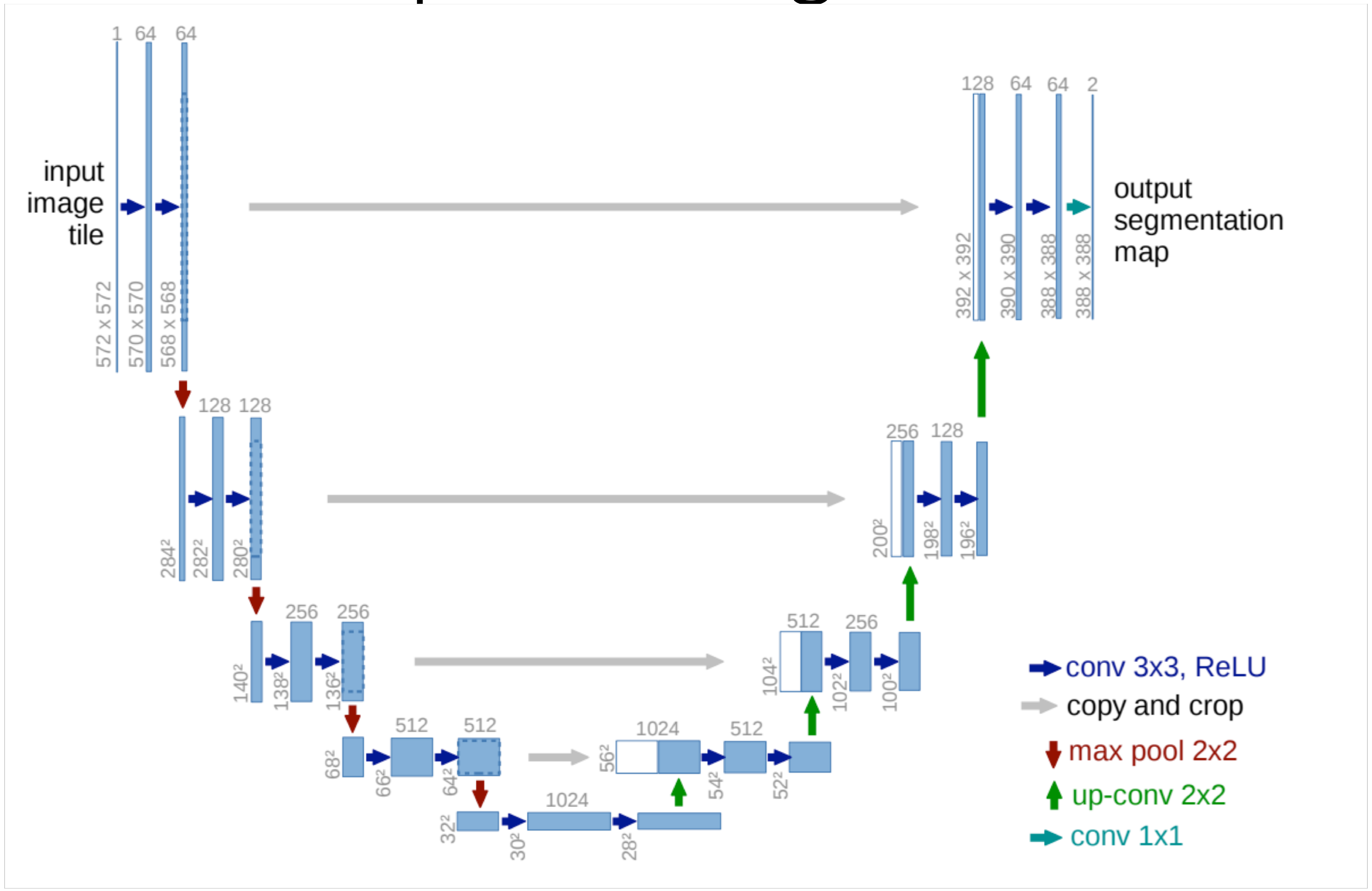
LDOF (Brox et al, 2009) (Error = 1.606)

EpicFlow (Revaud et al, 2015) (Error = 1.295)

Coarse-to-fine processing

- A specific instance of a general idea
  - Also coarse-to-fine versions of Lucas-Kanade
- Coarse scales:
  - Global / large structures
  - Long-range relationships
  - But: imprecise localization
- Fine scales:
  - Precise localization
  - But: aperture problem
- Idea: start from coarse scales, add fine scale detail

Coarse-to-fine processing



U-Net: Convolutional Networks for Biomedical Image Segmentation. Olaf Ronneberger, Philipp Fischer, and Thomas Brox. In *MICCAI,* 2015.