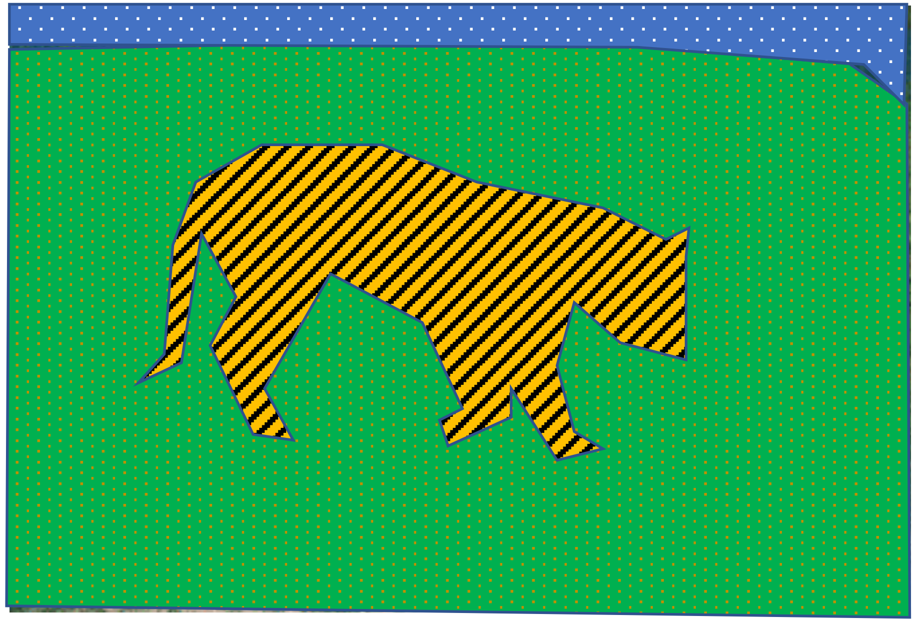
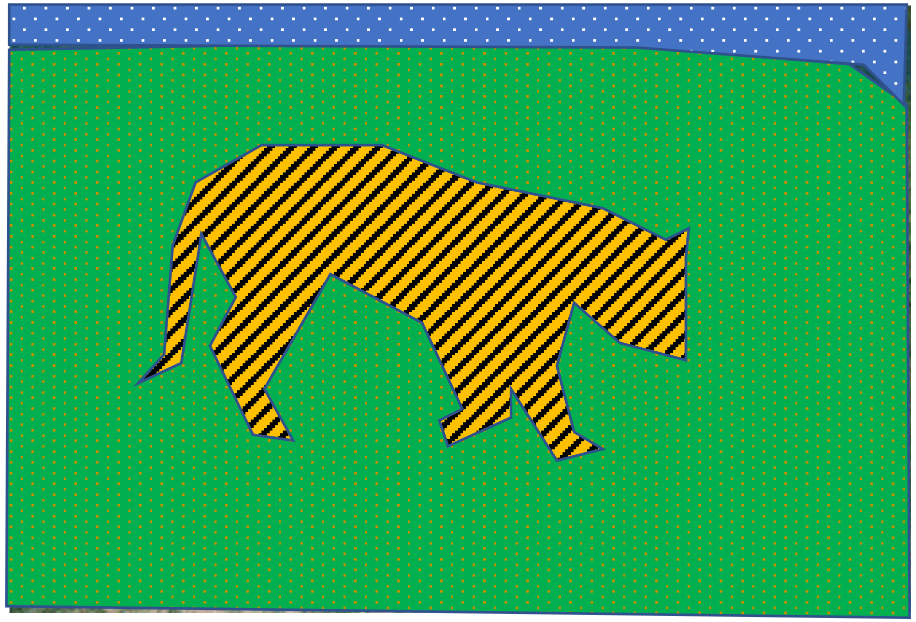
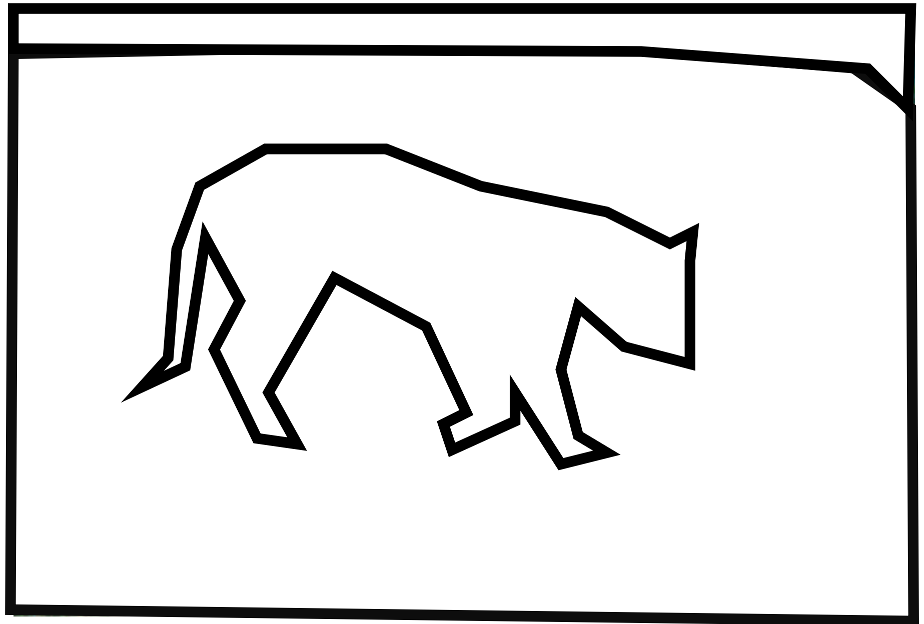
Grouping

What is grouping?



Regions ⟷ Boundaries





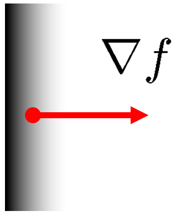
Why grouping?

- Pixels property of sensor, not world
- Reasoning at object level (might) make things easy:
  - objects at consistent depth
  - objects can be recognized
  - objects move as one

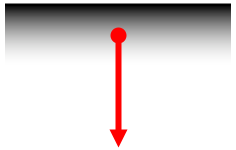
Image gradient

- The *gradient* of an image: $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$

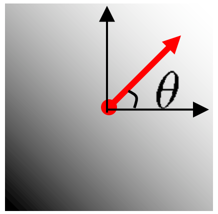
The gradient points in the direction of most rapid increase in intensity



$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$



$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$



$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$

The *edge strength* is given by the gradient magnitude:

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

The gradient direction is given by:

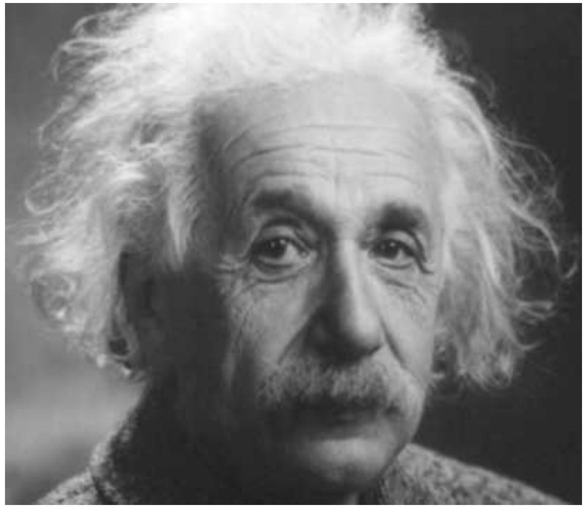
$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} \Big/ \frac{\partial f}{\partial x}\right)$$

- how does this relate to the direction of the edge?

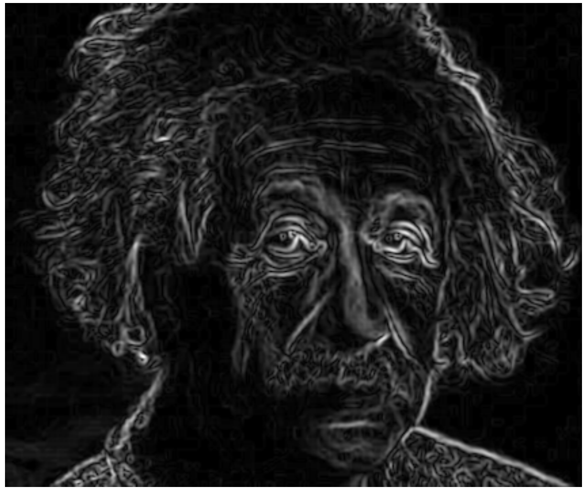
Source: Steve Seitz

Gradient magnitude and orientation

- Orientation is undefined at pixels with 0 gradient



Image

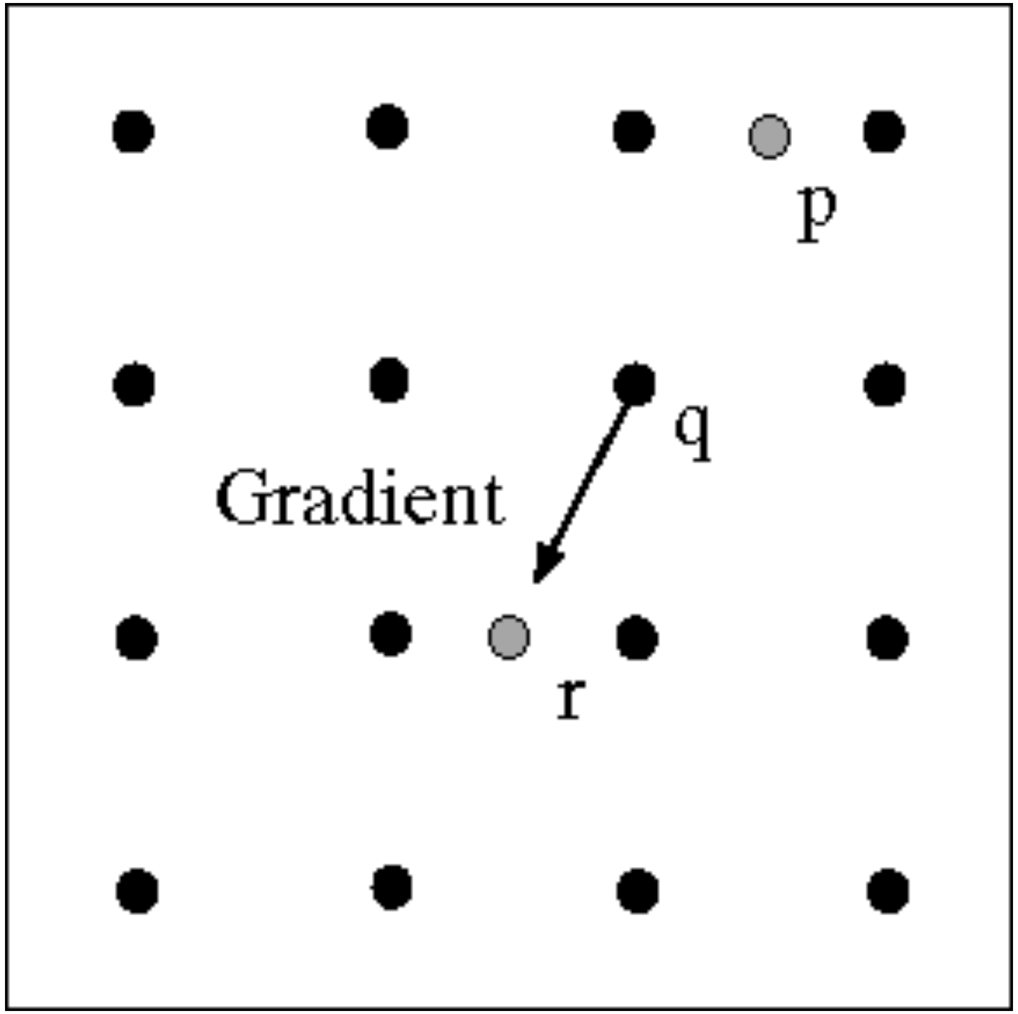


Magnitude

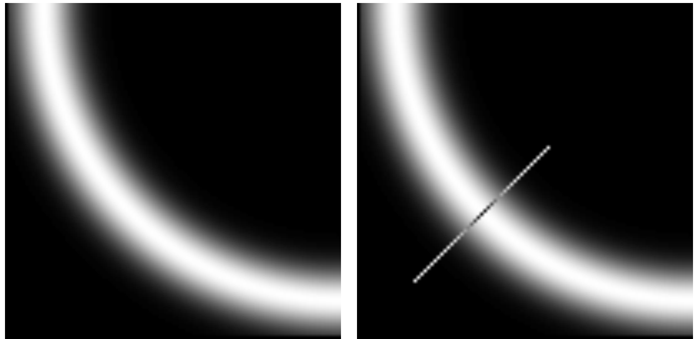
Orientation

theta = numpy.arctan2(gy, gx)

Non-maximum suppression for each orientation



At q, we have a maximum if the value is larger than those at both p and at r. Interpolate to get these values.



Source: D. Forsyth

Before Non-max Suppression

After Non-max Suppression

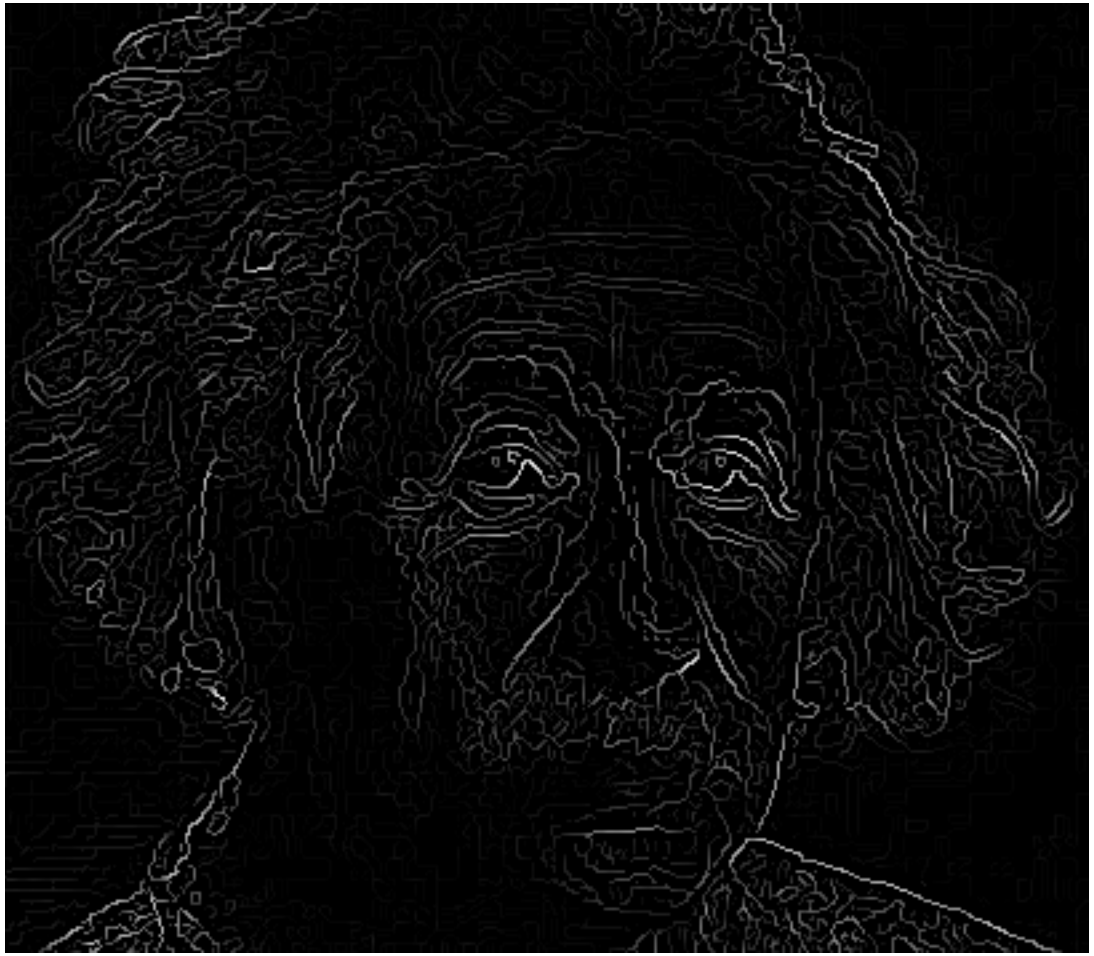
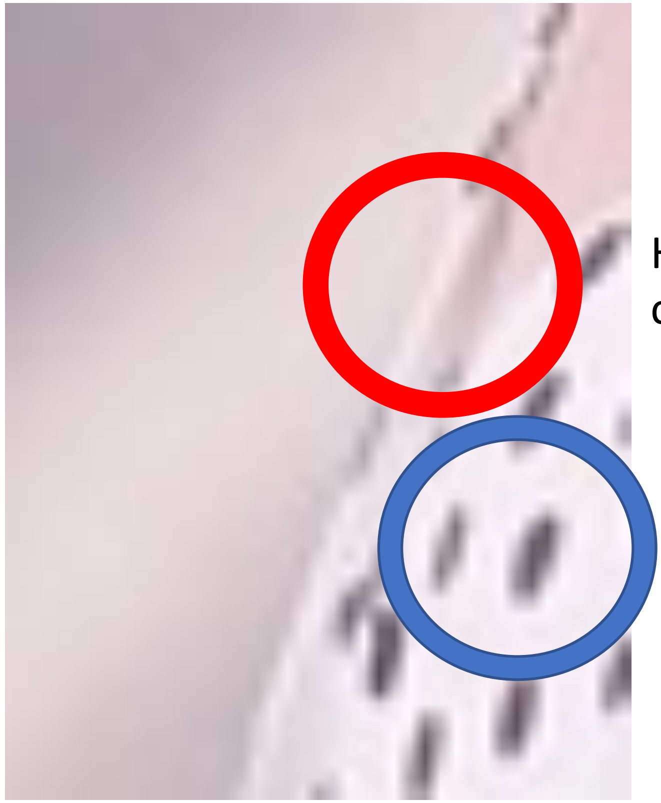


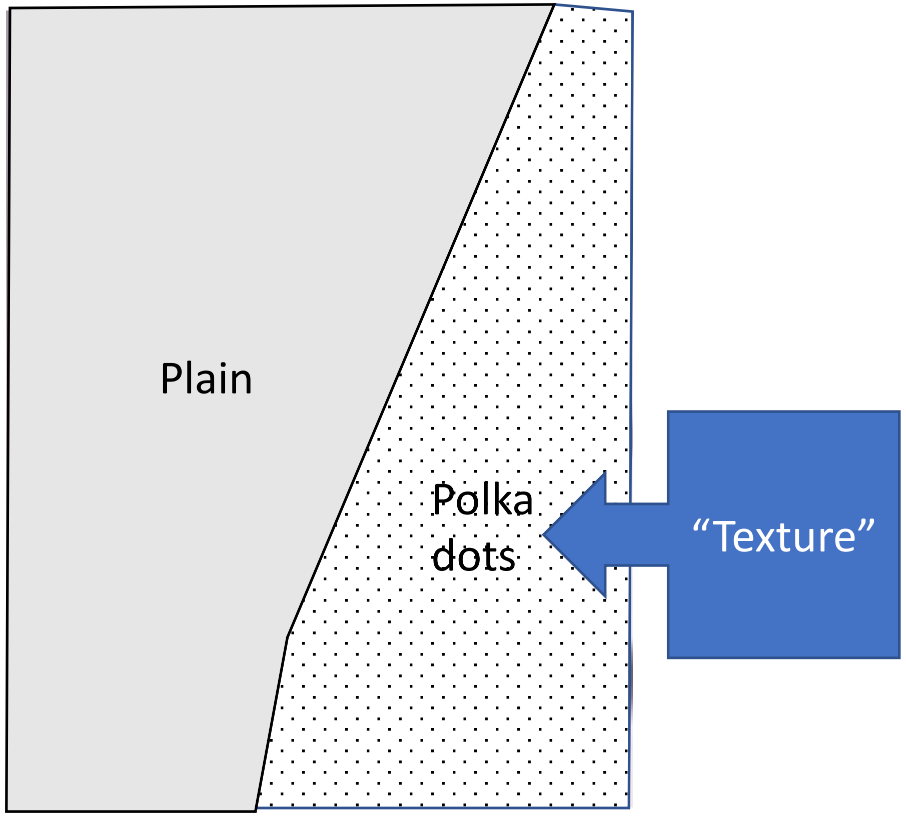
Image gradients are not enough



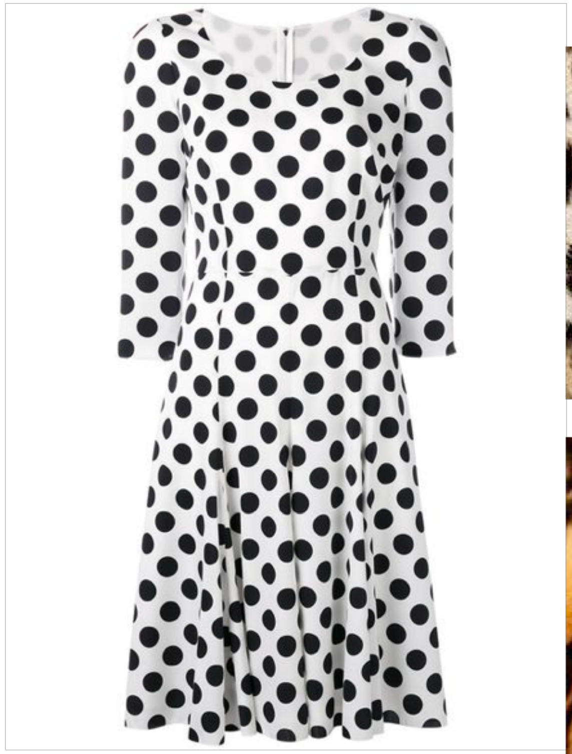
Hard-to-detect low contrast boundaries

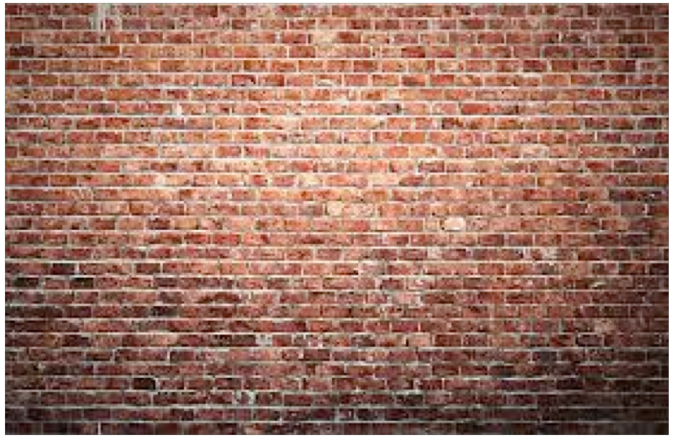
Strong internal image graadients

Image gradients are not enough



What is texture?





Same thing repeated over and over

What is texture?

Julesz's texton theory

- What is texture?
- Distributions of some elements
  - Elongated blobs of specific orientations, widths, lengths
  - Terminators (ends of line segments)
  - Crossings of line segments

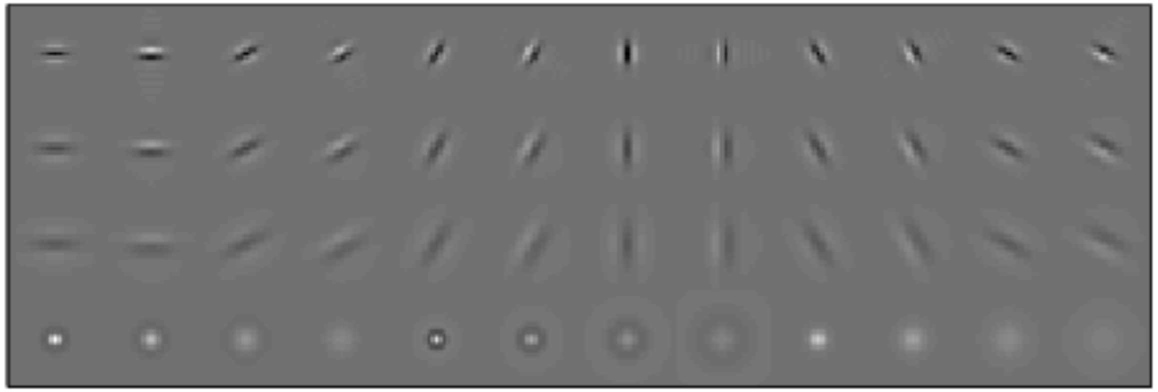
Textons

Bringing textons to computer vision

- Define a “vocabulary” of textons
- Describe texture by a distribution of different textons

Bringing textons to computer vision

- **Define a “vocabulary” of textons**
- Describe texture by a distribution of different textons

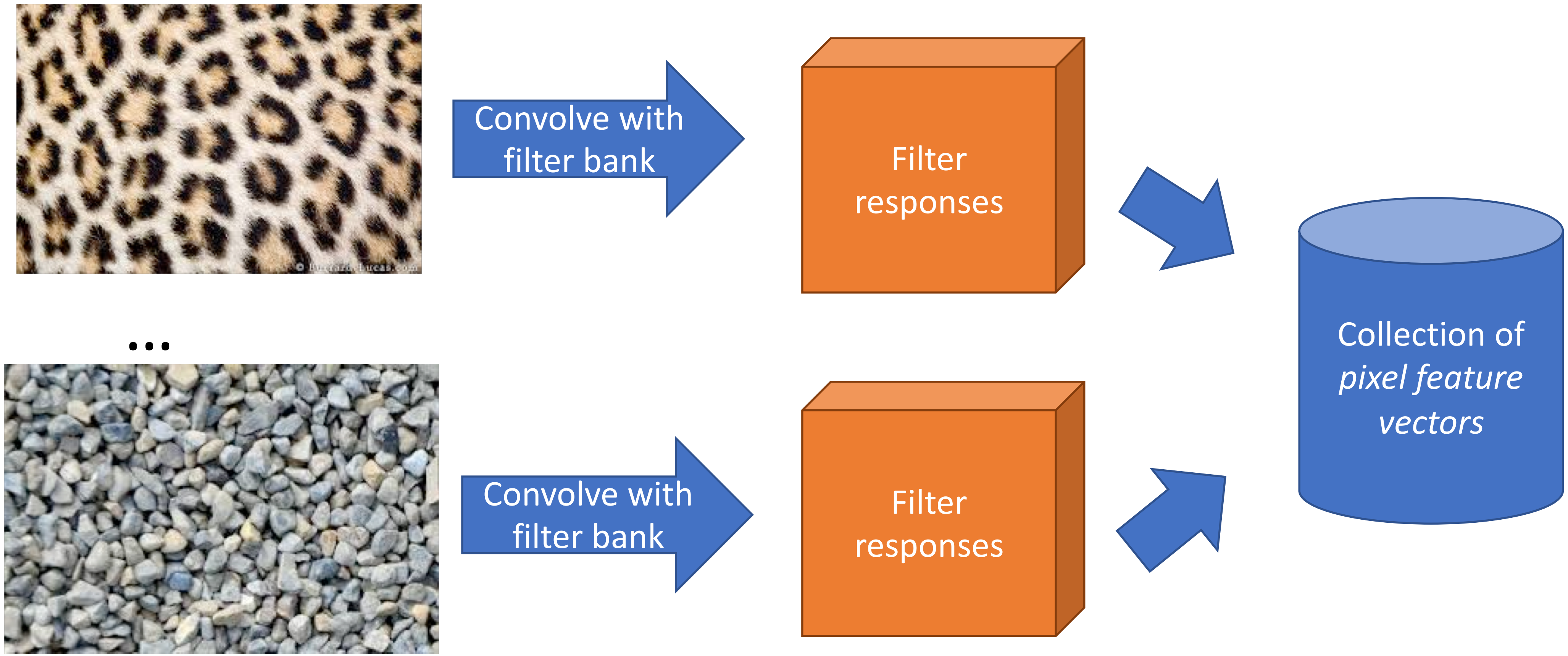


Convolve with filter bank

Filter responses

Bringing textons to computer vision

- **Define a "vocabulary" of textons**
- Describe texture by a distribution of different textons



Bringing textons to computer vision

- **Define a "vocabulary" of textons**
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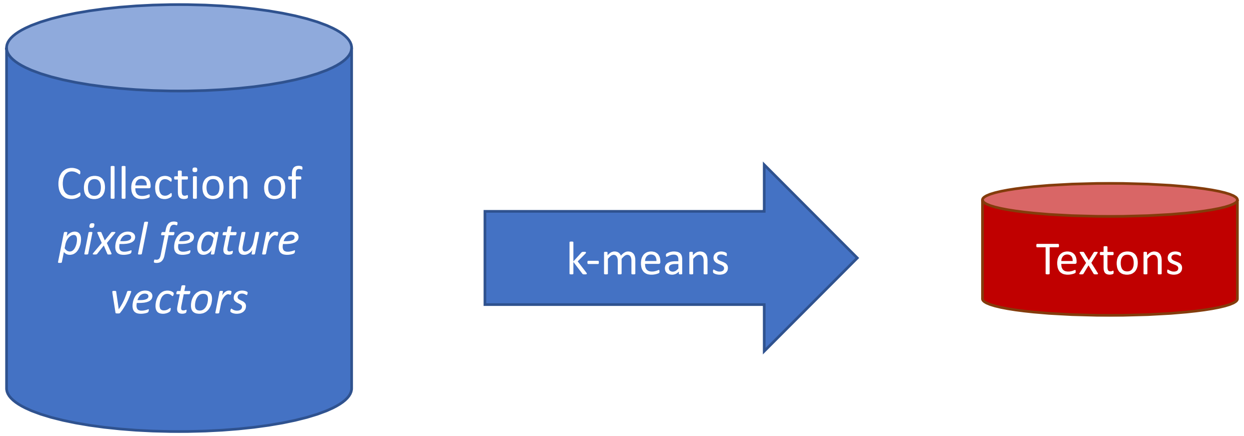
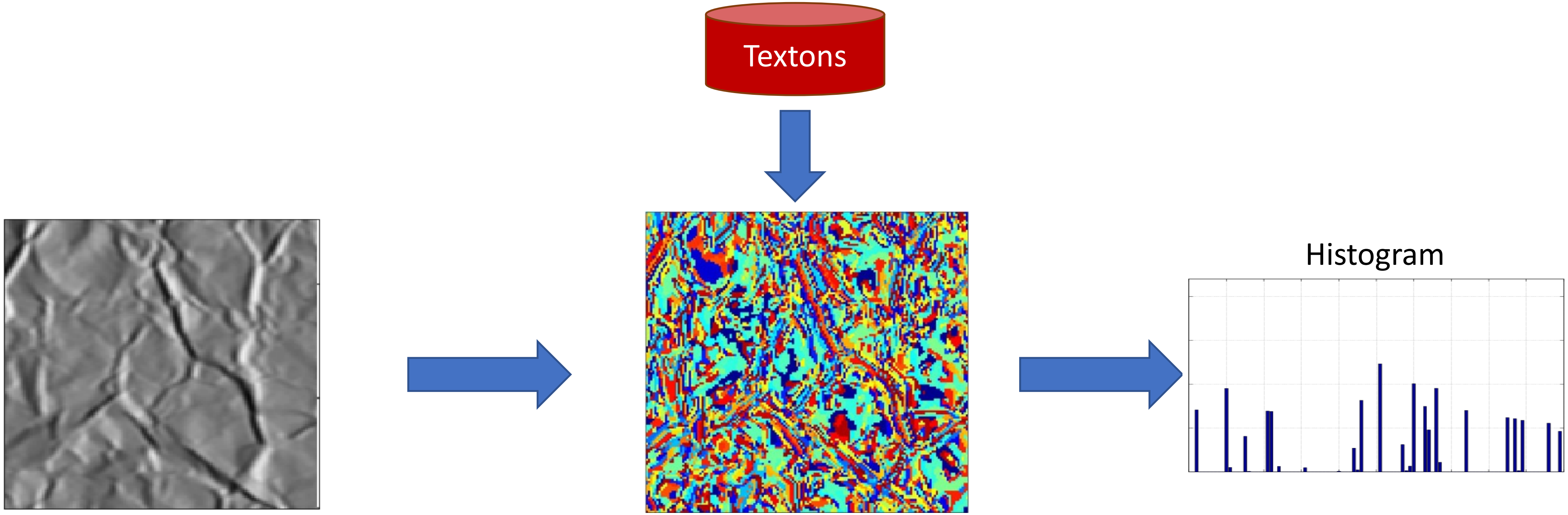


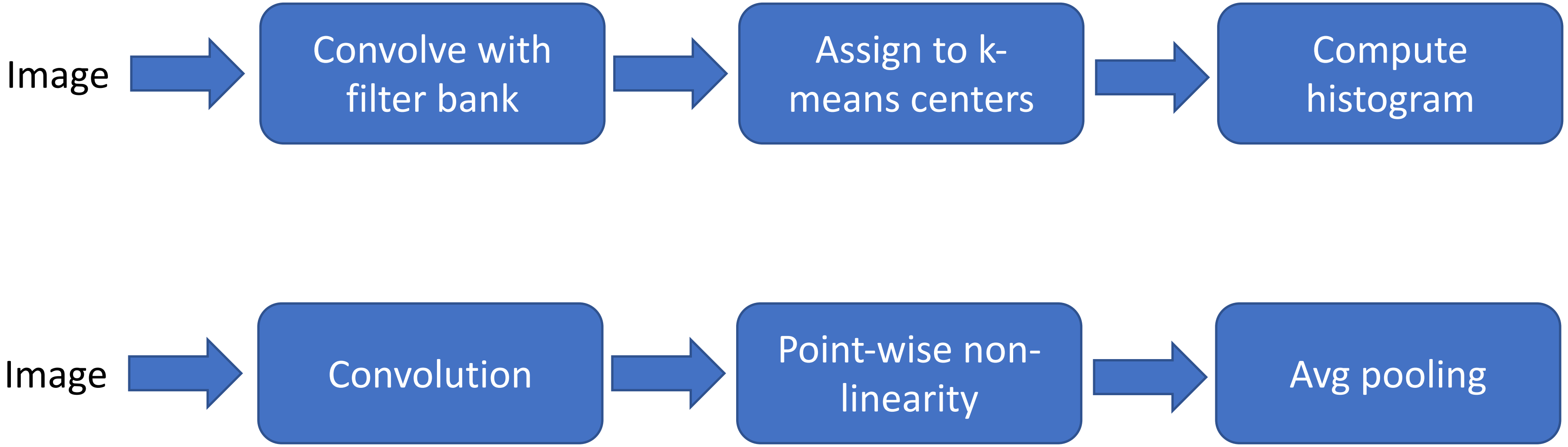
Bringing textons to computer vision

- Define a “vocabulary” of textons
- **Describe texture by a distribution of different textons**



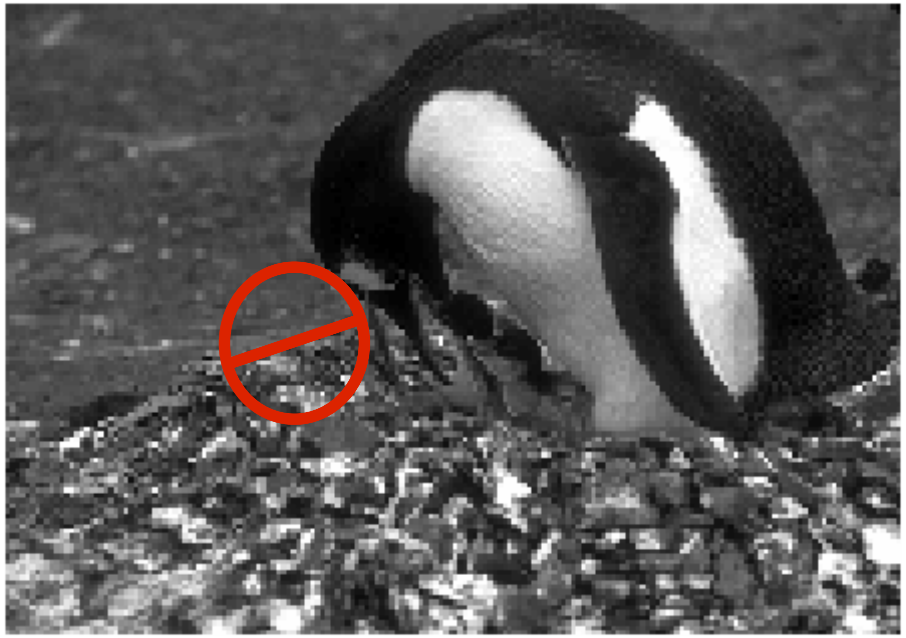
Leung, Thomas, and Jitendra Malik. "Representing and recognizing the visual appearance of materials using three-dimensional textons." *International journal of computer vision* 43.1 (2001): 29-44.

Textons in computer vision



Detecting texture boundaries

- Problem: gradient captures change from *pixel* to *pixel*
- *But texture property of region*
- Take region around pixel and divide into two halves based on hypothetical orientation



Martin, David R., Charless C. Fowlkes, and Jitendra Malik. "Learning to detect natural image boundaries using local brightness, color, and texture cues." *TPAMI (2004).*

Cue combination

Image

Boundary Cues

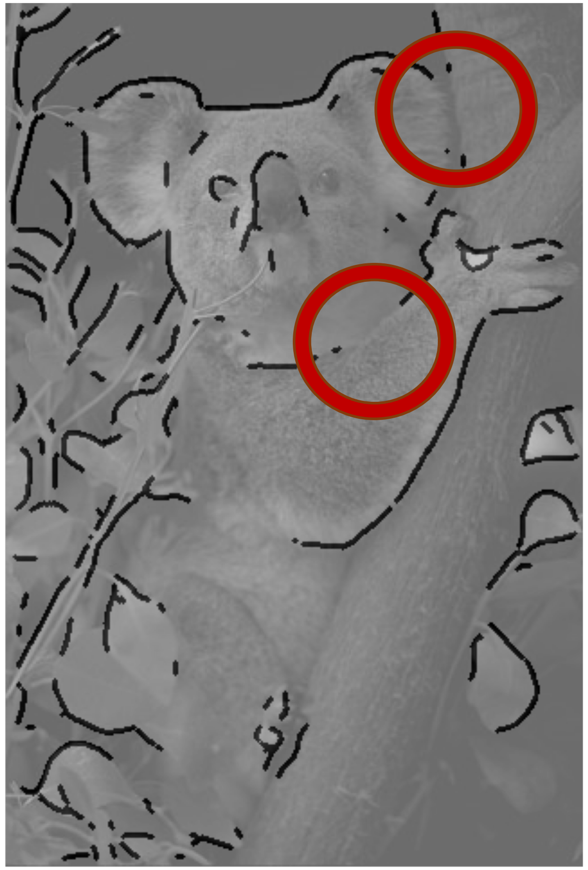
- **Brightness**
- **Color**
- **Texture**

Average

P_b

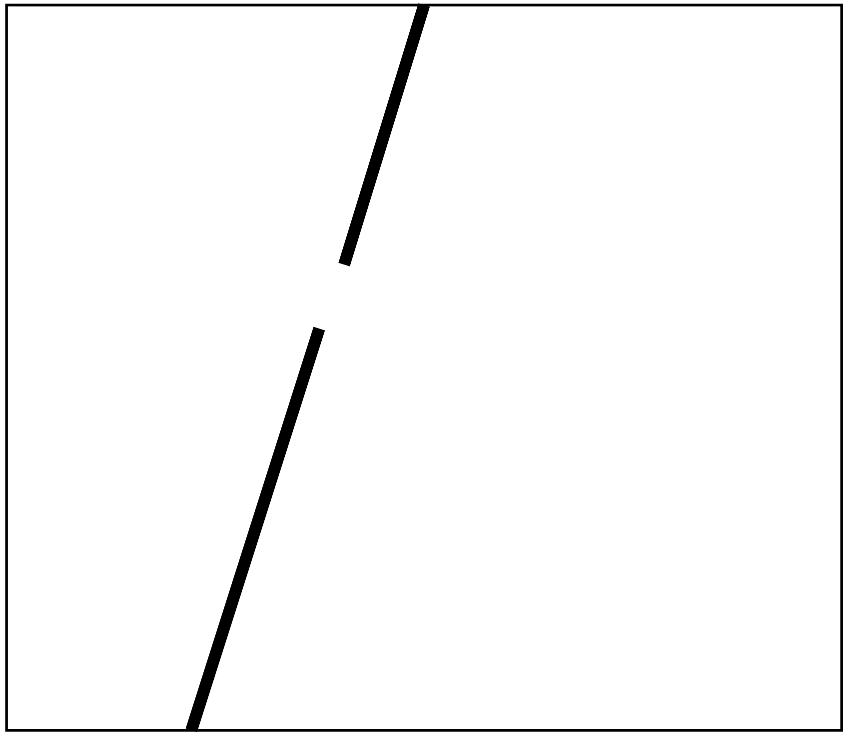
Martin, David R., Charless C. Fowlkes, and Jitendra Malik. "Learning to detect natural image boundaries using local brightness, color, and texture cues." *TPAMI (2004).*

Local computation not enough



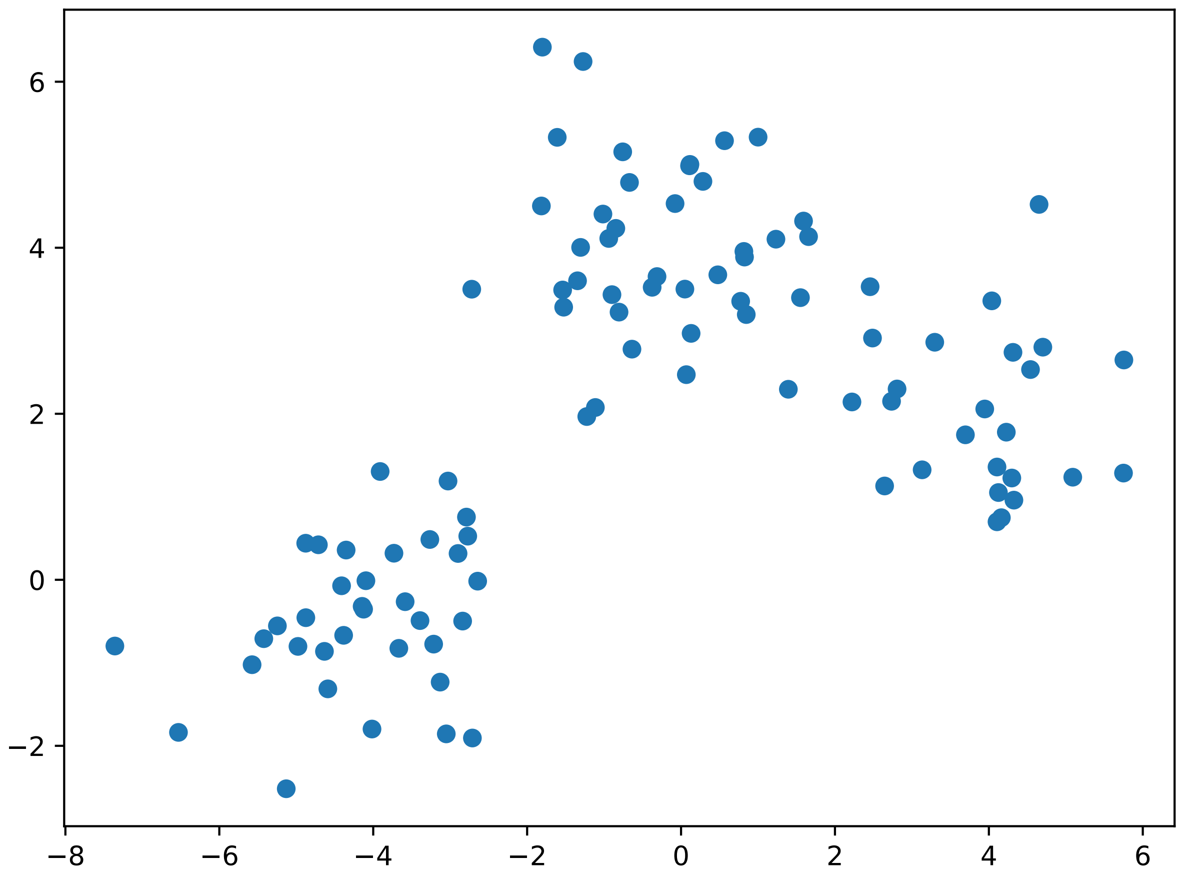
Local computation is not enough

- Key constraints:
  - Boundaries are continuous
  - They enclose a region
- How do we go from local, patchy contours to boundaries?



Grouping by clustering

- Idea: embed pixels into high-dimensional space (e.g. 3-dimensions)
- Each pixel is a point
- Group together points



K-means

- Assumption: each group is a Gaussian with different means and same standard deviation

$$P(x_i|\mu_j) \propto e^{-\frac{1}{2\sigma^2}\|x_i-\mu_j\|^2}$$

- Suppose we know all μ_j. Which group should a point x_i belong to?
  - The j with highest $P(x_i|\mu_j)$
  - = The j with smallest $||x_i - \mu_j||^2$

K-means

- Problem: means are not known
- What if we know a set of points from each cluster?
- $x_{k_1}, x_{k_2}, \ldots, x_{k_n}$ belong to cluster k
- What should be μ_k?

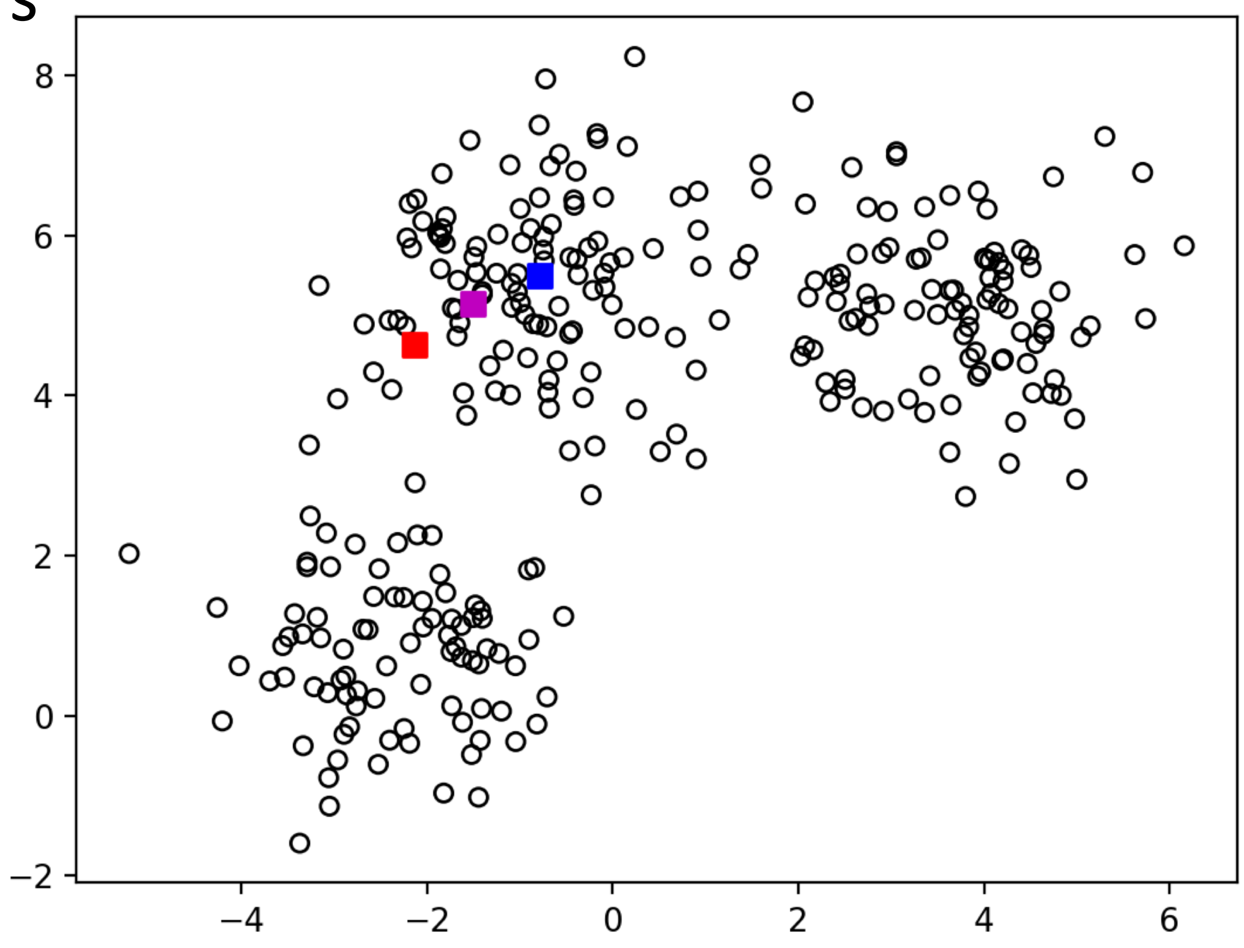
$$\mu_k = \frac{(x_{k_1} + x_{k_2} + \ldots + x_{k_n})}{n}$$

K-means

- Given means, can assign points to clusters
- Given assignments, can compute means
- Idea: iterate!

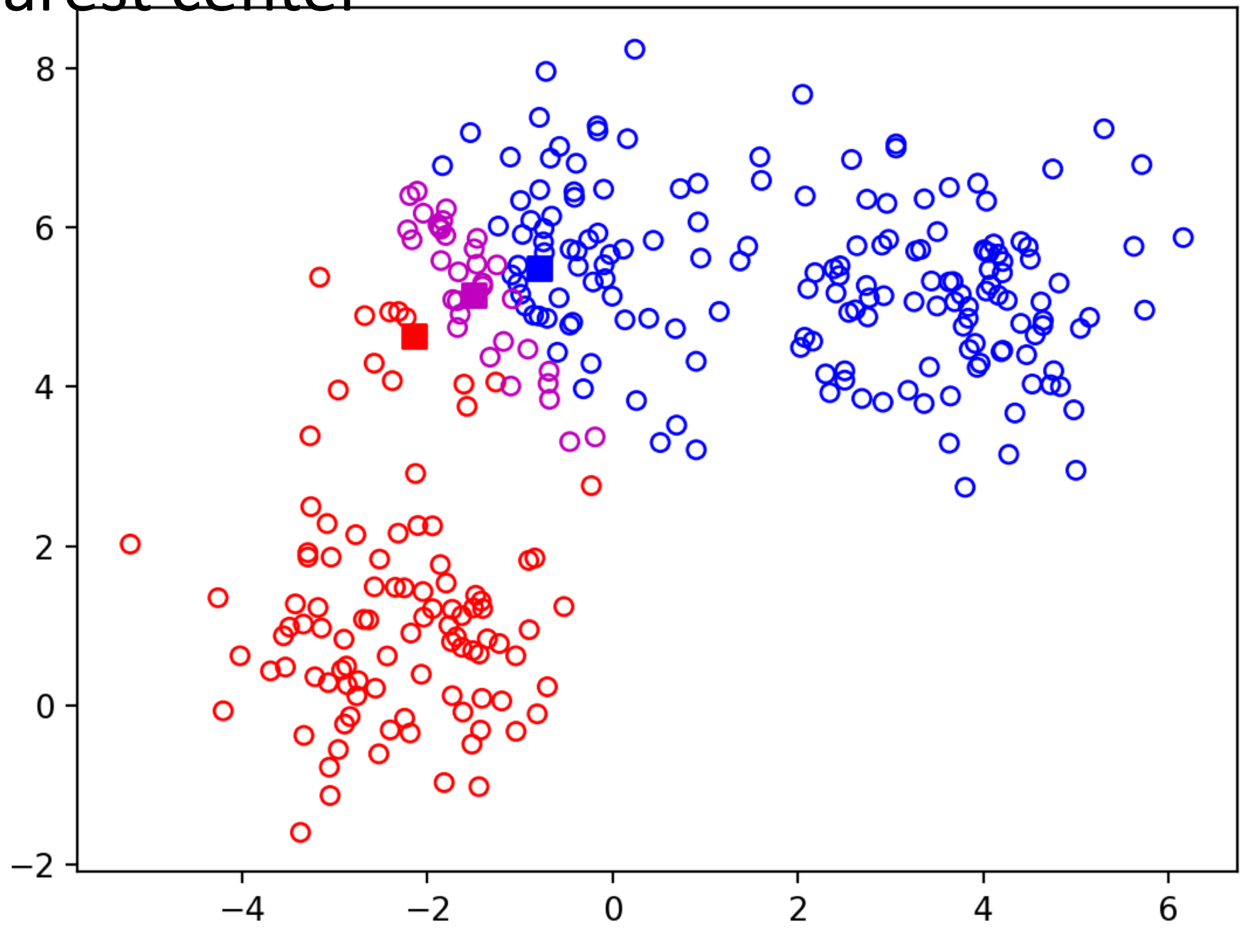
K-means

- Step-1 : randomly pick k centers



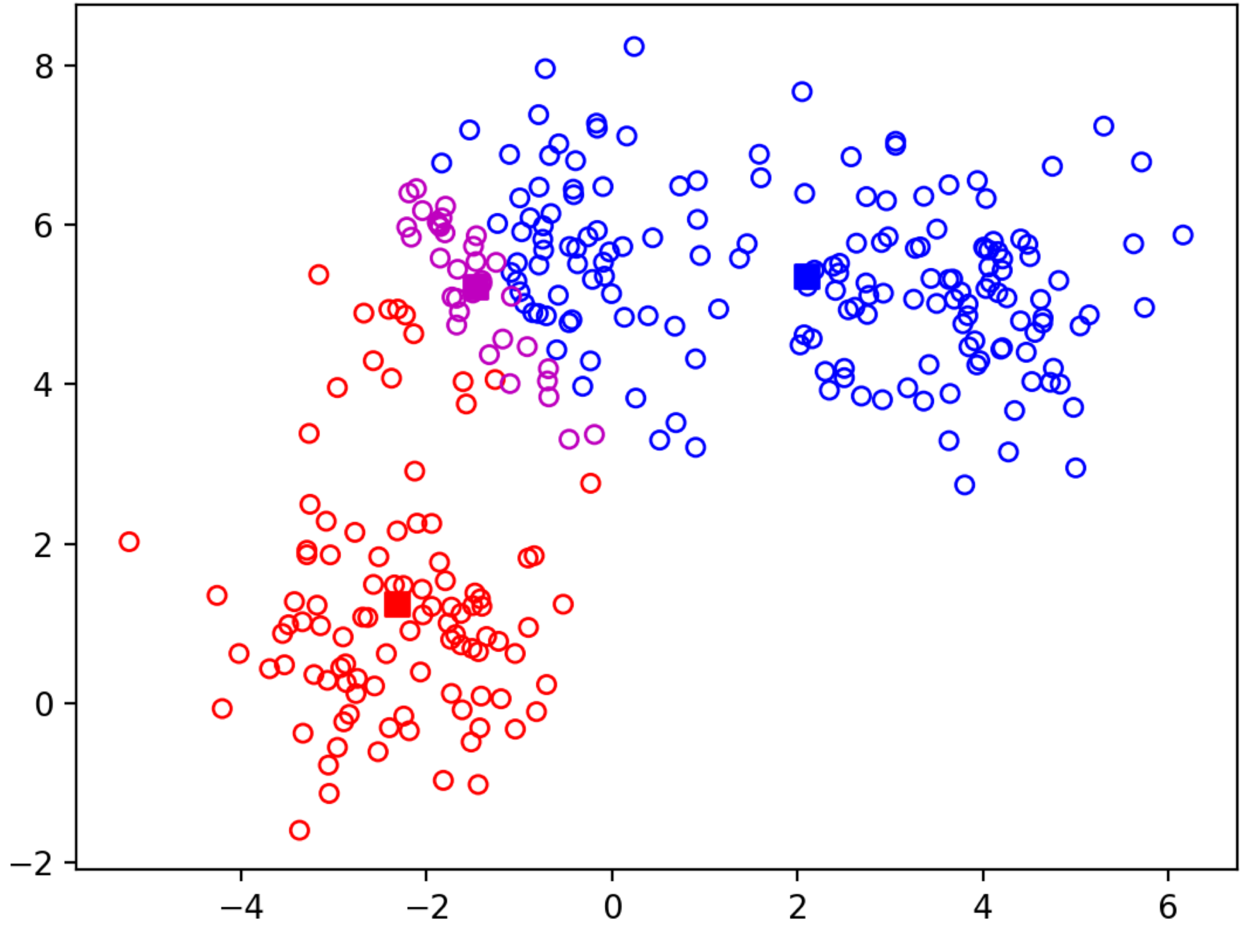
K-means

- Step 2: Assign each point to nearest center



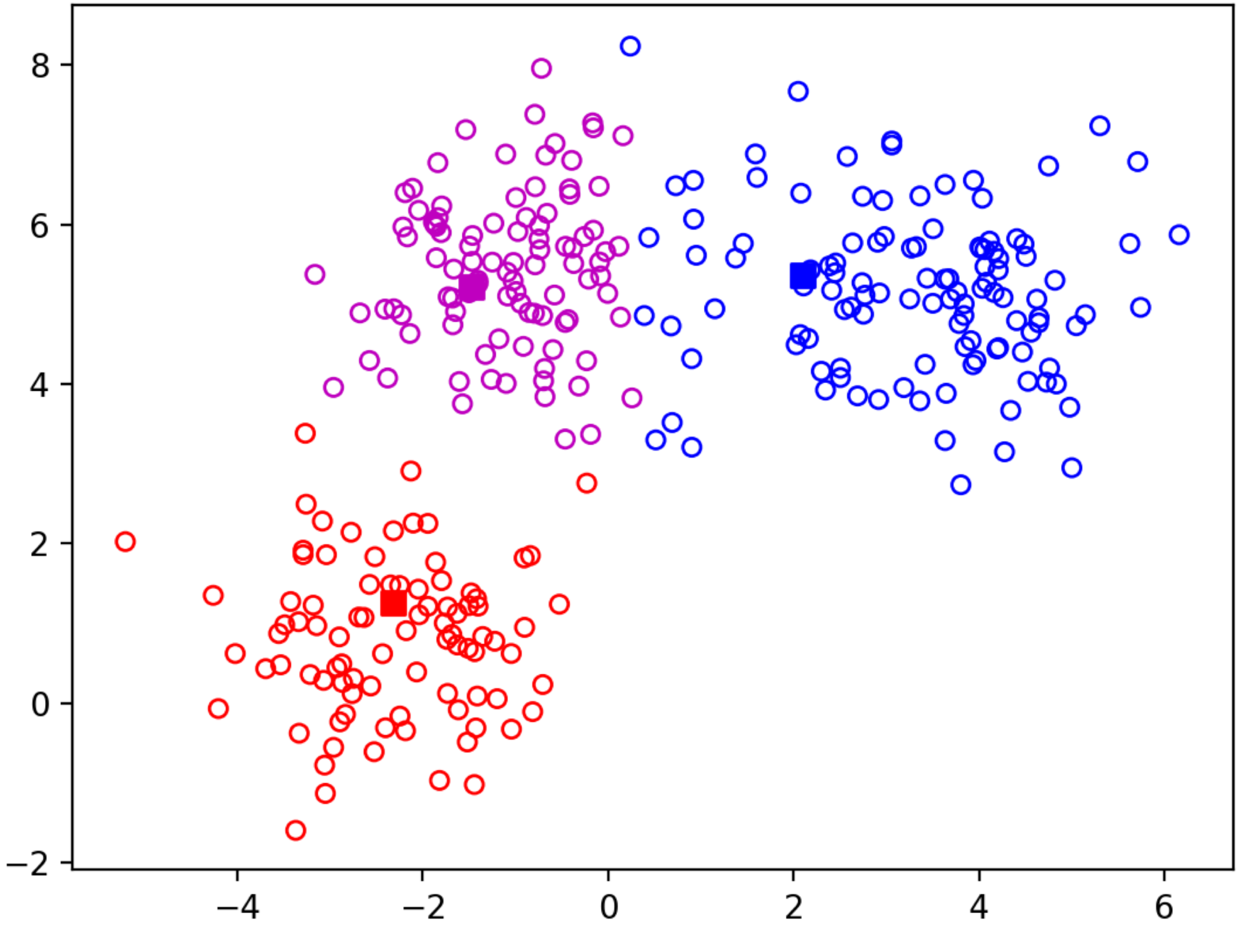
K-means

- Step 3: re-estimate centers



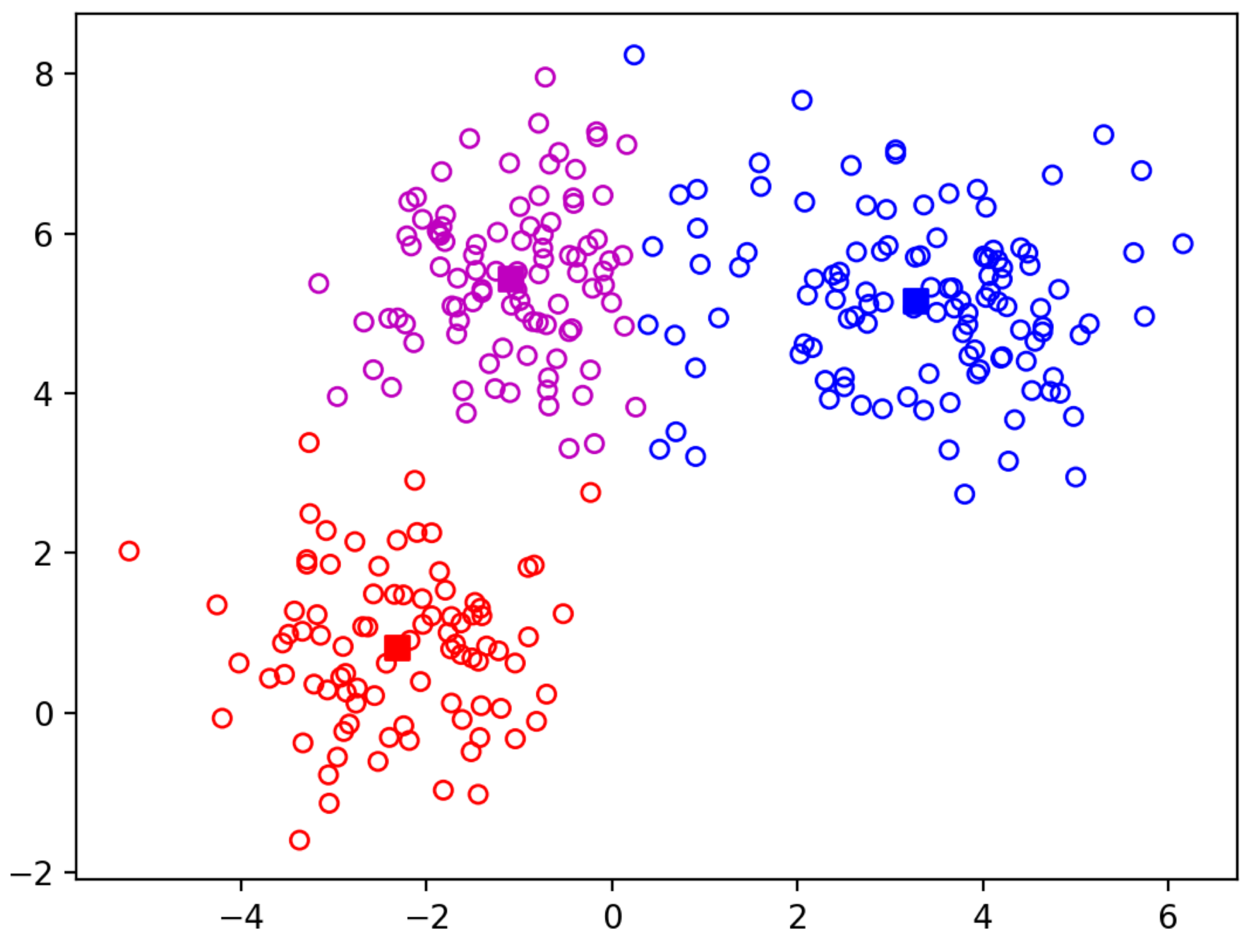
K-means

- Step 4: Repeat



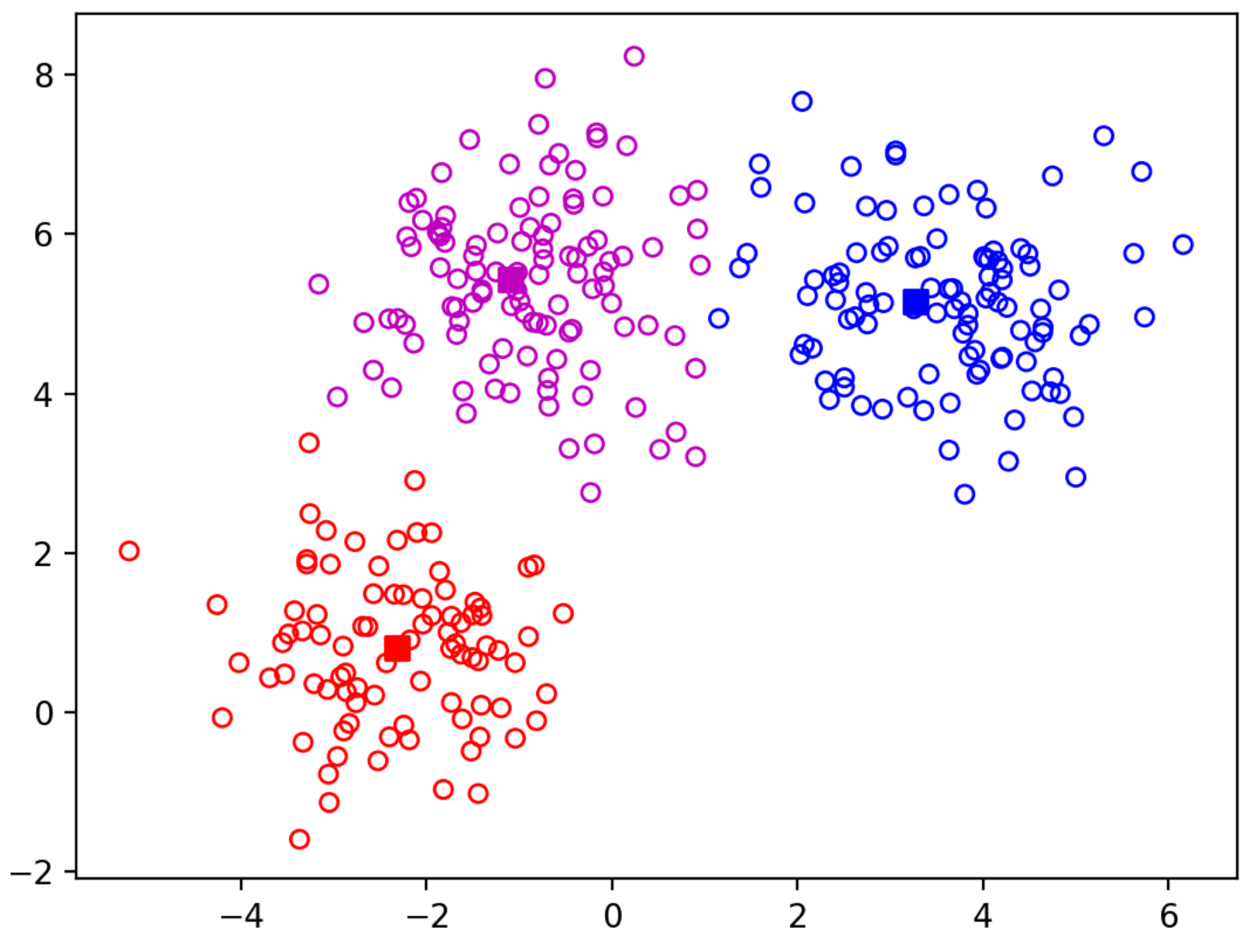
K-means

- Step 4: Repeat



K-means

- Step 4: Repeat



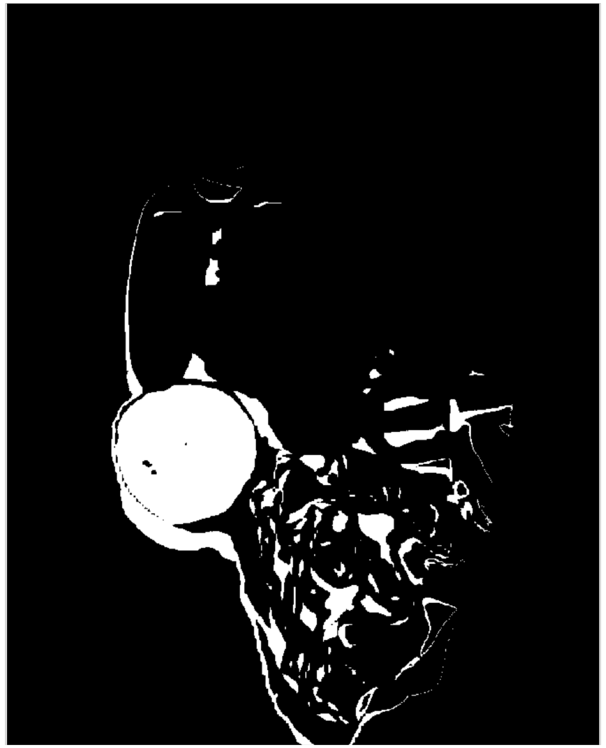
K-means on image pixels

K-means on image pixels

Picture courtesy David Forsyth

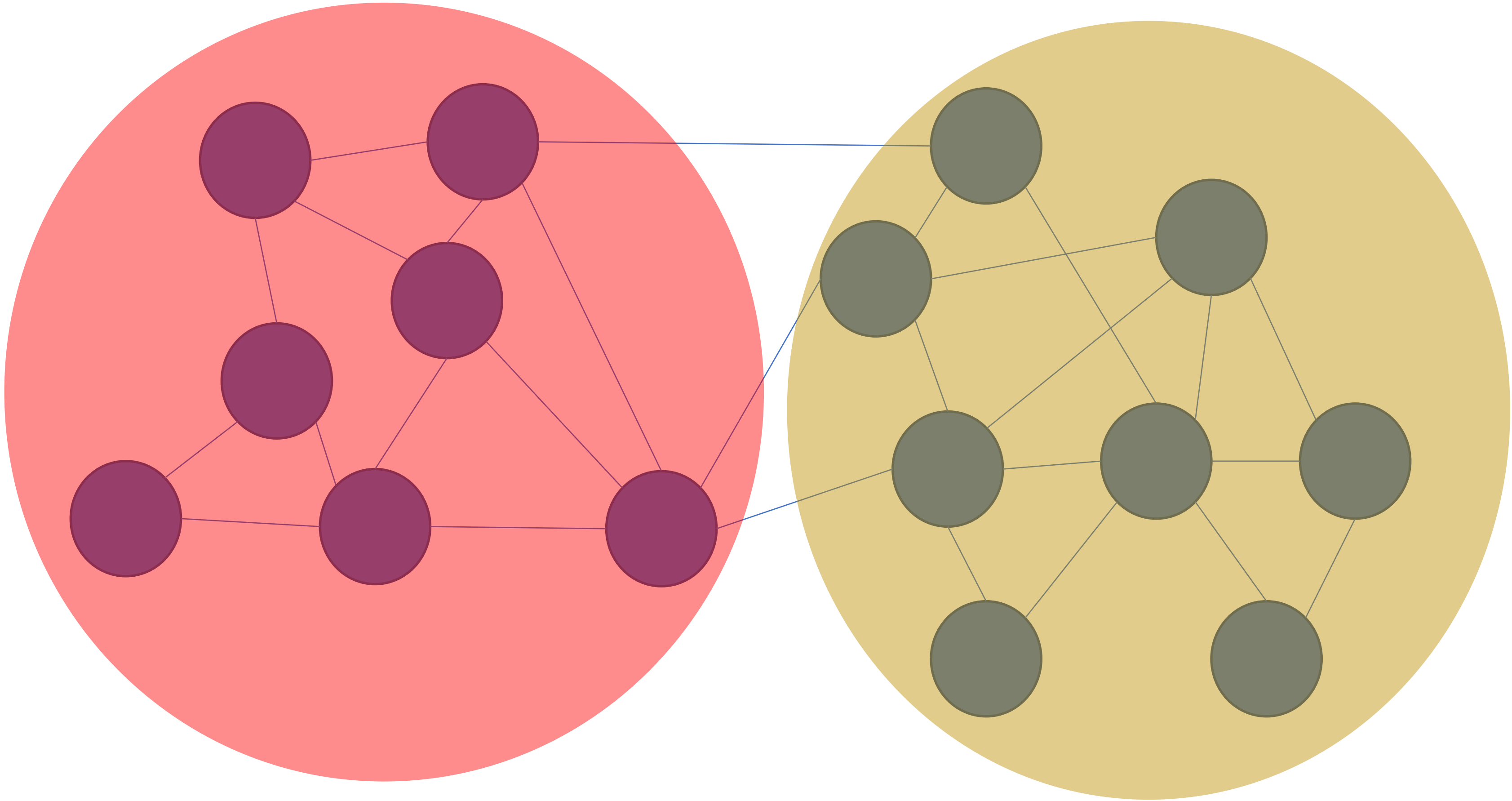
One of the clusters from k-means

K-means on image pixels+position

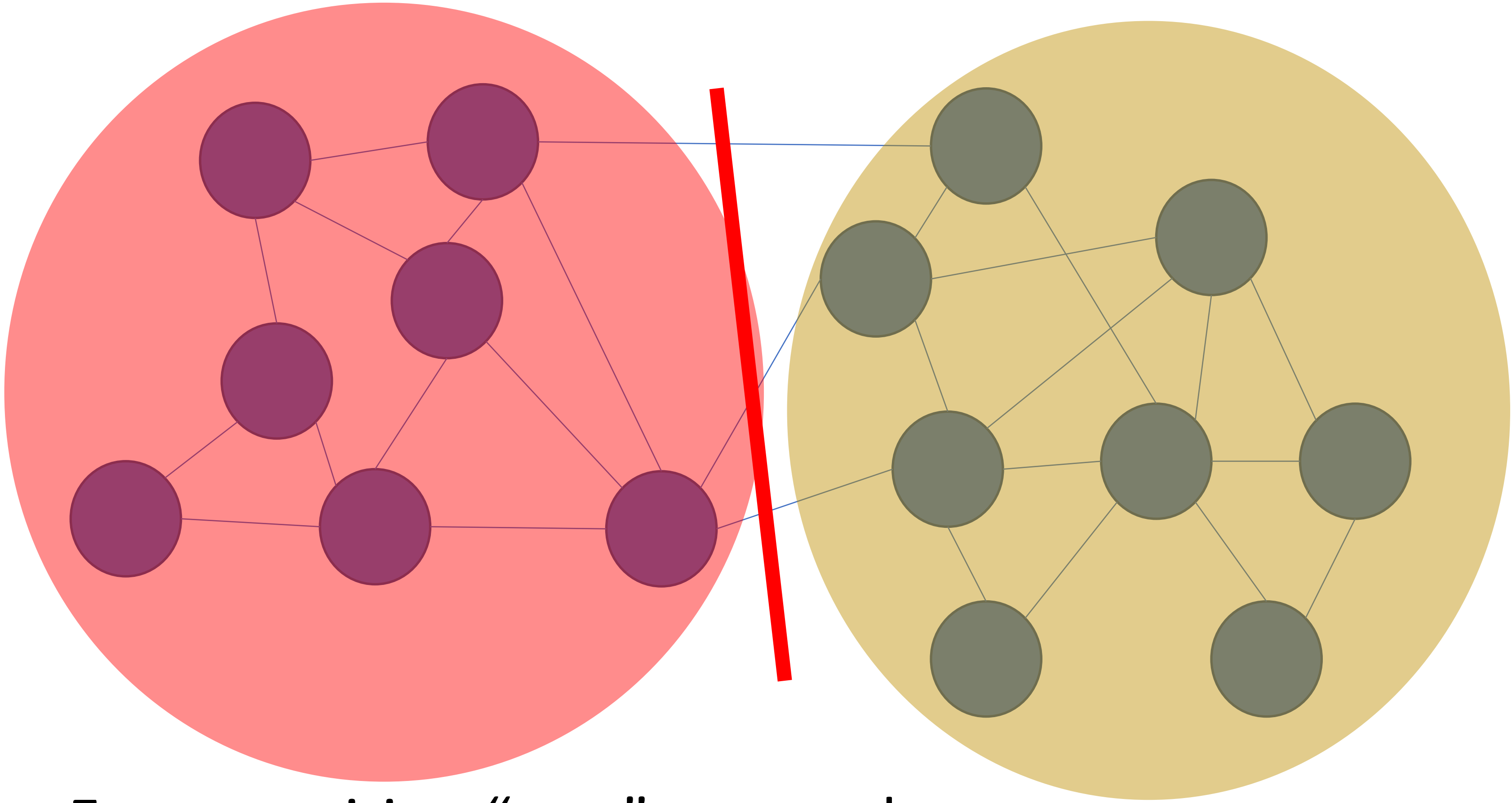


- Groups pixels together, but does not produce compact regions

Segmentation is graph partitioning

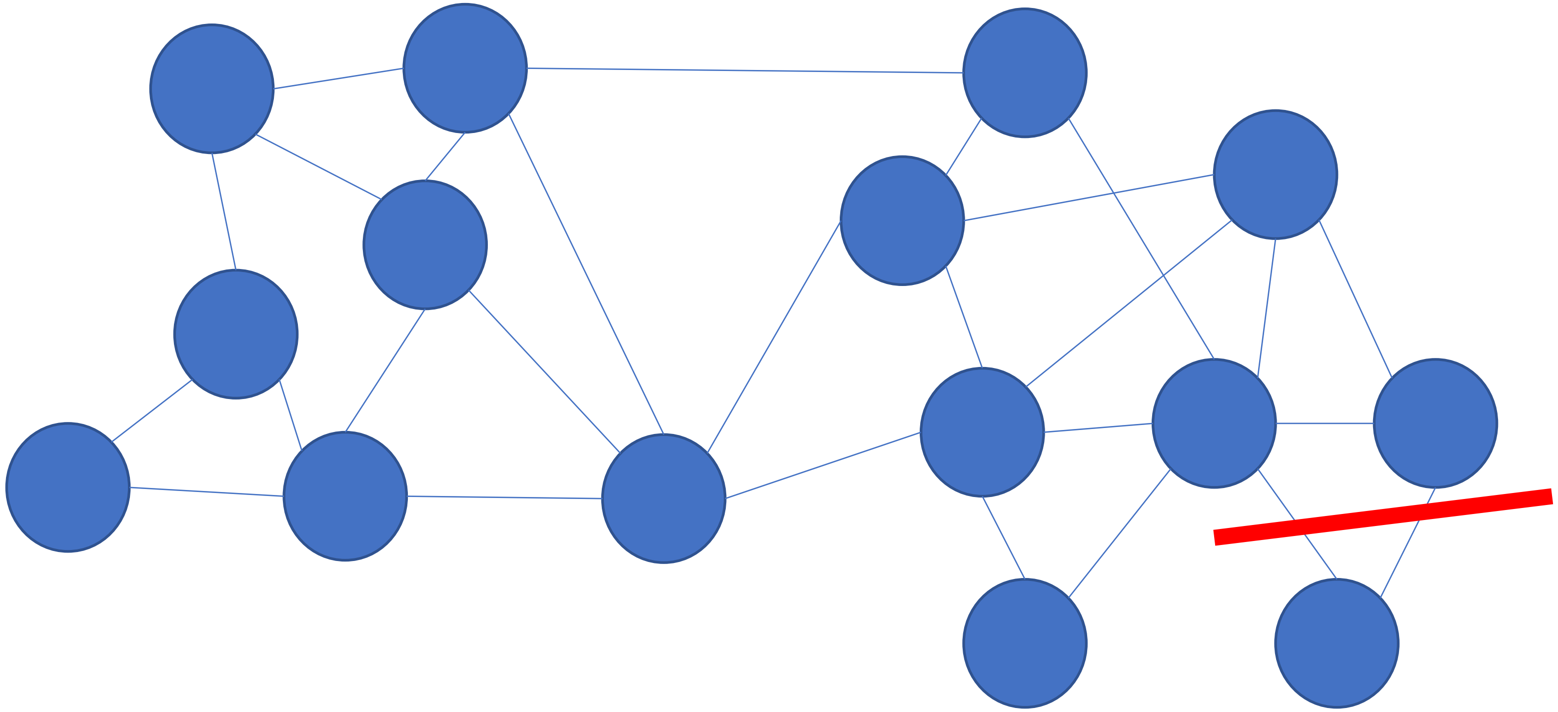


Segmentation is graph partitioning



- Every partition "cuts" some edges
- Idea: minimize total weight of edges cut!

Criterion: Min-cut?

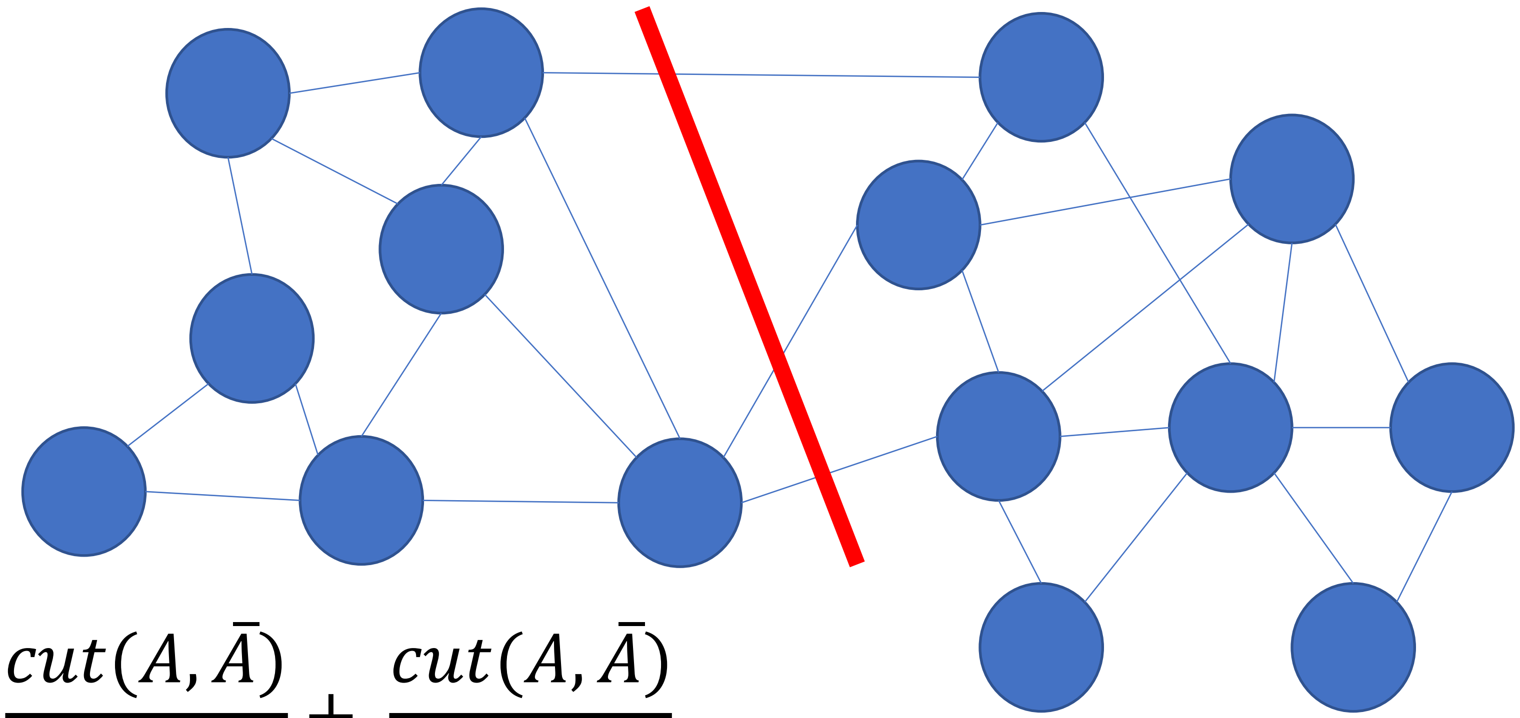


- Min-cut carves out small isolated parts of the graph
- In image segmentation: individual pixels

Normalized cuts

- "Cut" = total weight of cut edges
- Small cut means the groups don't "like" each other
- But need to normalize w.r.t how much they like *themselves*
- *"Volume"* of a subgraph = total weight of edges within the subgraph

Normalized cut



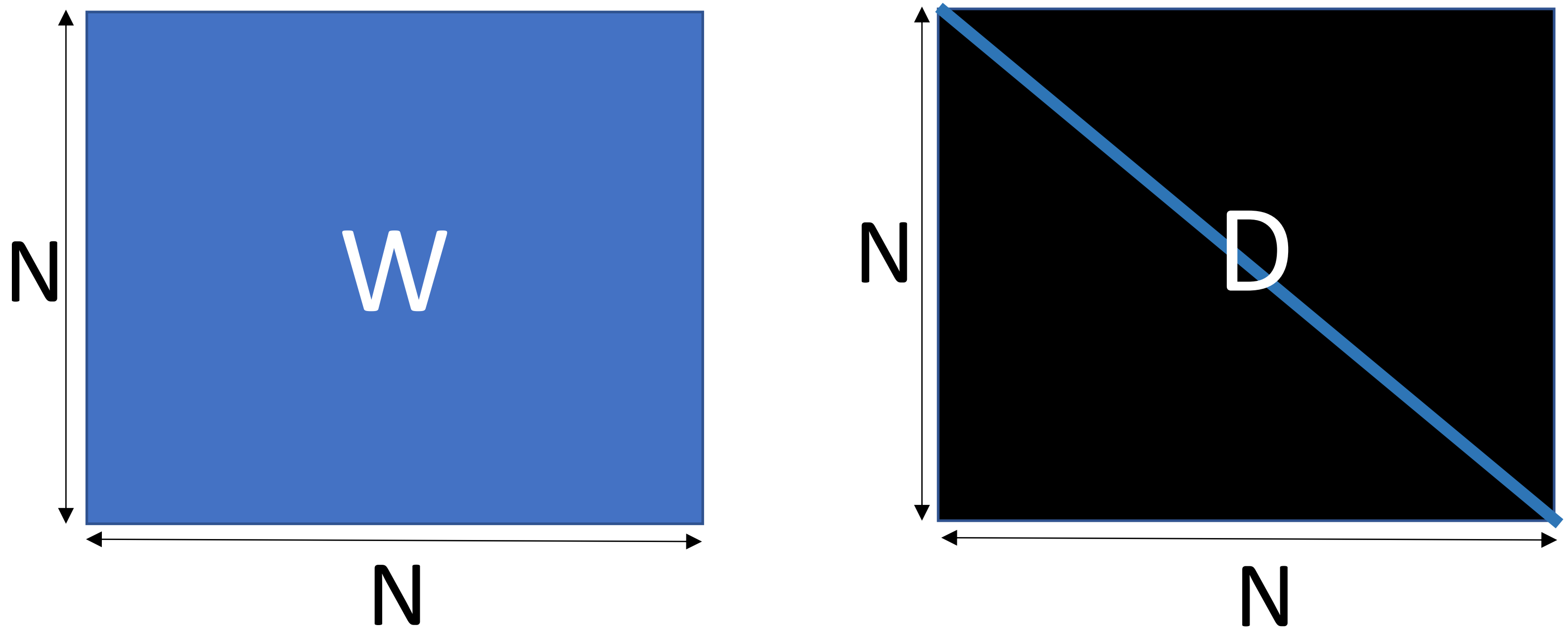
$$\frac{cut(A,\bar{A})}{vol(A)} + \frac{cut(A,\bar{A})}{vol(\bar{A})}$$

Min-cut vs normalized cut

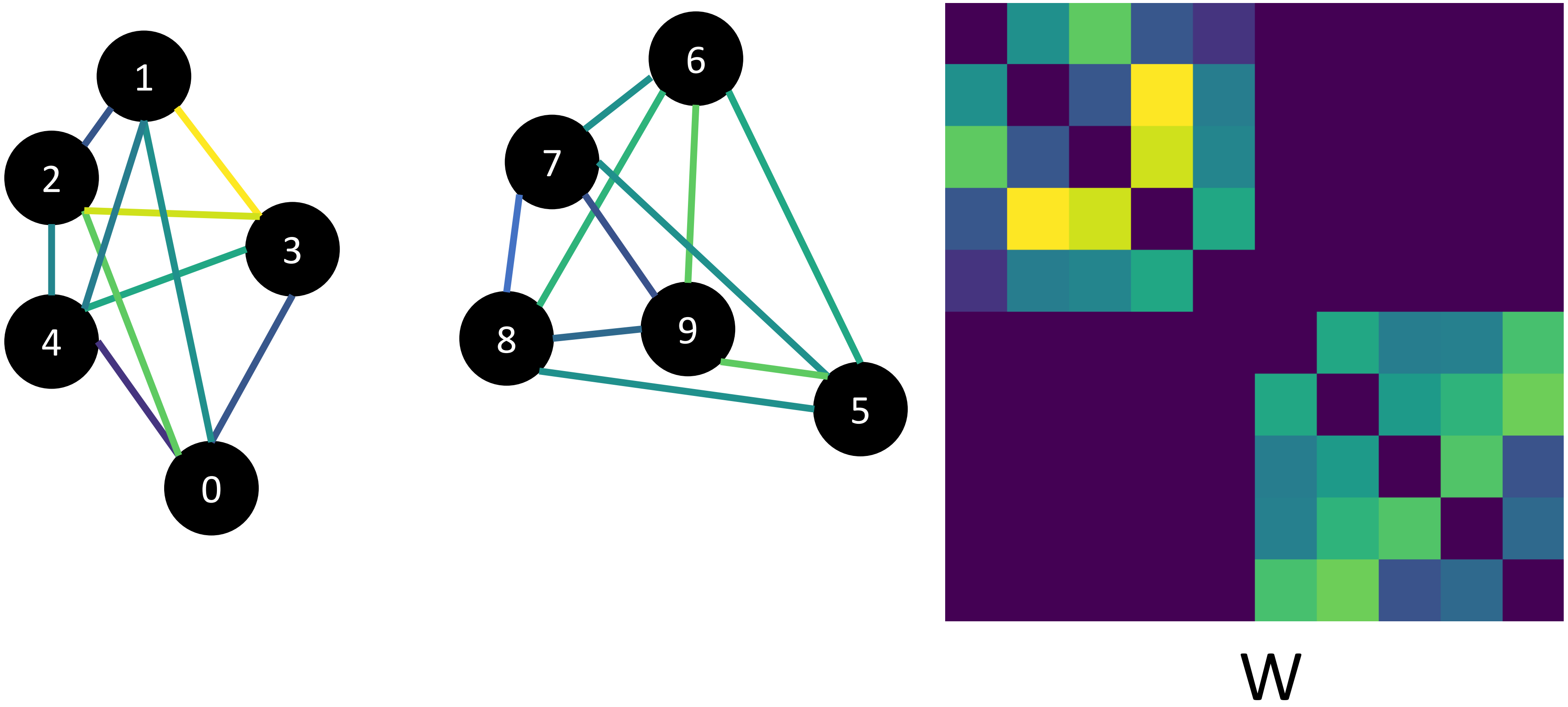
- Both rely on interpreting images as graphs
- By itself, min-cut gives small isolated pixels
  - But can work if we add other constraints
- min-cut can be solved in polynomial time
  - Dual of max-flow
- N-cut is NP-hard
  - But approximations exist!

Graphs and matrices

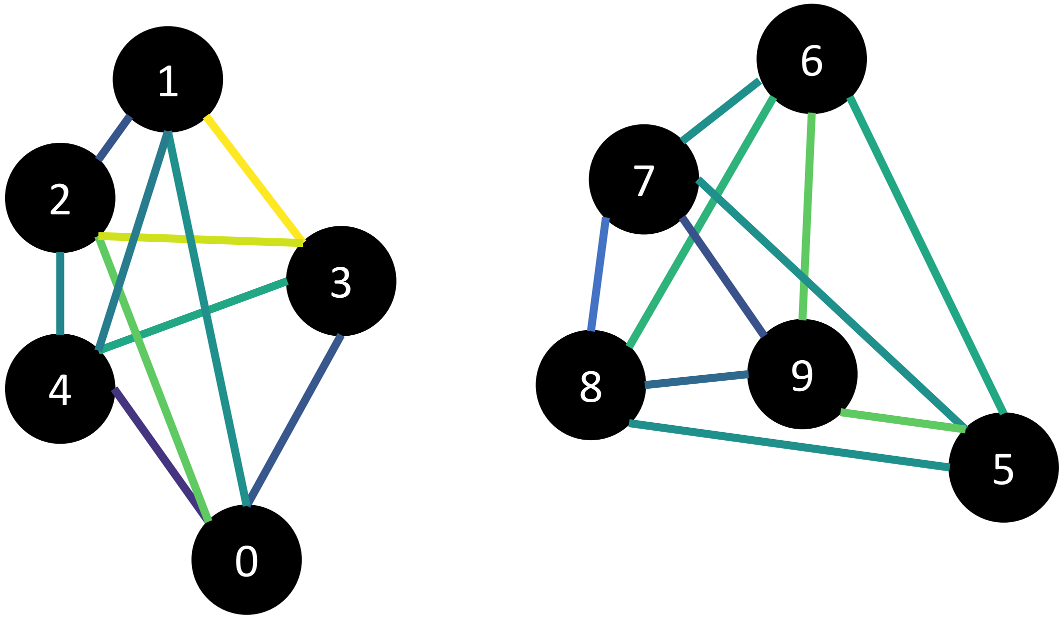
- w(i,j) = weight between i and j (*Affinity matrix)*
- d(i) = degree of i =$\sum_j w(i,j)$
- D = diagonal matrix with d(i) on diagonal



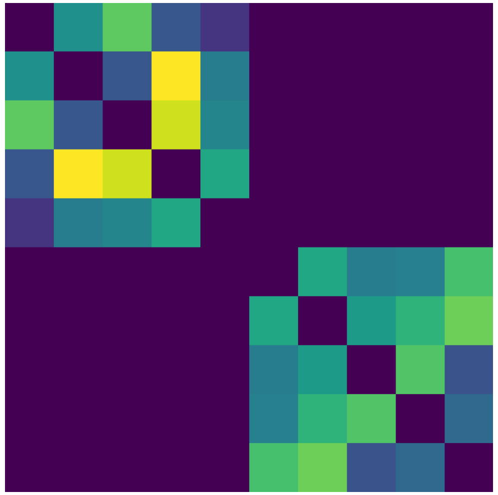
Graphs and matrices



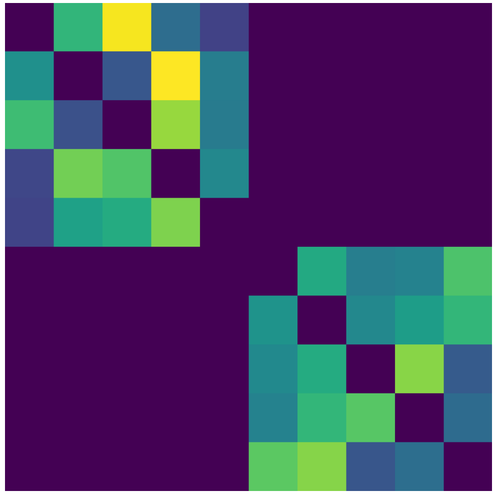
Graphs and matrices



$$E_{ij} = \frac{w_{ij}}{\sum_k w_{ik}}$$



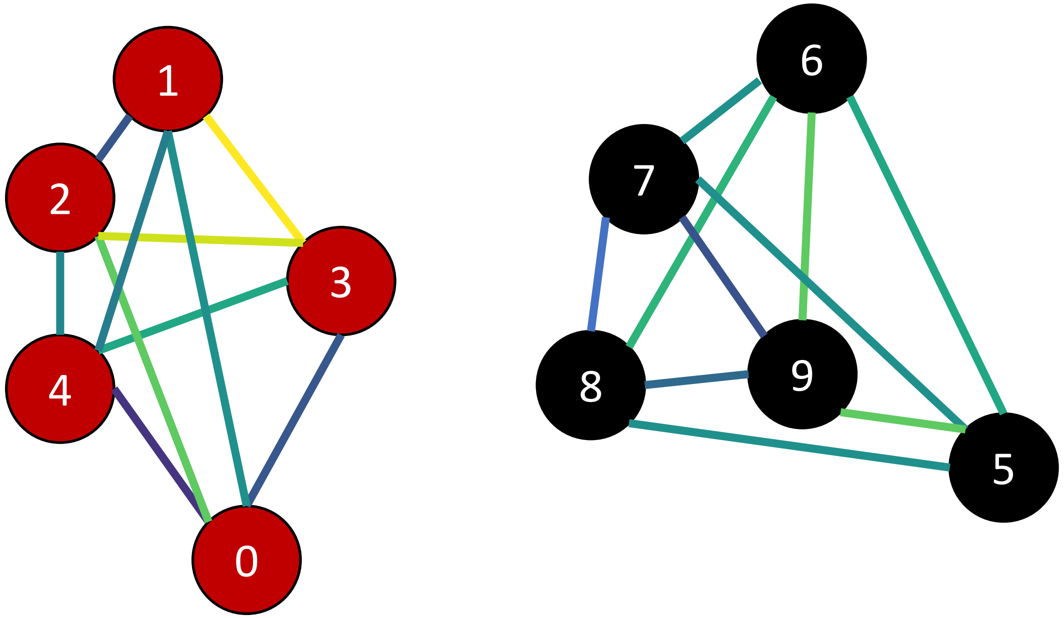
W



$E = D^{-1}W$

Graphs and matrices

- How do we represent a clustering?
- A label for N nodes
  - 1 if part of cluster A, 0 otherwise
- An N-dimensional vector!



	v_1
0:	1
1:	1
2:	1
3:	1
4:	1
5:	0
6:	0
7:	0
8:	0
9:	0

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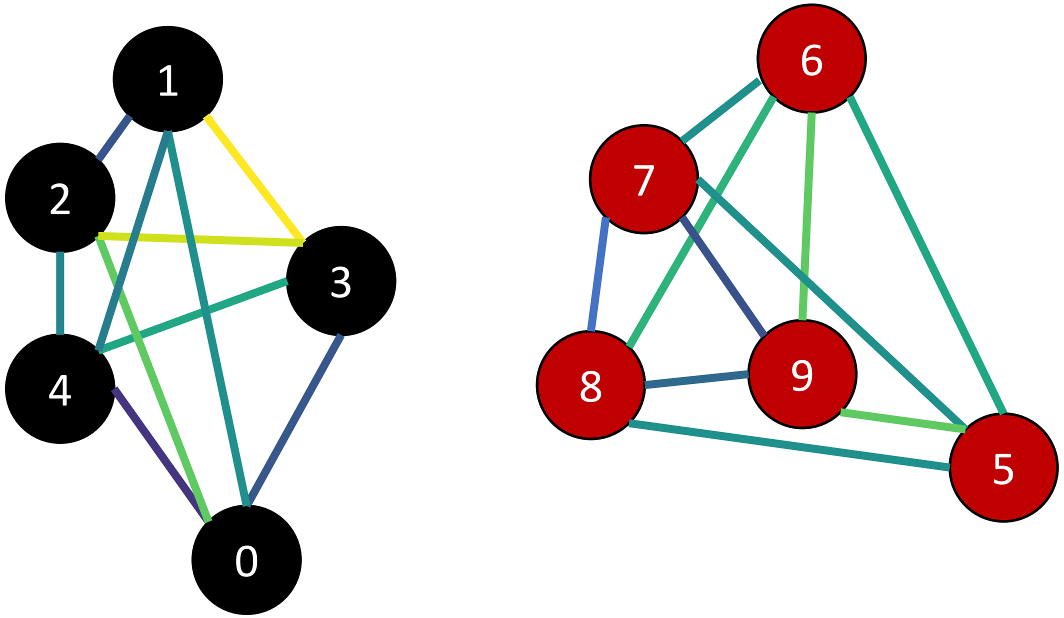

	v_1	v_2
0:	1	0
1:	1	0
2:	1	0
3:	1	0
4:	1	0
5:	0	1
6:	0	1
7:	0	1
8:	0	1
9:	0	1

Graphs and matrices

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- A label for N nodes
  - 1 if part of cluster A, 0 otherwise
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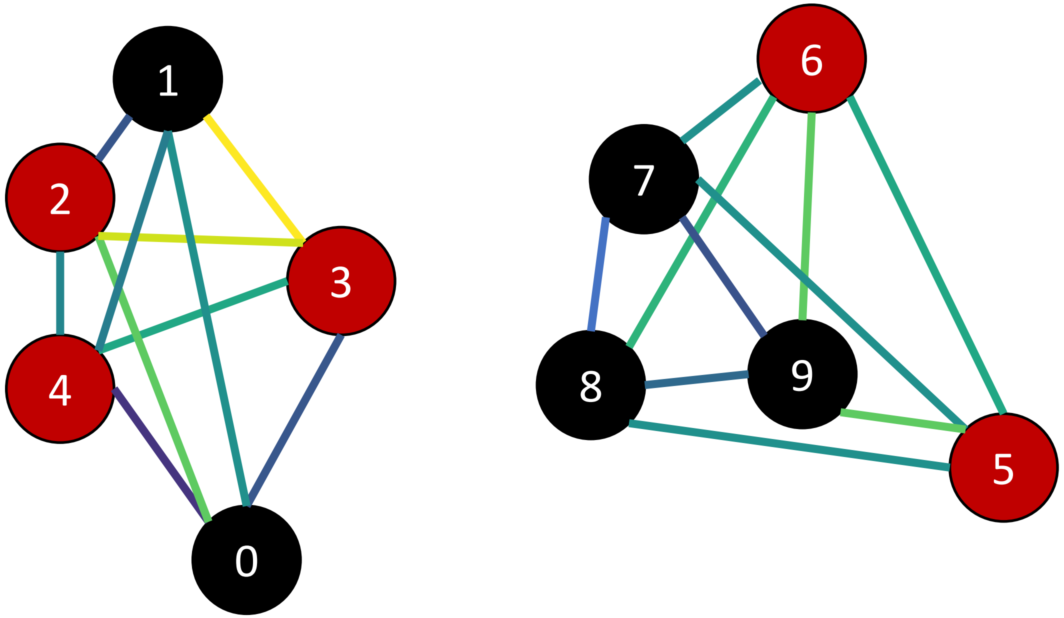
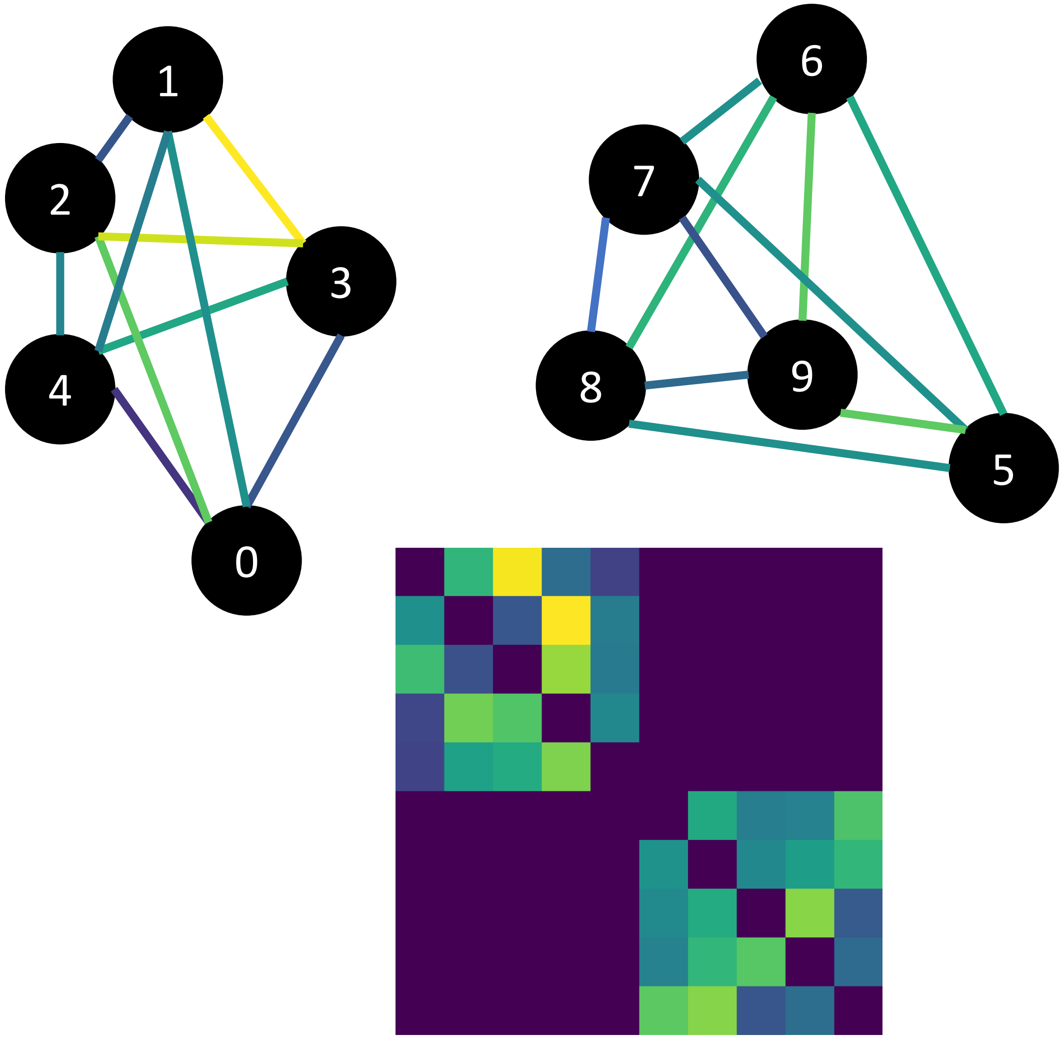


	v_1	v_2	v_3
0:	1	0	0
1:	1	0	0
2:	1	1	1
3:	1	1	1
4:	1	1	1
5:	0	1	1
6:	0	1	1
7:	0	0	0
8:	0	0	0
9:	0	0	0

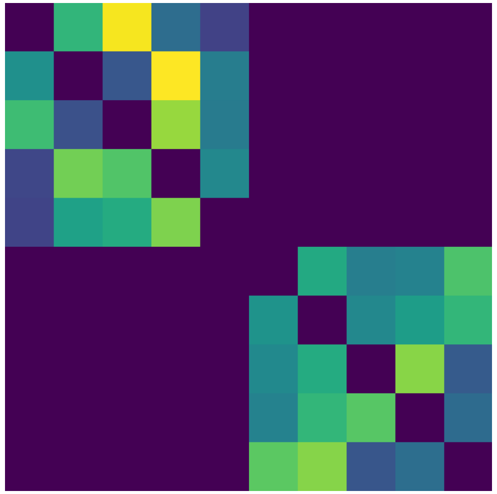
Graphs and matrices



$E = D^{-1}W$

	v_1
0:	1
1:	1
2:	1
3:	1
4:	1
5:	0
6:	0
7:	0
8:	0
9:	0

Graphs and matrices

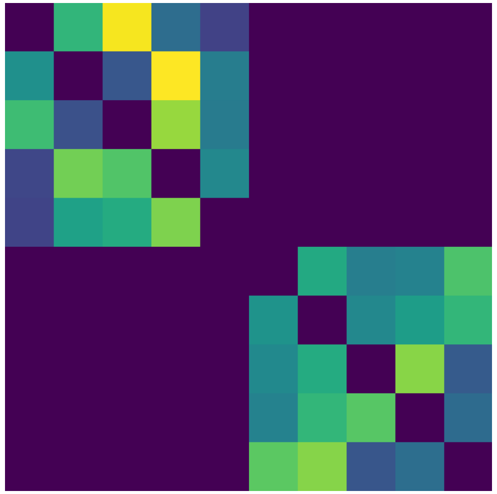


$E = D^{-1}W$

$$E_{ij} = \frac{w_{ij}}{\sum_k w_{ik}}$$

	v_1	Ev_1
0:	1	1
1:	1	1
2:	1	1
3:	1	1
4:	1	1
5:	0	0
6:	0	0
7:	0	0
8:	0	0
9:	0	0

Graphs and matrices

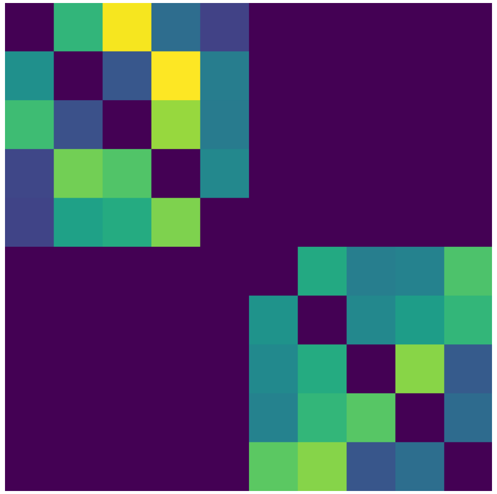


$E = D^{-1}W$

$$E_{ij} = \frac{w_{ij}}{\sum_k w_{ik}}$$

	v_2	Ev_2
0:	0	0
1:	0	0
2:	0	0
3:	0	0
4:	0	0
5:	1	1
6:	1	1
7:	1	1
8:	1	1
9:	1	1

Graphs and matrices

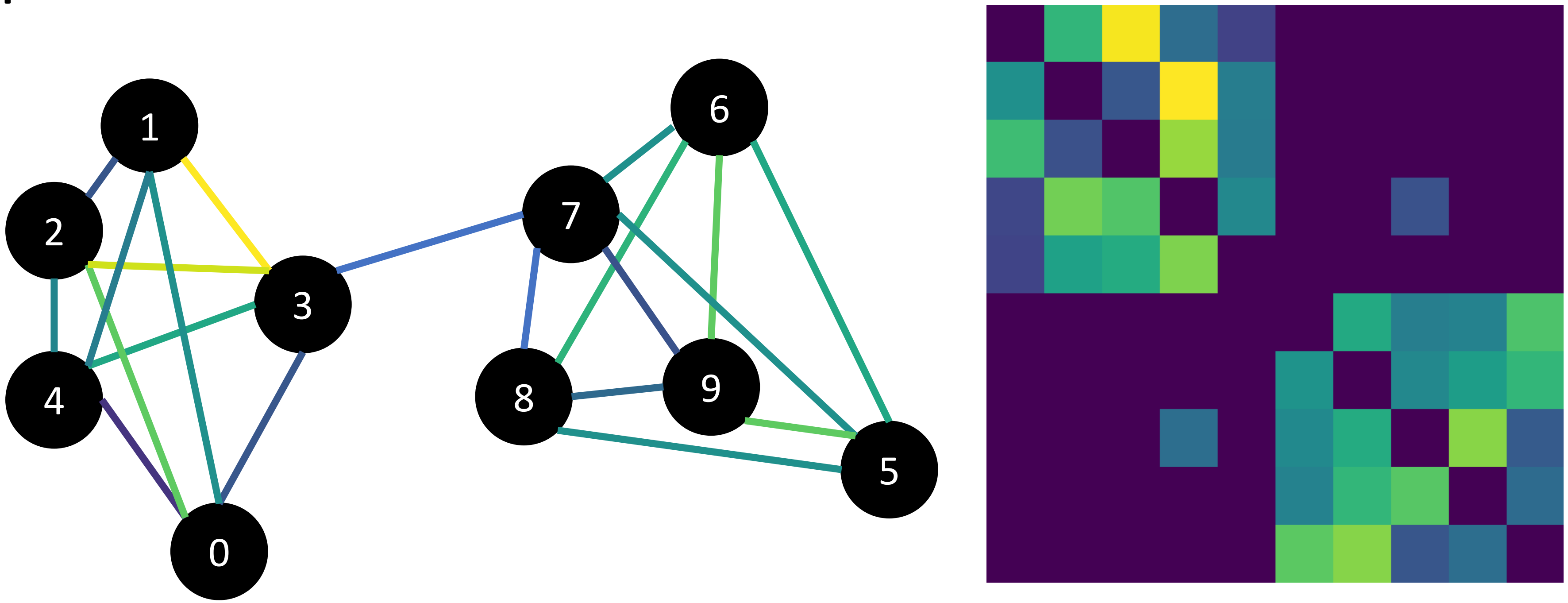


$E = D^{-1}W$

$$E_{ij} = \frac{w_{ij}}{\sum_k w_{ik}}$$

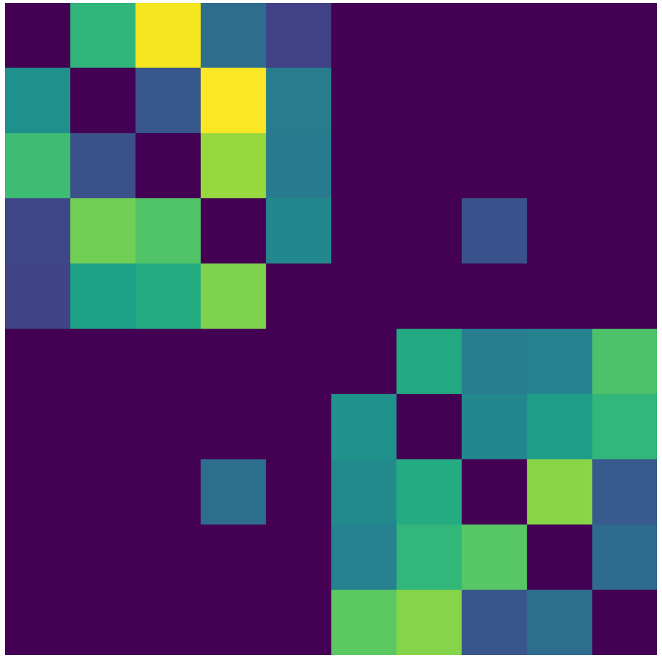
	v_3	Ev_3
0:	0	0.7
1:	0	0.8
2:	1	0.6
3:	1	0.5
4:	1	0.6
5:	1	0.3
6:	1	0.2
7:	0	0.5
8:	0	0.5
9:	0	0.7

Graphs and matrices



$E = D^{-1}W$

Graphs and matrices



E = $D^{-1}W$

$$E_{ij} = \frac{w_{ij}}{\sum_k w_{ik}}$$

	v_1	Ev_1
0:	1	1
1:	1	1
2:	1	1
3:	1	1
4:	1	1
5:	0	0
6:	0	0
7:	0	0.2
8:	0	0
9:	0	0

Graphs and matrices

$$D^{-1}Wy \approx y$$

Define z so that $y = D^{-\frac{1}{2}}z$

$$D^{-1}WD^{-\frac{1}{2}}z \approx D^{-\frac{1}{2}}z$$

$$\Rightarrow D^{-\frac{1}{2}}WD^{-\frac{1}{2}}z \approx z$$

$$\Rightarrow (I - D^{-\frac{1}{2}}WD^{-\frac{1}{2}})z \approx 0$$

Graphs and matrices

$$\Rightarrow (I - D^{-\frac{1}{2}} W D^{-\frac{1}{2}})z \approx 0$$
$$\Rightarrow \mathcal{L}z \approx 0$$

$$\mathcal{L} = I - D^{-\frac{1}{2}} W D^{-\frac{1}{2}}$$

is called the
Normalized Graph
Laplacian

Graphs and matrices

$$\mathcal{L} = I - D^{-\frac{1}{2}} W D^{-\frac{1}{2}}$$

- We want $\mathcal{L}z \approx 0$
- Trivial solution: all nodes of graph in one cluster, nothing in the other
- To avoid trivial solution, look for the *eigenvector with the second smallest eigenvalue*

$$\mathcal{L}z = \lambda z$$
$$\lambda_1 < \lambda_2 < \ldots < \lambda_N$$

- Find z s.t. $\mathcal{L}z = \lambda_2 z$

Normalized cuts

- Approximate solution to normalized cuts
- Construct matrix W and D
- Construct normalized graph laplacian

$$\mathcal{L} = I - D^{-\frac{1}{2}} W D^{-\frac{1}{2}}$$

- Look for the second smallest eigenvector

$$\mathcal{L}z = \lambda_2 z$$

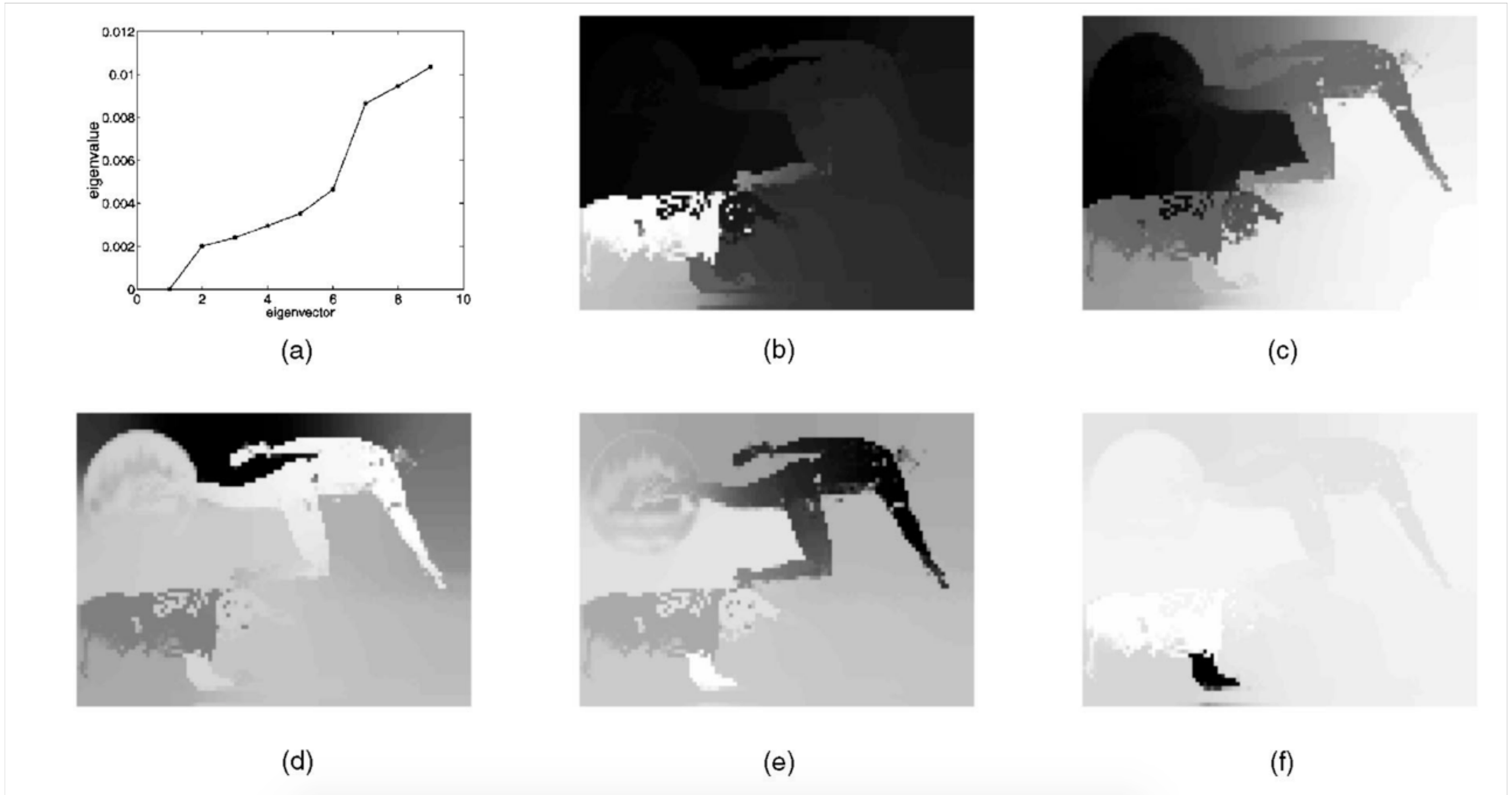
- Compute $y = D^{-\frac{1}{2}} z$
- *Threshold y to get clusters*
  - Ideally, sweep threshold to get lowest N-cut value

Eigenvectors of images

- The eigenvector has as many components as pixels in the image

Eigenvectors of images

- The eigenvector has as many components as pixels in the image



Another example

2^{nd} eigenvector

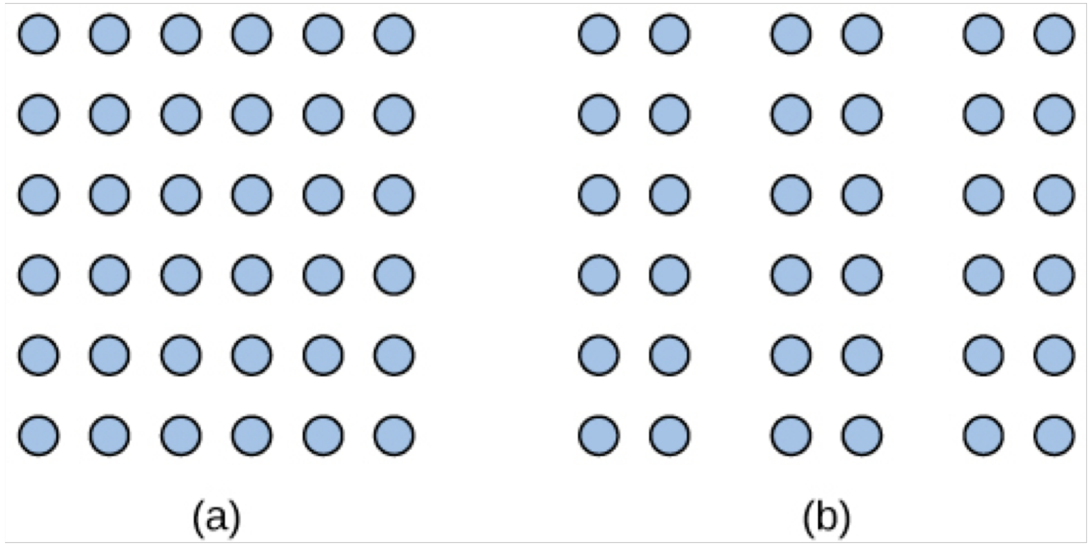
3^{rd} eigenvector

4^{th} eigenvector

Eigenvectors of images

How do we group things?

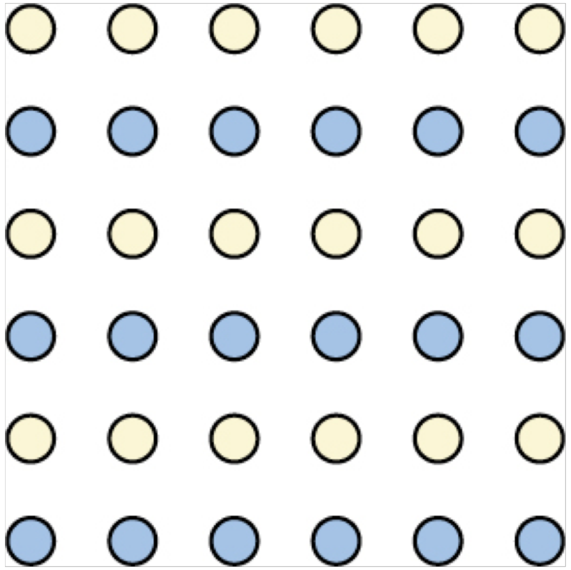
- *Gestalt* principles
- Principle of *proximity*



https://courses.lumenlearning.com/wsu-sandbox/chapter/gestalt-principles-of-perception/

How do we group things?

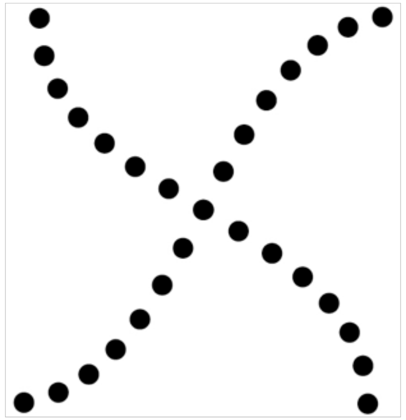
- Gestalt principles
- Principle of *similarity*



https://courses.lumenlearning.com/wsu-sandbox/chapter/gestalt-principles-of-perception/

How do we group things?

- Gestalt principles
- Principle of *continuity* and *closure*



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How do we group things?

- Gestalt principles
- Principle of *common fate*

Gestalt principles in the context of images

- Principle of proximity: nearby pixels are part of the same object
- Principle of similarity: similar pixels are part of the same object
    - Look for differences in color, intensity, or texture across the boundary
- Principle of closure and continuity: contours are likely to continue
- High-level knowledge?