Image processing

Today

- Consequences of image formation

- Some basic primitives needed for computer vision problems
  - Edge detection
  - Image resizing

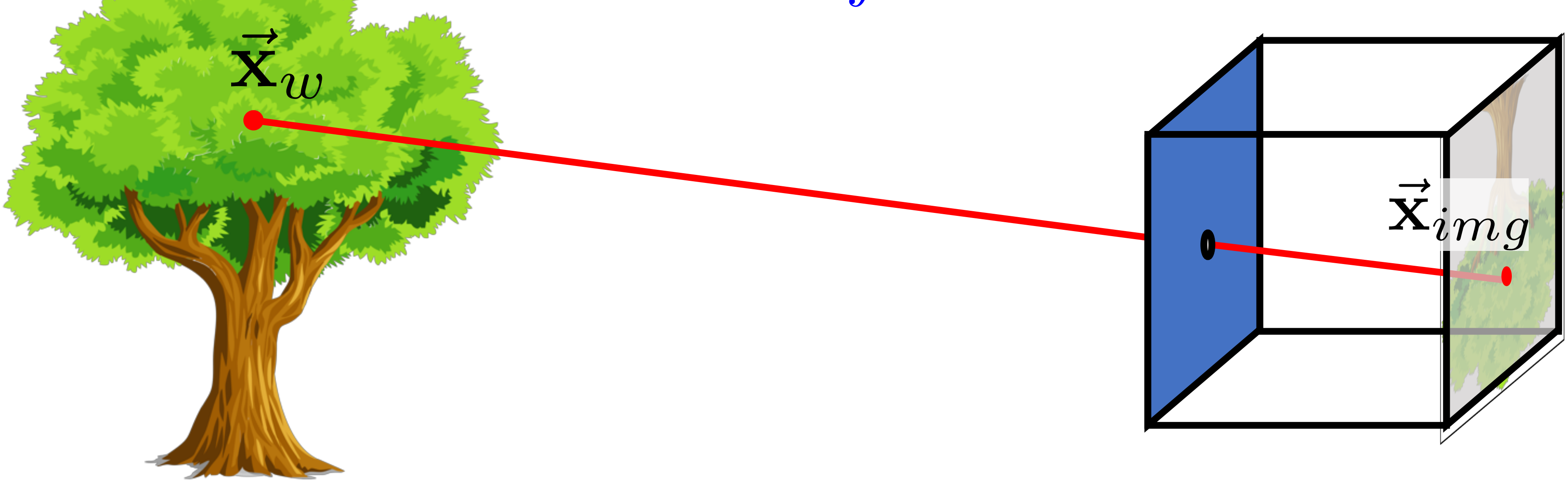
- Convolution as a basic operation
- Image pyramids as a basic structure

Recap

- Geometry: $\vec{\mathbf{x}}_{img} \equiv K \begin{bmatrix} R & \mathbf{t} \end{bmatrix} \vec{\mathbf{x}}_w$

- Color (Lambertian assumptipn):

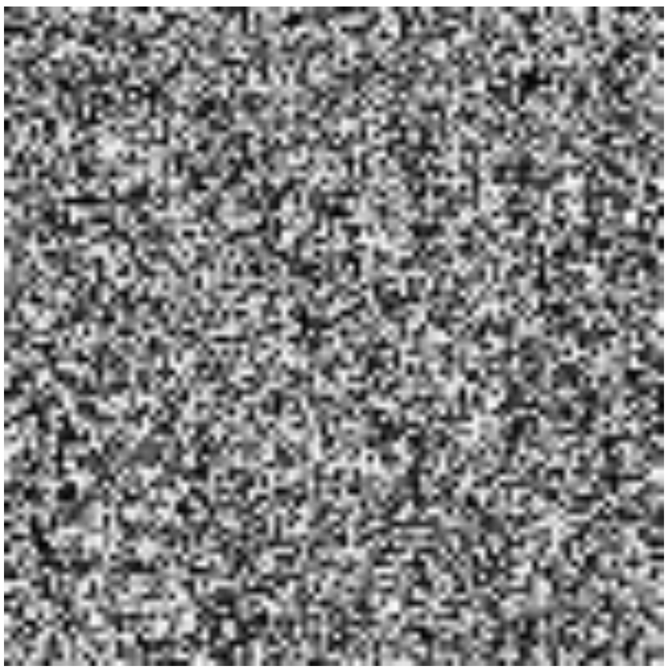
$$I(\vec{\mathbf{x}}_{img}) = \rho(\vec{\mathbf{x}}_w) \int L(\vec{\mathbf{x}}_w, \Omega) \cos\theta(\vec{\mathbf{x}}_w, \Omega) d\Omega$$

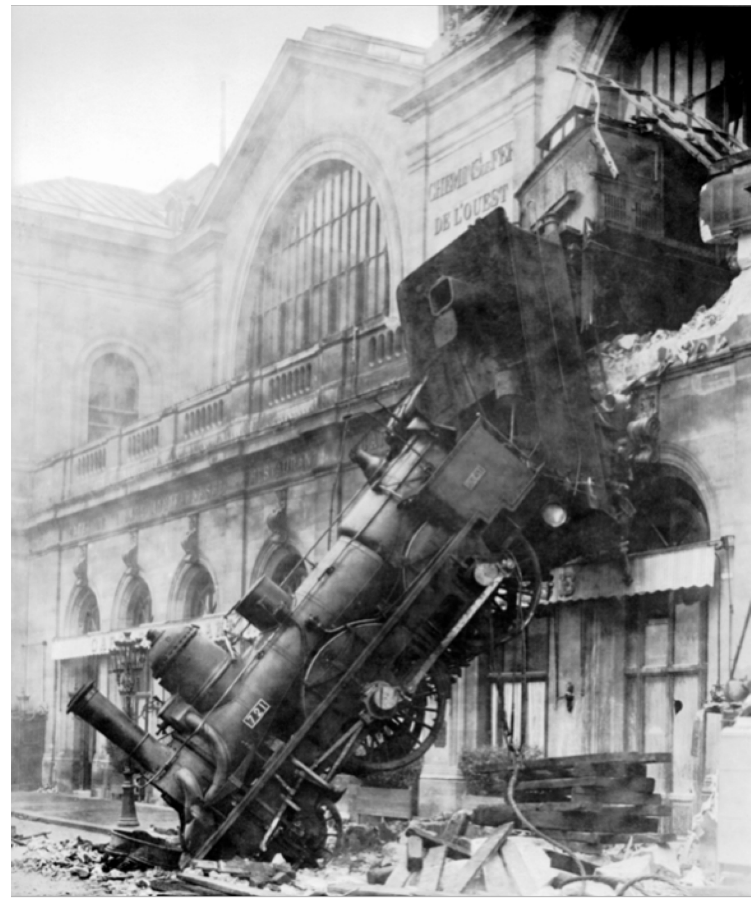


Consequences of image formation

- Nearby objects appear larger
- Parallel lines and planes converge
- Information lost: distance from camera
- Pixel color depends on light intensity, light direction and surface normal and paint on object
- So objects in images
  - can appear in many different sizes and many positions
  - can have very different color

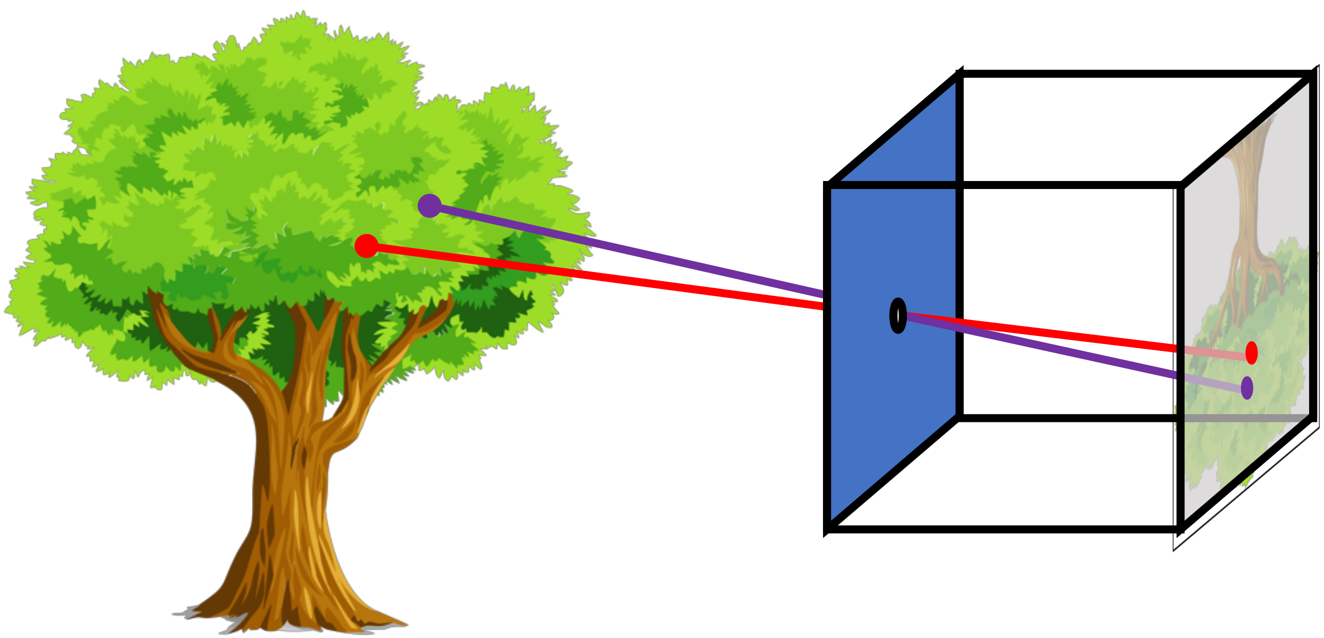
Consequence 1: nearby pixels are similar





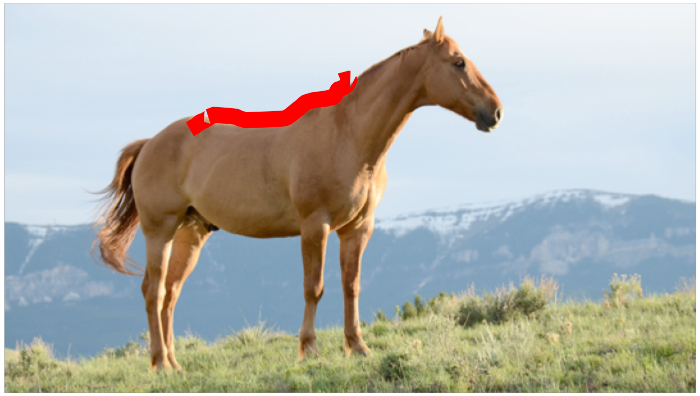
Consequence 1: nearby pixels are similar

- Why?
- Nearby pixels in pinhole camera lead to nearby rays
- Nearby rays *mostly* fall on the same object
- Objects have *mostly* smooth surfaces and *mostly* uniform color
- Lighting is *mostly* uniform



Consequence 1: nearby pixels are similar

- Nearby pixels that are *not* similar tend to have different depth, surface normal, paint or lighting
- Idea: *Abrupt changes in color can delineate objects, be a clue to shape, or be distinctive marks*



Depth discontinuities

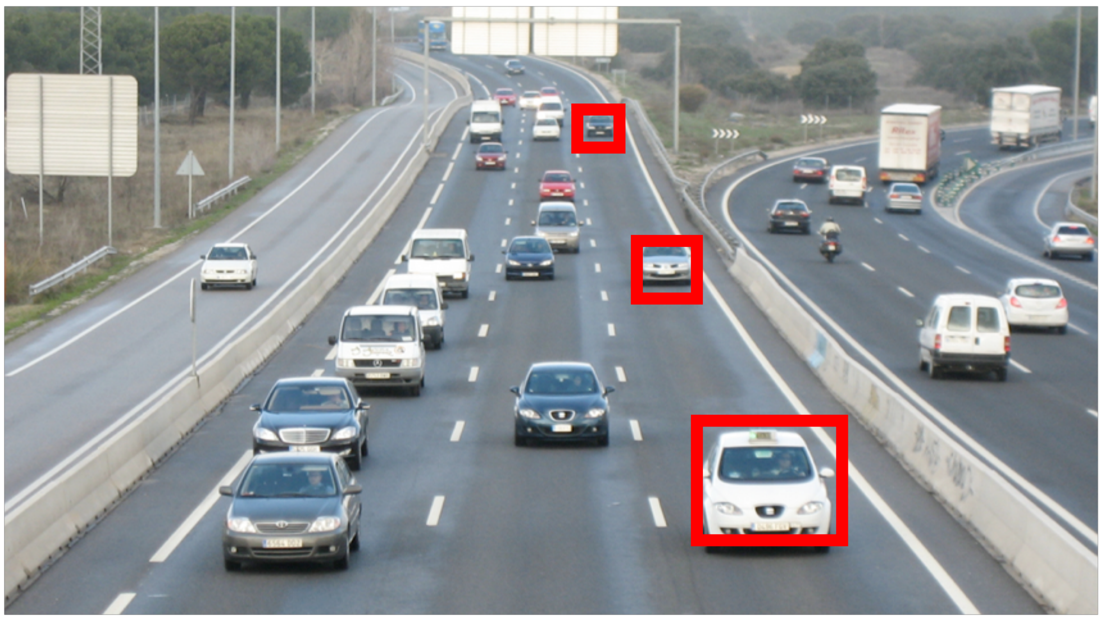
Changes in albedo

Normal discontinuities

Key primitive: edge detection

Consequence 2: Farther away objects appear smaller

Key primitive: Image resizing



- May need to match objects/patches across different scales.

Some primitives

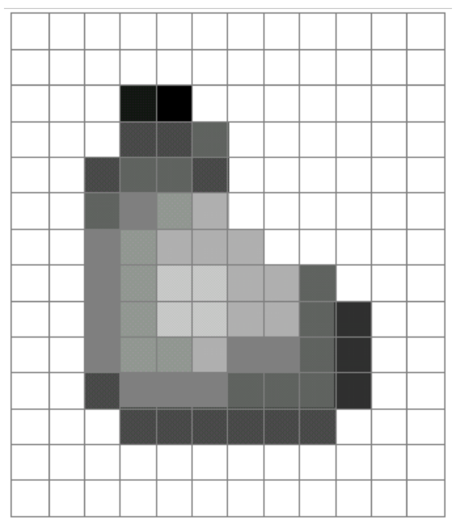
- Edge detection: identifying where pixels change color
  - Cue to object boundary
  - Cue to shape
  - More resilient to lighting than pixel color
- Image resizing: downsizing or upscaling images
  - Allows searching over scales
- Basic operation: *convolution*

Convolution

Image denoising

What is an image?

- A grid (matrix) of intensity values: 1 color or 3 colors



=

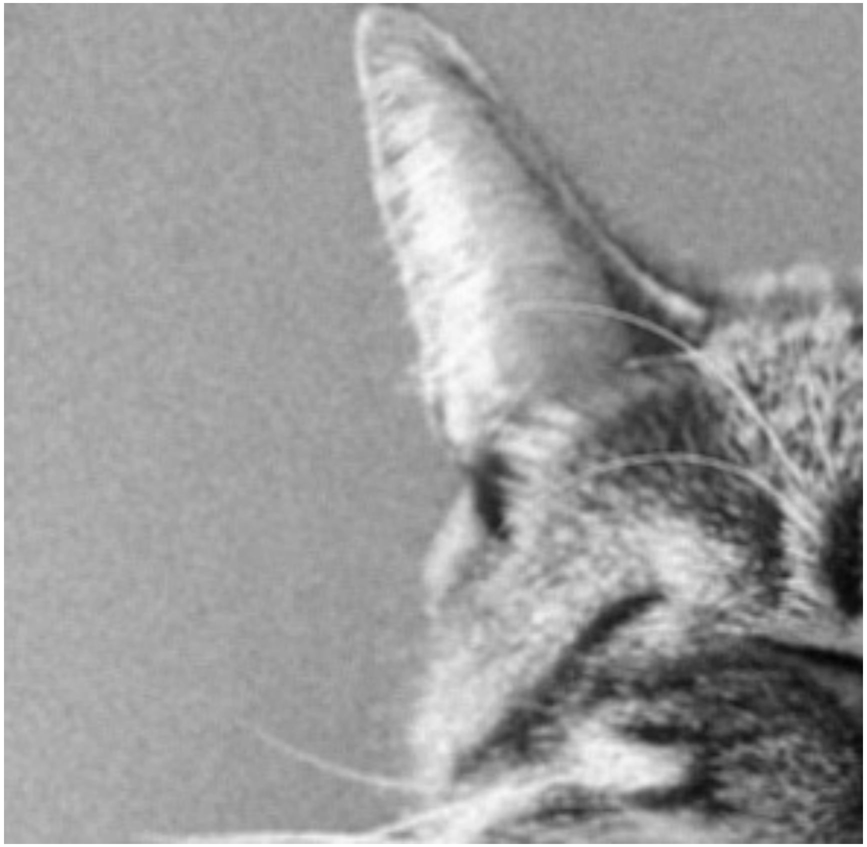
255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	20	0	255	255	255	255	255	255	255
255	255	255	75	75	75	255	255	255	255	255	255
255	255	75	95	95	75	255	255	255	255	255	255
255	255	96	127	145	175	255	255	255	255	255	255
255	255	127	145	175	175	175	255	255	255	255	255
255	255	127	145	200	200	175	175	95	255	255	255
255	255	127	145	200	200	175	175	95	47	255	255
255	255	127	145	145	175	127	127	95	47	255	255
255	255	74	127	127	127	95	95	95	47	255	255
255	255	255	74	74	74	74	74	74	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255

Mean filtering: replace pixel by mean of neighborhood

0	0	0	0	0	0	0	0	0	0
0	0	0	10	10	10	0	0	0	0
0	0	10	20	20	20	10	40	0	0
0	10	20	30	0	20	10	0	0	0
0	10	0	30	40	30	20	10	0	0
0	10	20	30	40	30	20	10	0	0
0	10	20	10	40	30	20	10	0	0
0	10	20	30	30	20	10	0	0	0
0	0	10	20	20	0	10	0	20	0
0	0	0	10	10	10	0	0	0	0

$$(0 + 0 + 0 + 10 + 40 + 0 + 10 + 0 + 0)/9 = 6.66$$

Noise reduction using mean filtering



A more general version

0	10	5	7	0
5	11	6	8	3
9	22	4	5	1
2	9	14	6	7
3	10	15	12	9

Local image data

		7		

Kernel size = 2k+1

$$S[f](m,n) = \sum_{i=-k}^{k} \sum_{j=-k}^{k} w(i,j) f(m+i, n+j)$$

Convolution and cross-correlation

- Cross correlation

$$S[f] = w \otimes f$$

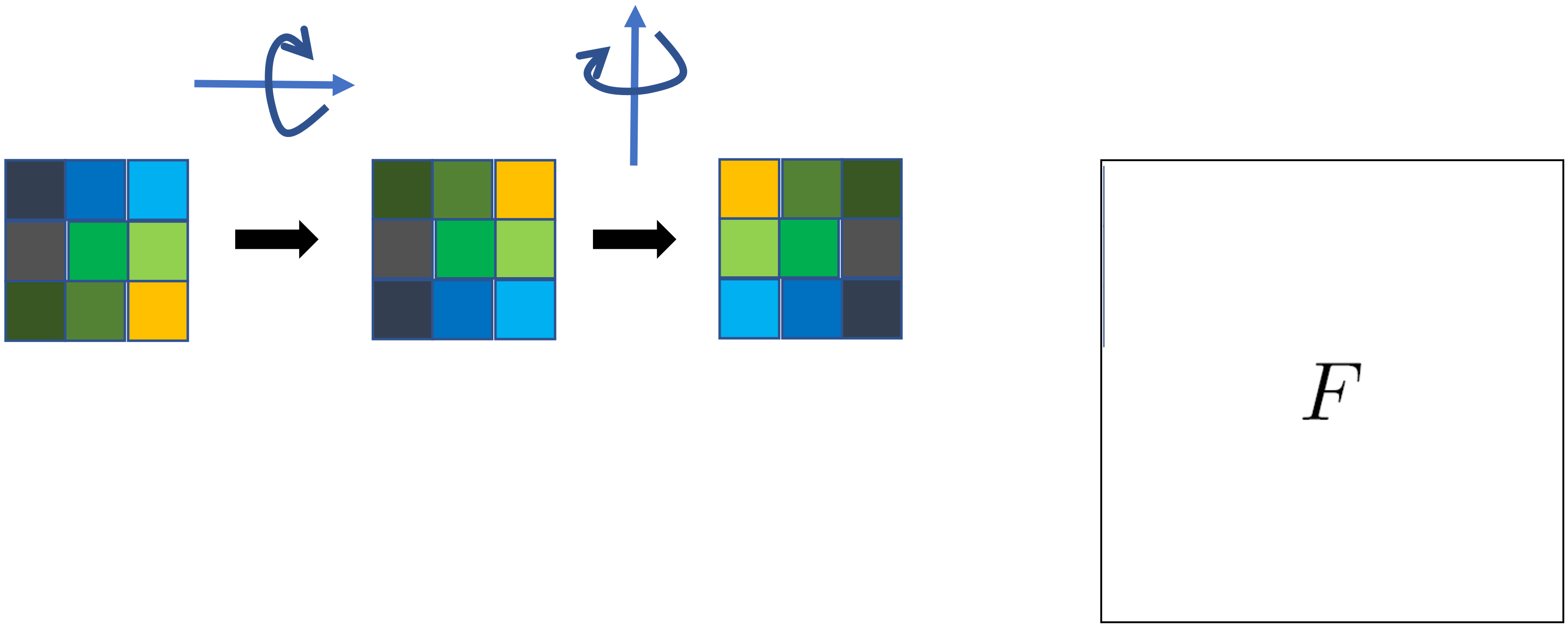
$$S[f](m,n) = \sum_{i=-k}^{k} \sum_{j=-k}^{k} w(i,j) f(m+i, n+j)$$

- Convolution

$$S[f] = w * f$$

$$S[f](m,n) = \sum_{i=-k}^{k} \sum_{j=-k}^{k} w(i,j) f(m-i, n-j)$$

Convolution



Adapted from F. Durand

Properties: Linearity

$$(w \otimes f)(m,n) = \sum_{i=-k}^{k} \sum_{j=-k}^{k} w(i,j) f(m+i, n+j)$$

$$f' = af + bg$$

$$w \otimes f' = a(w \otimes f) + b(w \otimes g)$$

Properties: Linearity

$$(w \otimes f)(m,n) = \sum_{i=-k}^{k} \sum_{j=-k}^{k} w(i,j) f(m+i, n+j)$$

$$w' = aw + bv$$

$$w' \otimes f = a(w \otimes f) + b(v \otimes f)$$

Properties: Shift invariance

$$(w \otimes f)(m,n) = \sum_{i=-k}^{k} \sum_{j=-k}^{k} w(i,j) f(m+i, n+j)$$

$$f'(m,n) = f(m-m_0, n-n_0)$$

f

f'

Shift invariance

$$(w \otimes f)(m,n) = \sum_{i=-k}^{k} \sum_{j=-k}^{k} w(i,j) f(m+i, n+j)$$

$$f'(m,n) = f(m - m_0, n - n_0)$$

$$(w \otimes f')(m,n) = \sum_{i=-k}^{k} \sum_{j=-k}^{k} w(i,j) f'(m+i, n+j)$$

$$= \sum_{i=-k}^{k} \sum_{j=-k}^{k} w(i,j) f(m+i-m_0, n+j-n_0)$$

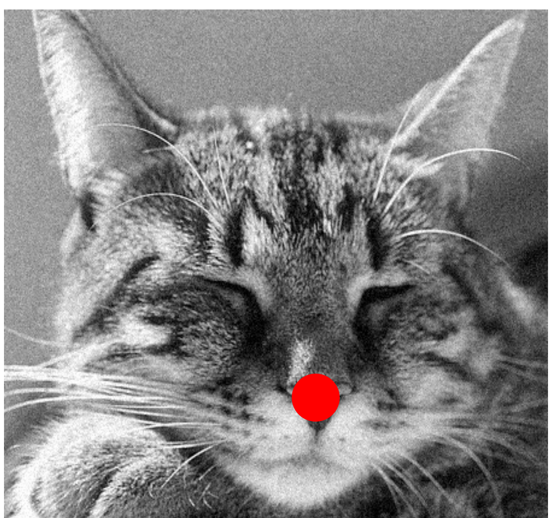
$$= (w \otimes f)(m - m_0, n - n_0)$$

Shift invariance

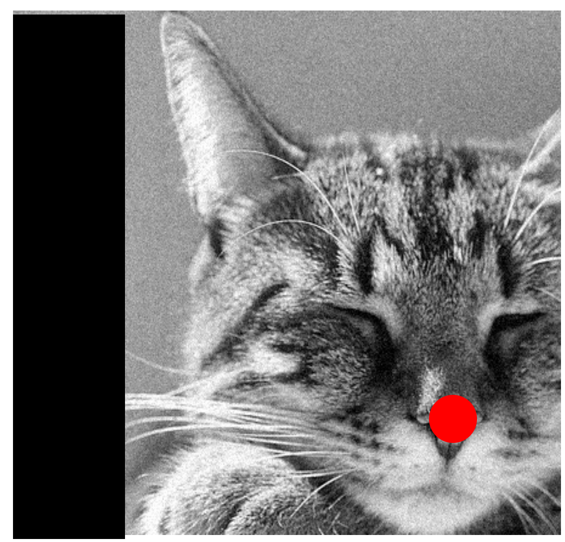
$$f'(m,n) = f(m-m_0, n-n_0)$$

$$(w \otimes f')(m,n) = (w \otimes f)(m-m_0, n-n_0)$$

- Shift, then convolve = convolve, then shift
- Convolution does not depend on where the pixel is



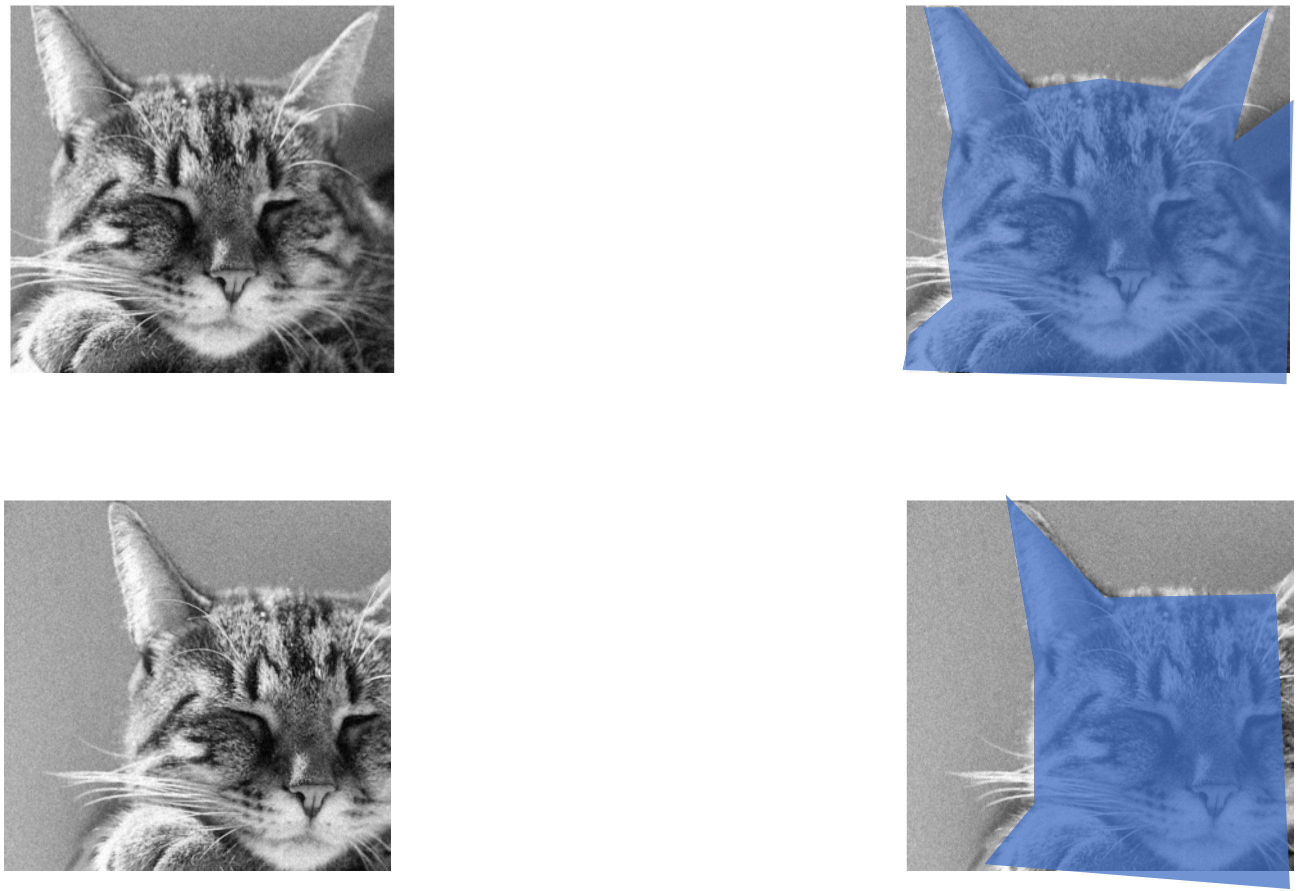
f



f'

Why is convolution important?

- Shift invariance is a crucial property



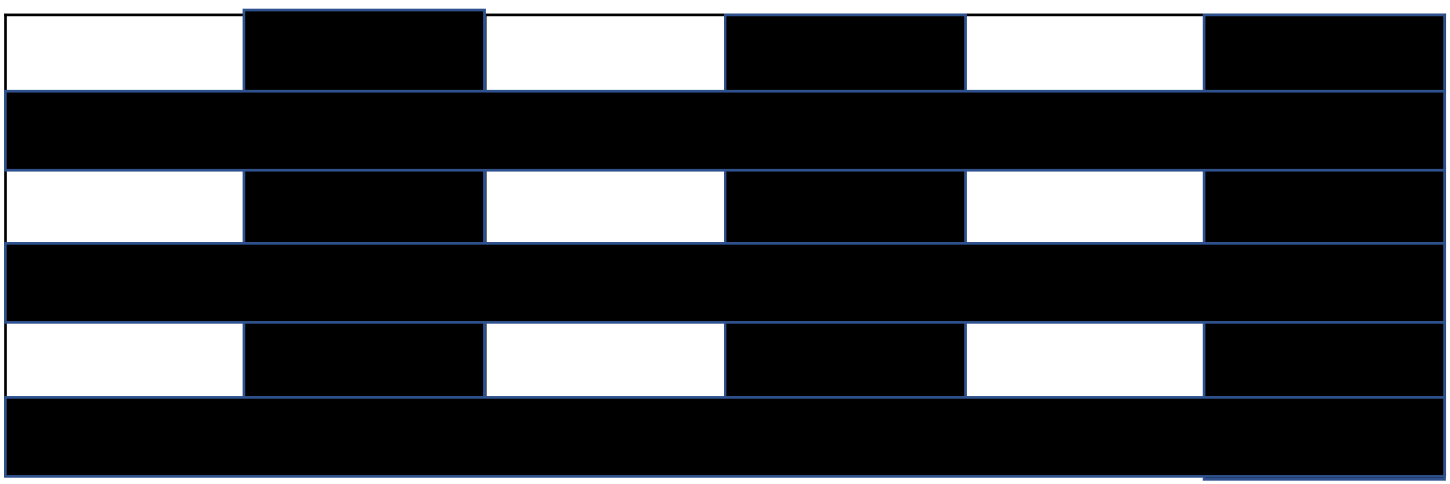
Why is convolution important?

- We *like* linearity
  - Linear functions behave predictably when input changes
  - Lots of theory just easier with linear functions
- *All linear shift-invariant systems can be expressed as a convolution*
- Basic primitive in computer vision

Image resizing

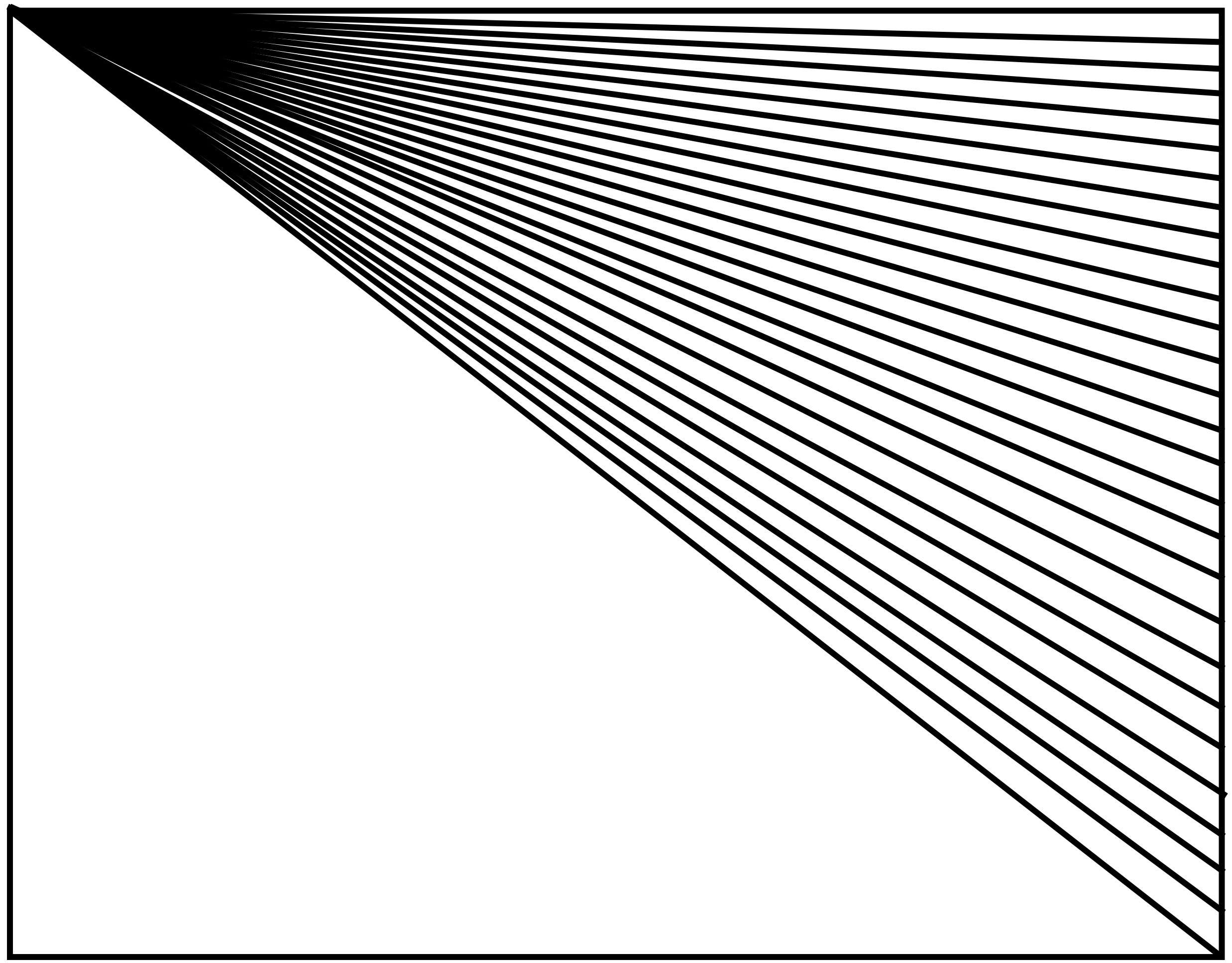
Why is resizing hard?

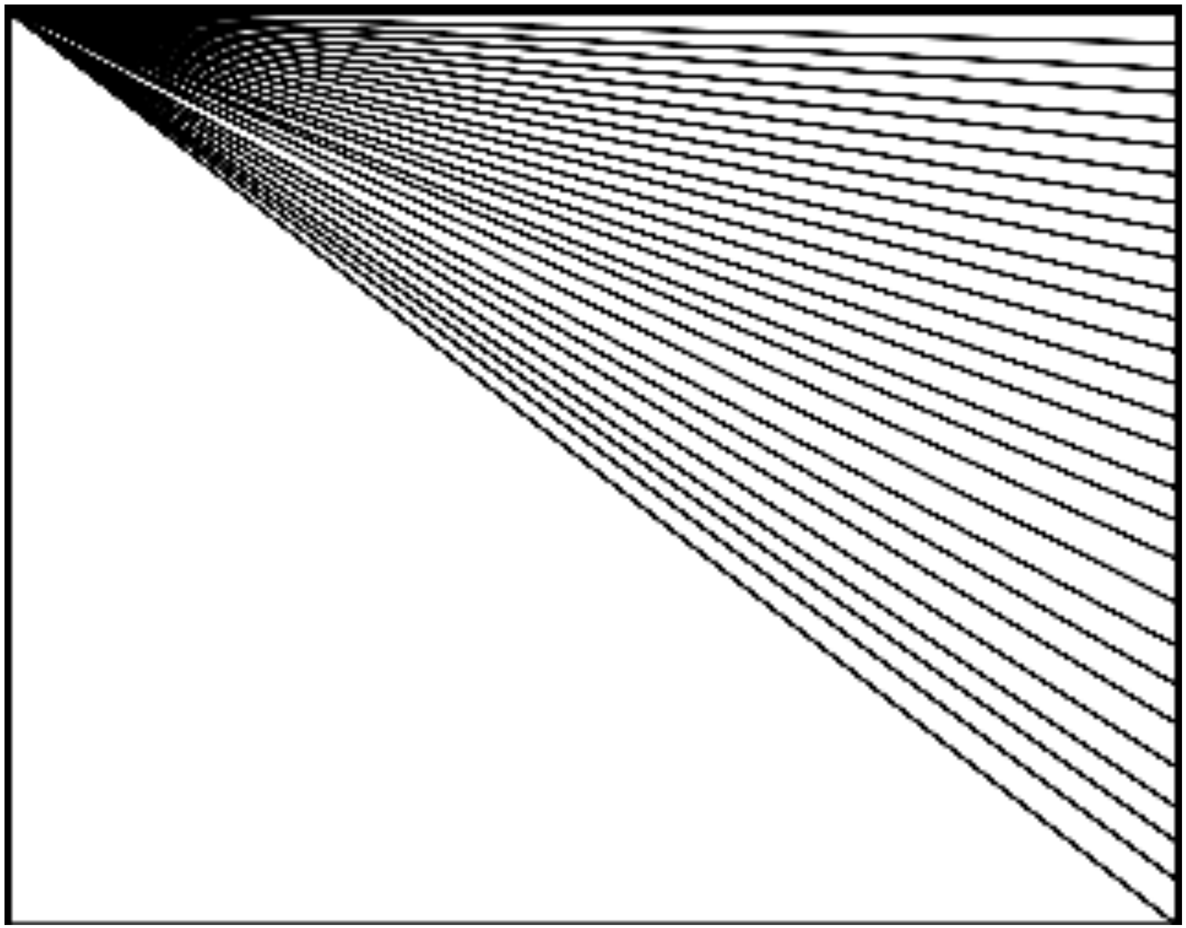
- E.g, consider reducing size by a factor of 2
- Simple solution: subsampling
- Example: subsampling by a factor of 2



Why is resizing hard?

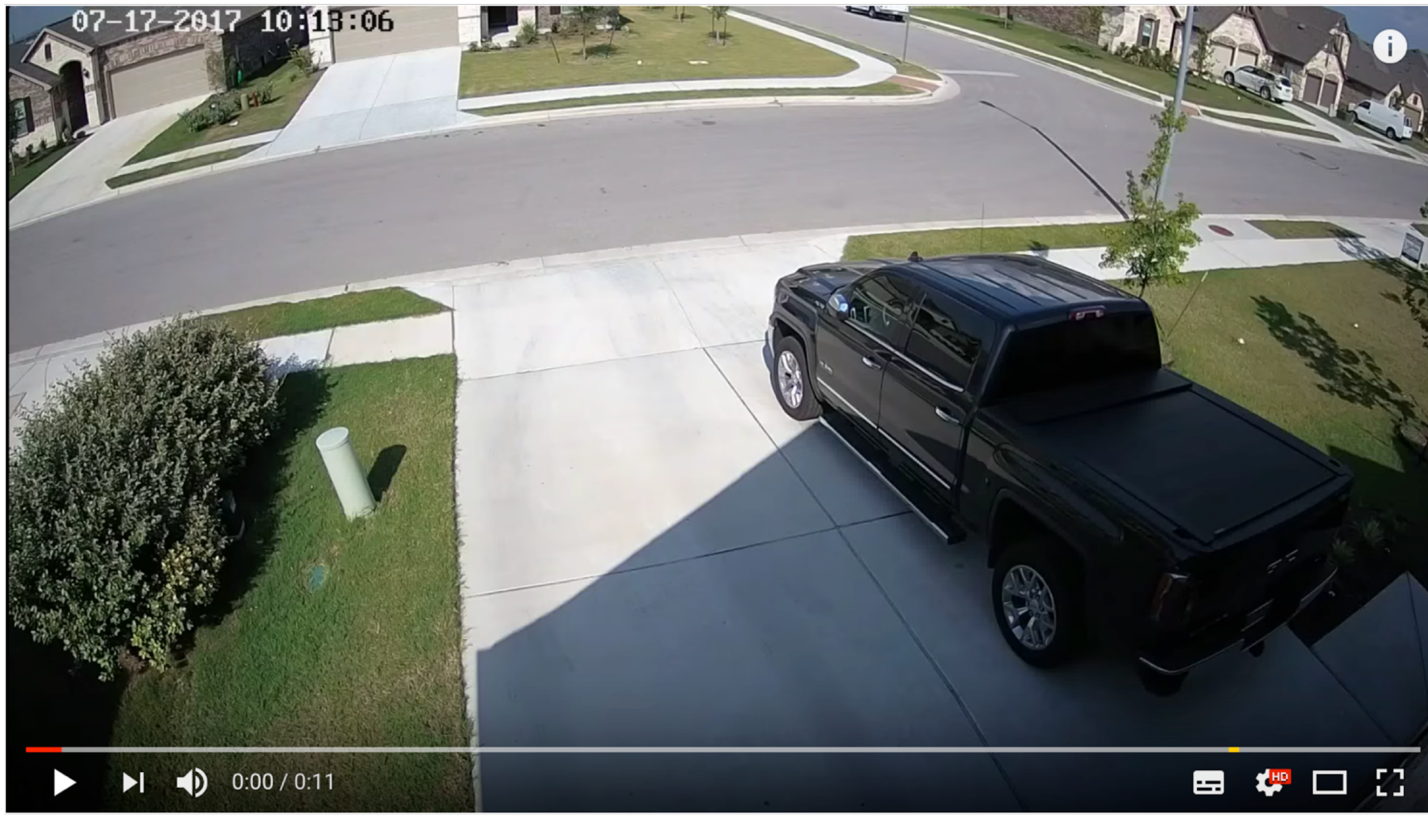
- Dropping pixels causes problems





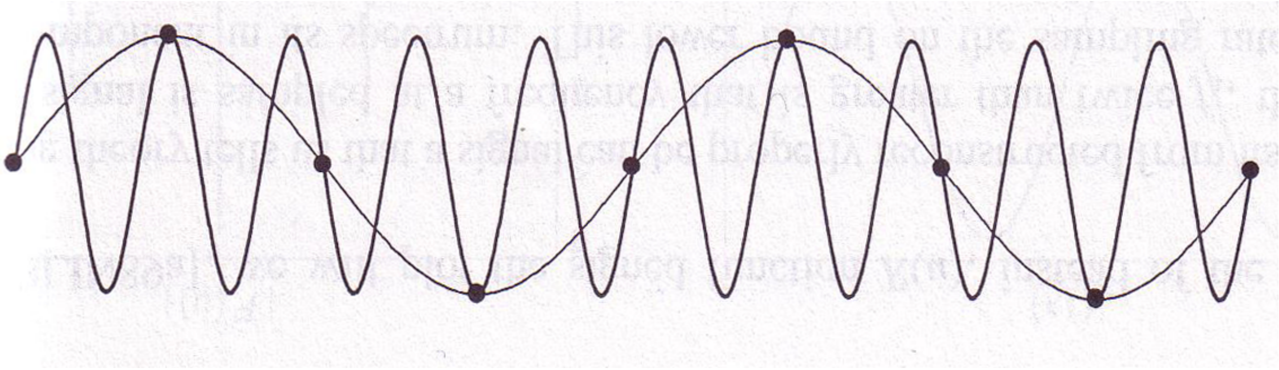
Aliasing in time

Aliasing in time



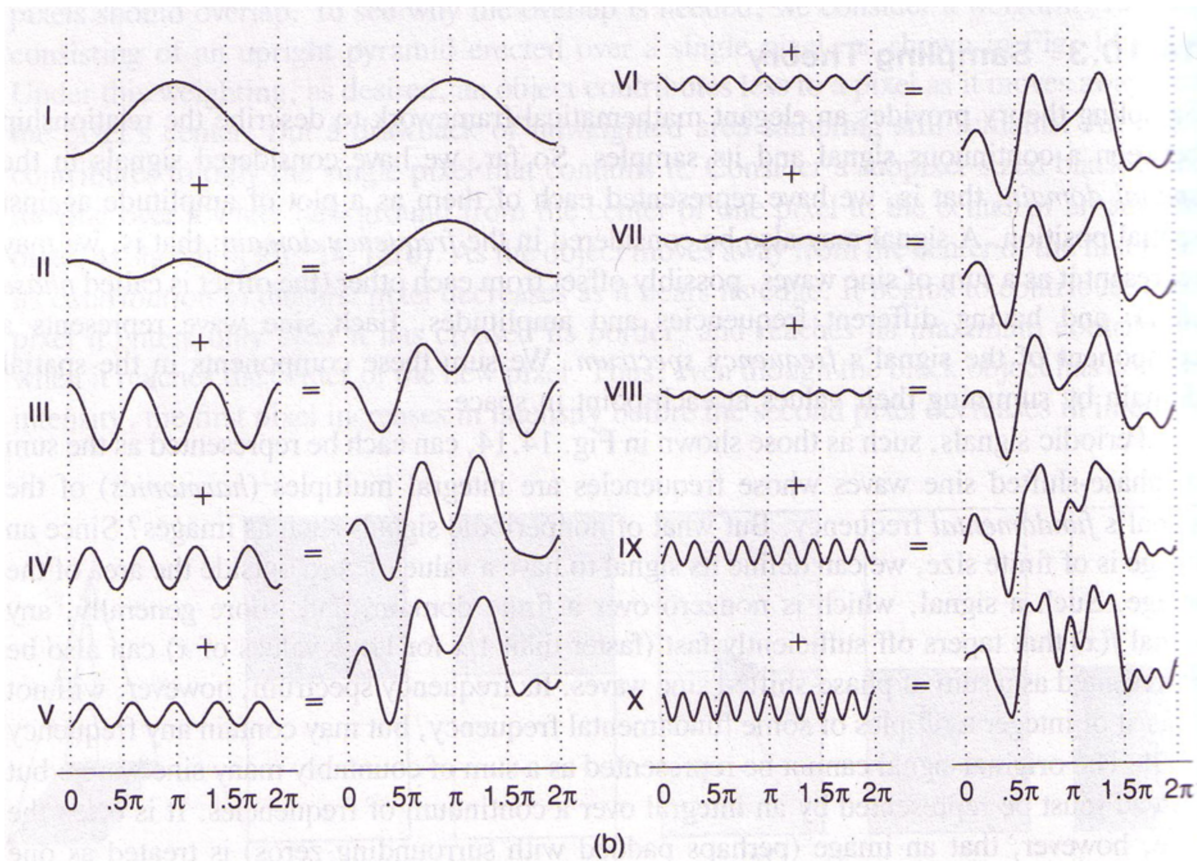
Why does aliasing happen?

- We ”miss” things between samples
- High frequency signals might appear as low frequency signals
- Called “aliasing”



What about the general case?

- Every signal (doesn't matter what it is)
  - Sum of sine/cosine waves
  - Fourier transform



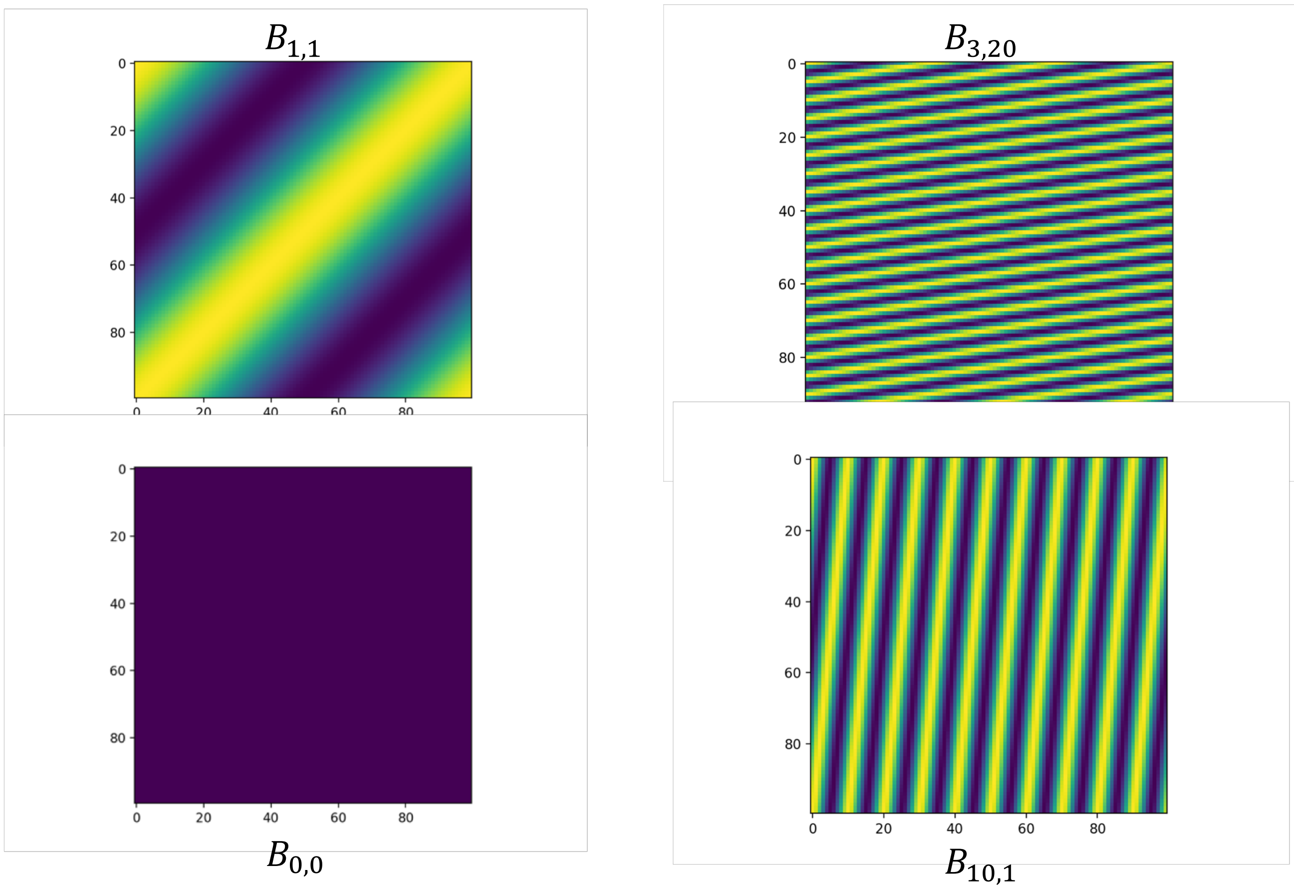
Fourier transform

- Represent each signal as a linear combination of sines and cosines
- Equivalent to a *change of basis*
- *Fourier transform = representation of signal in Fourier basis*

Fourier transform for images

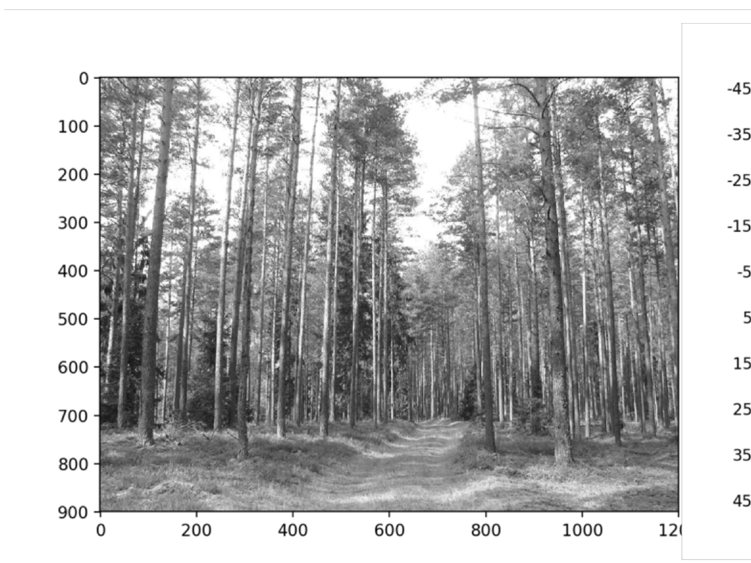
- Images are 2D arrays
- Fourier basis elements are indexed by 2 spatial frequencies
- (i,j)th Fourier basis for N x N image
  - Has period N/i along x
  - Has period N/j along y
- $B_{k,l}(x, y) = e^{\frac{2\pi ikx}{N} + \frac{2\pi ily}{N}}$
  $= \cos\left(\frac{2\pi kx}{N} + \frac{2\pi ly}{N}\right) + i \sin\left(\frac{2\pi kx}{N} + \frac{2\pi ly}{N}\right)$

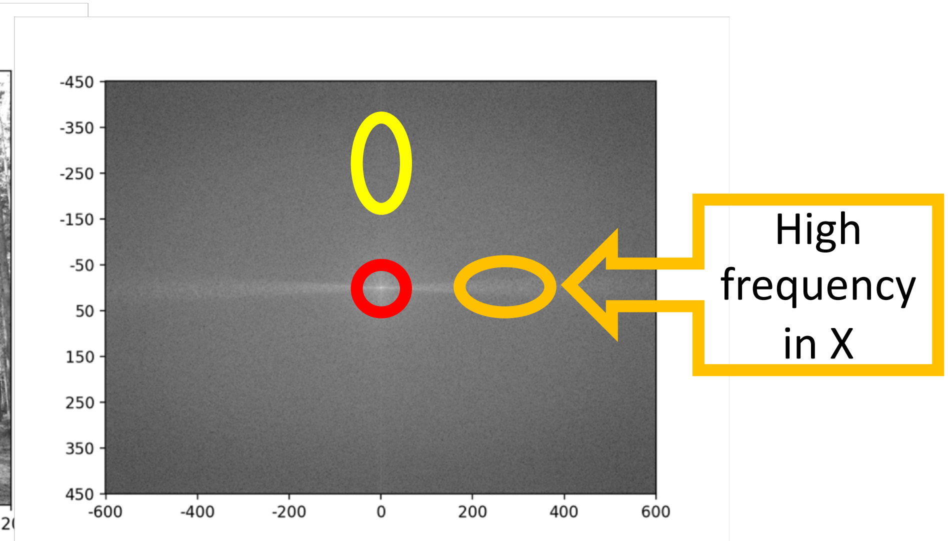
Visualizing the Fourier basis for images



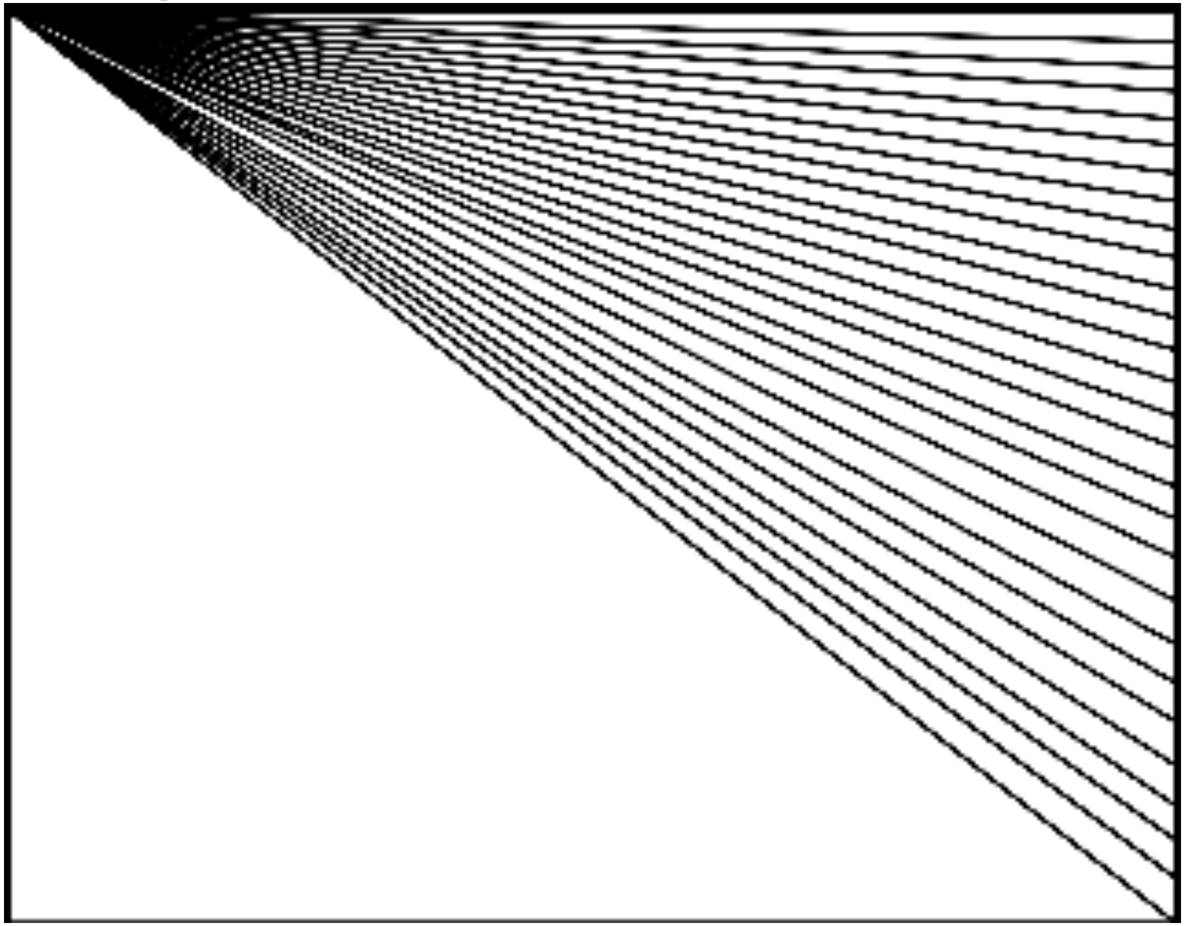
Visualizing the Fourier transform

- Given NxN image, there are NxN basis elements
- Fourier coefficients can be represented as an NxN image





Aliasing



Aliasing

- Image = linear combination of high frequency and low frequency components
- Subsampling: high frequency components *alias* as low frequency
- First smooth the image to remove high frequency components
- How should we smooth?
  - Mean filtering?

Convolution and Fourier transforms

- Image: Spatial domain

- Fourier Transform: Frequency domain
  - Amplitudes are called spectrum

- For any transformations we do in spatial domain, there are corresponding transformations we can do in the frequency domain
- *And vice-versa*

Convolution and Fourier transforms

- *Convolution* in spatial domain = *Point-wise multiplication* in frequency domain
  - $h = f * g \Rightarrow h(m,n) = \sum_{ij} f(i,j)g(m-i,n-j)$
  - $H = F \cdot G \Rightarrow H(k,l) = F(k,l)\, G(k,l)$

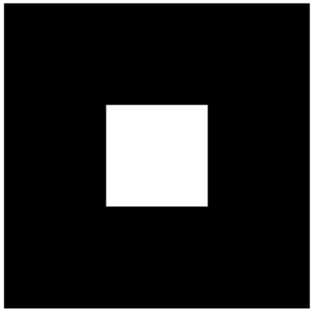
- *Convolution* in frequency domain = *Point-wise multiplication* in spatial domain

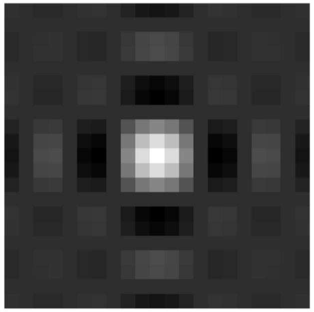
Smoothing and Fourier transforms

- Mean filter = convolving with a "box" filter

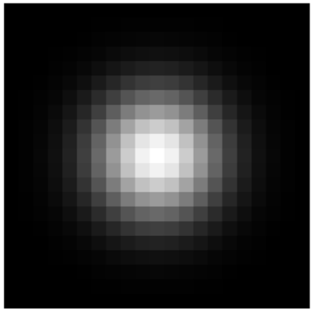
Filter

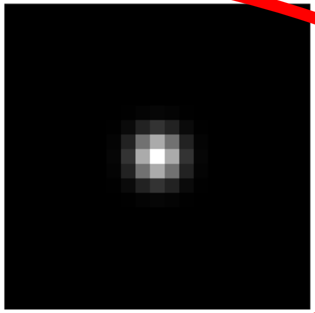
Fourier transform





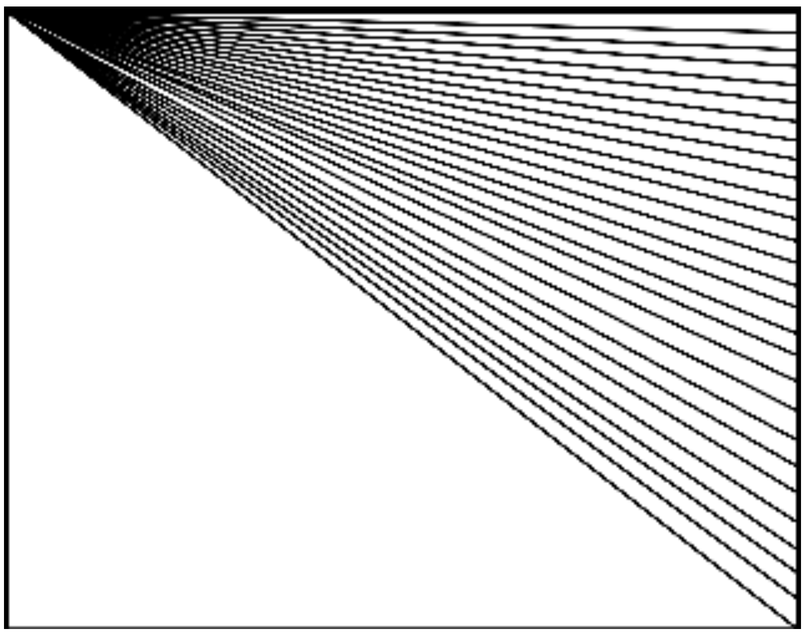
Box/mean filter



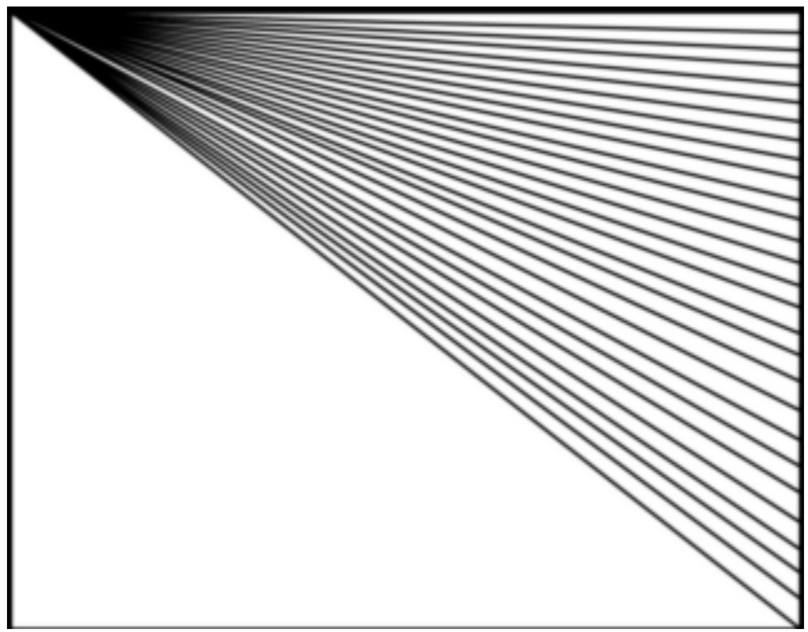


Gaussian filter

Subsampling before and after smoothing



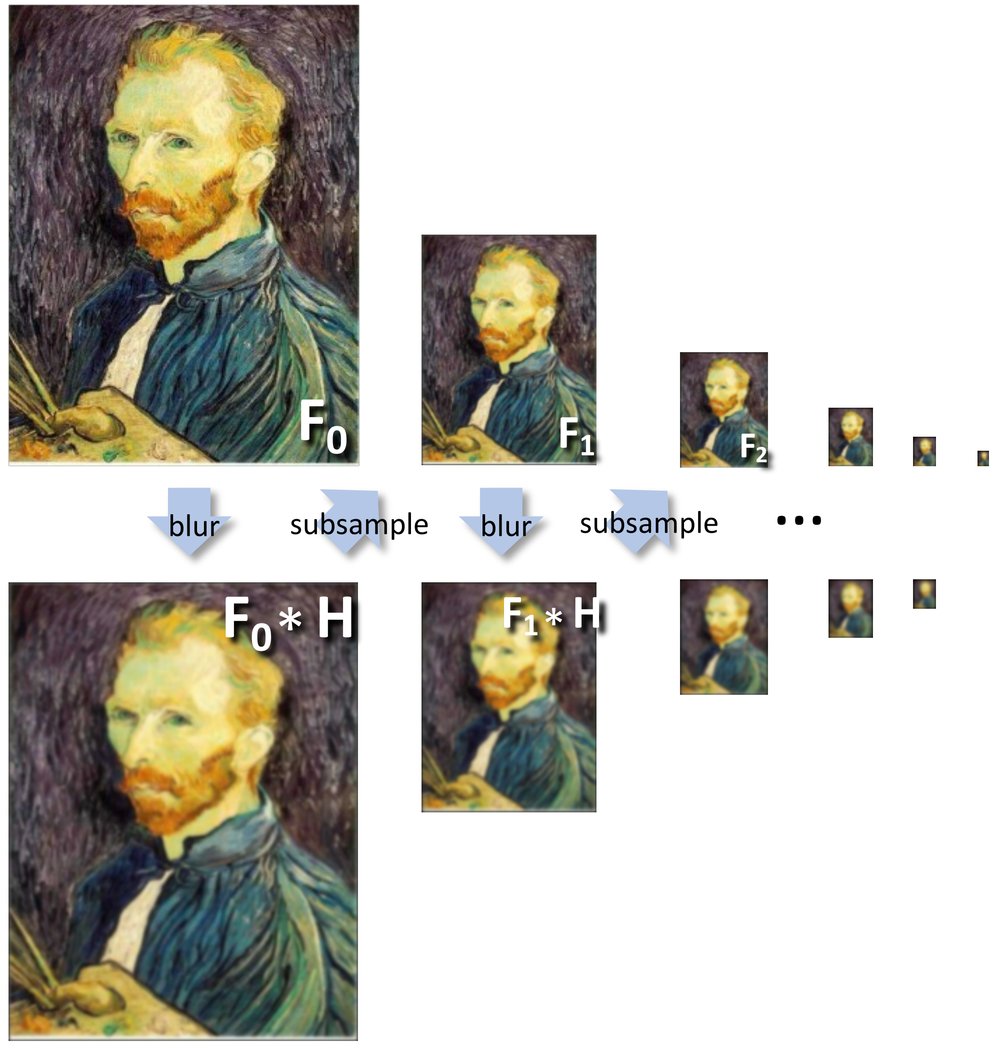
Before

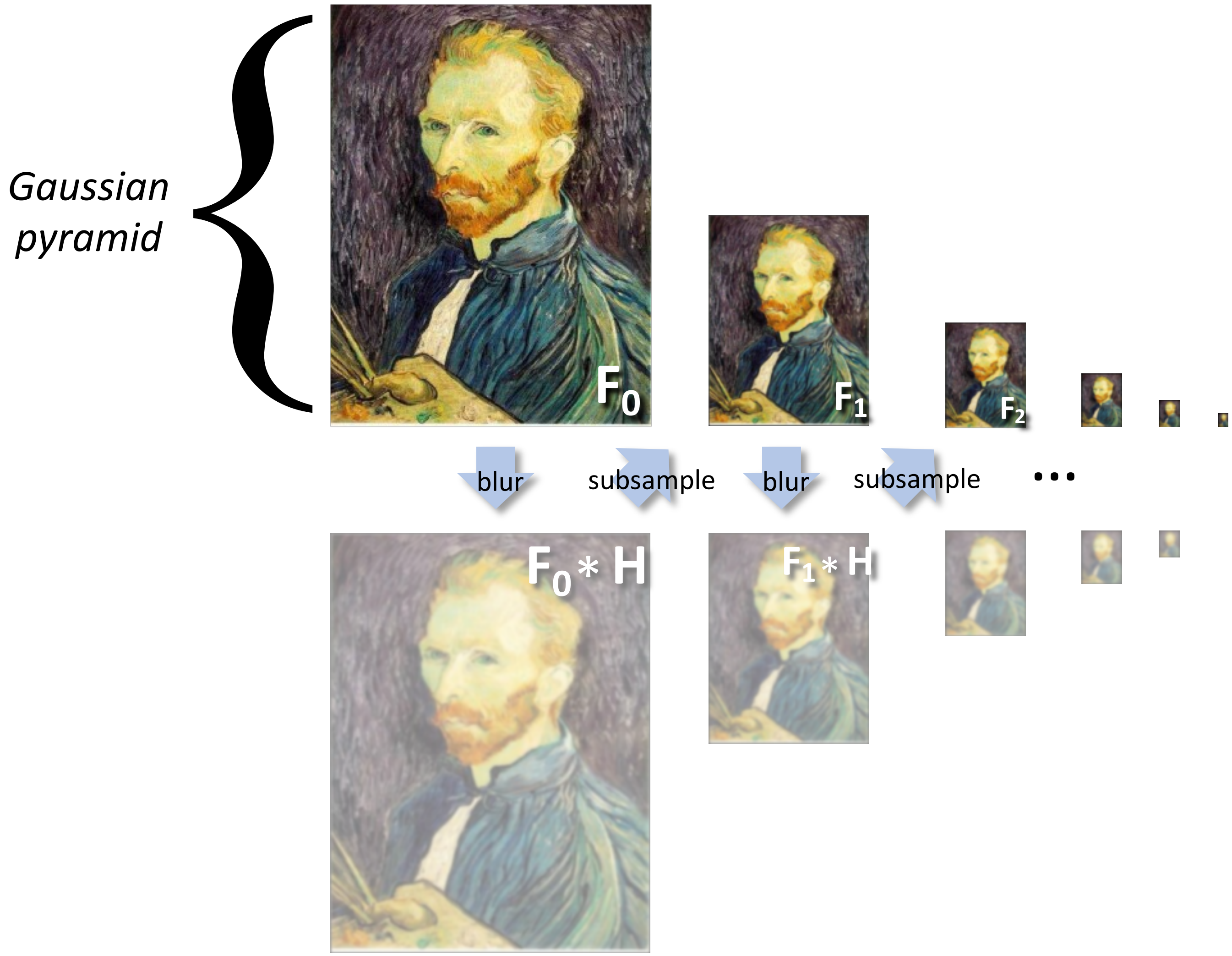


After

Gaussian pre-filtering

- Solution: filter the image, *then* subsample




Gaussian pyramid
F0
F1
F2
blur
subsample
blur
subsample
...
F0 * H
F1 * H

Anti-aliasing circa 2019

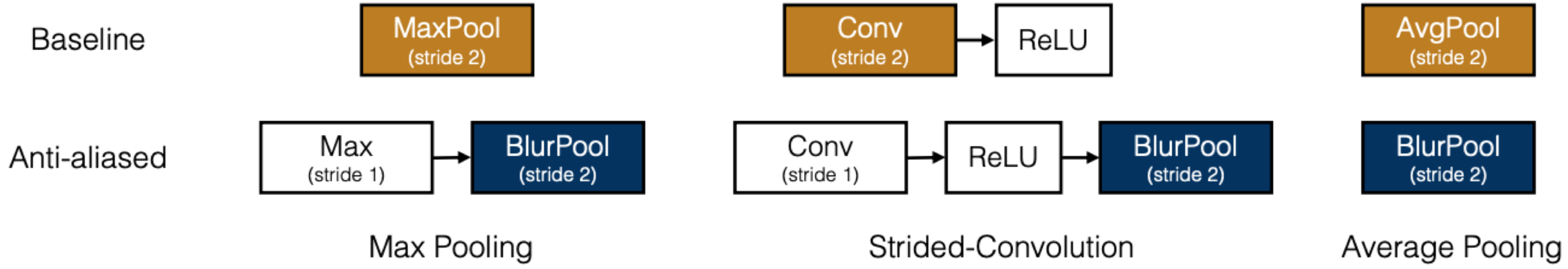


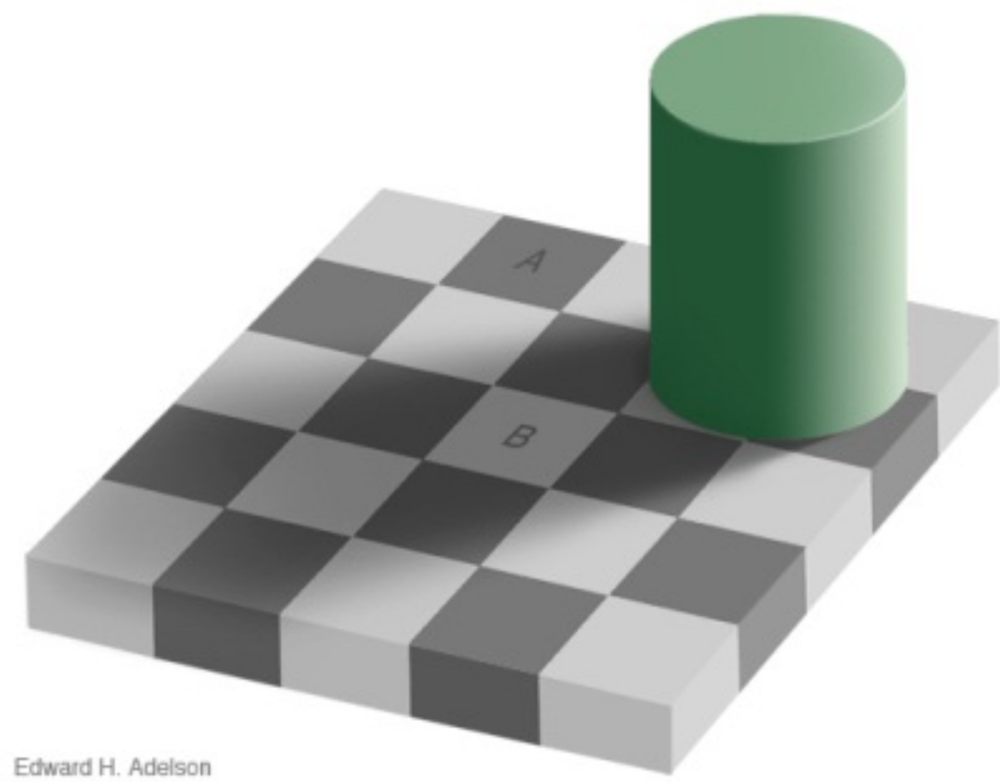
Figure 2. **Anti-aliasing common downsampling layers. (Top)** Max-pooling, strided-convolution, and average-pooling can each be better antialiased **(bottom)** with our proposed architectural modification. An example on max-pooling is shown below.

R. Zhang. Making convolutional networks shift-invariant again. In *ICML,* 2019.

Edge detection

Edges

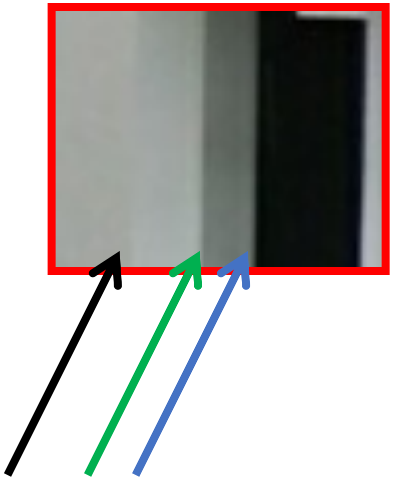
- Edges are curves in the image, across which the brightness changes "a lot"
- Corners/Junctions



Closeup of edges

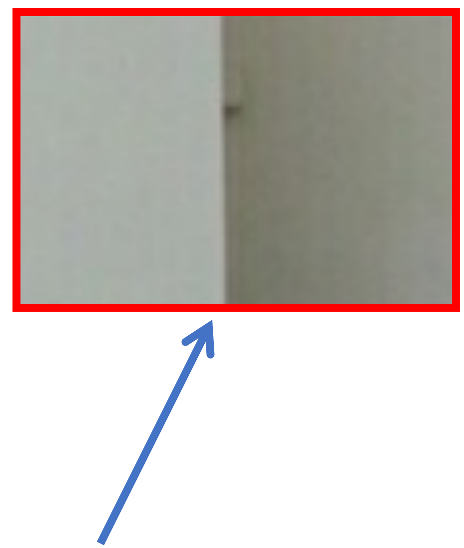
Source: D. Hoiem

Closeup of edges



Source: D. Hoiem

Closeup of edges



Source: D. Hoiem

Closeup of edges

Source: D. Hoiem

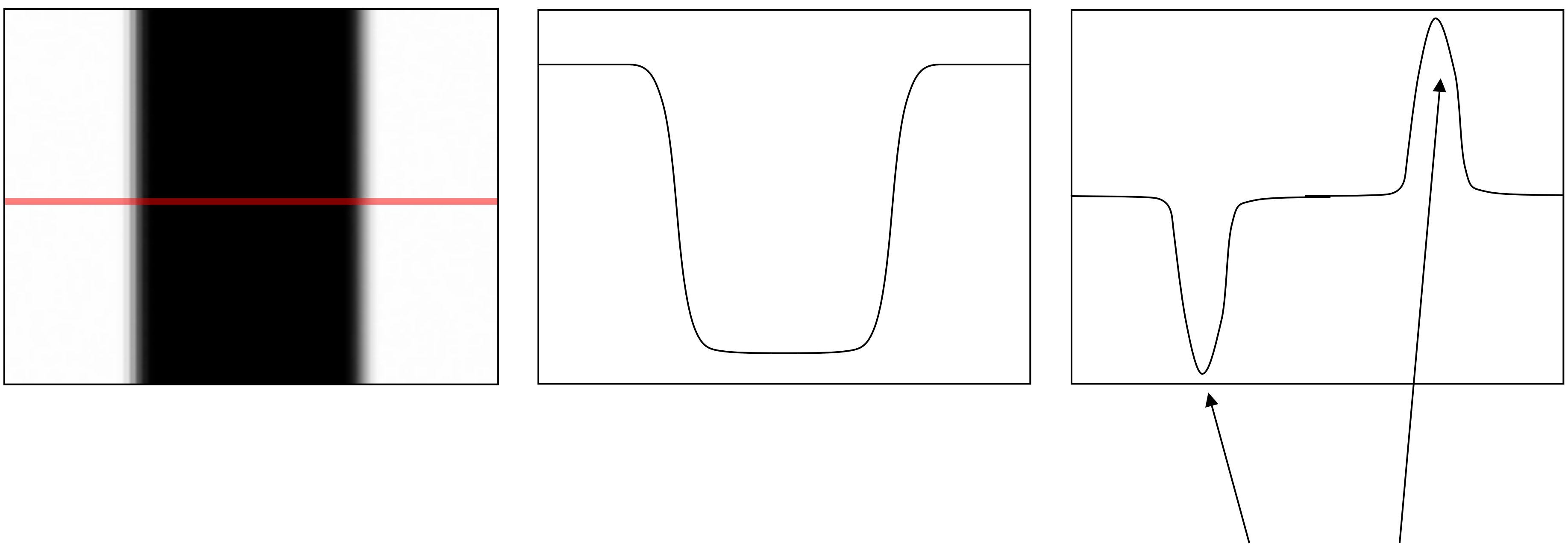
Characterizing edges

- An edge is a place of *rapid change* in the image intensity function

image

intensity function
(along horizontal scanline)

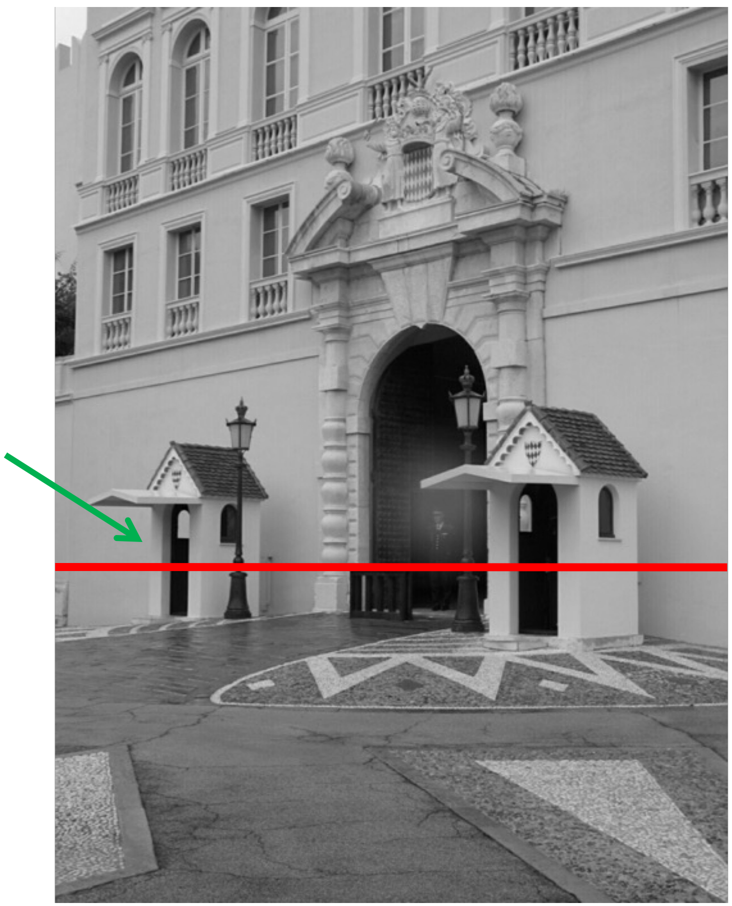
first derivative

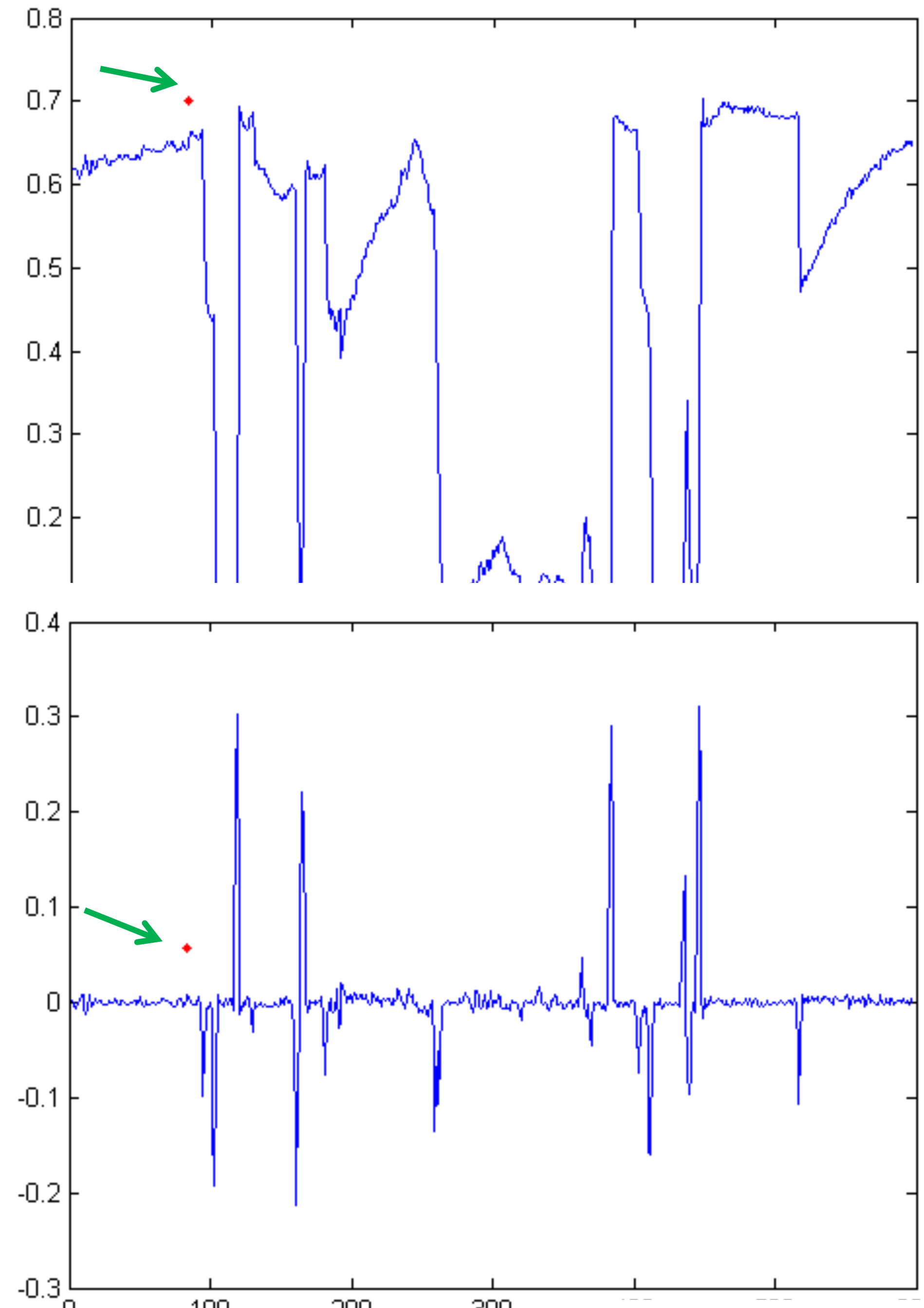


edges correspond to
extrema of derivative

Source: L. Lazebnik

Intensity profile





Source: D. Hoiem

Derivatives and convolution

- Differentiation is *linear*

$$\frac{\partial(af(x) + bg(x))}{\partial x} = a\frac{\partial f(x)}{\partial x} + b\frac{\partial g(x)}{\partial x}$$

- Differentiation is *shift-invariant*
  - Derivative of shifted signal is shifted derivative

- Hence, differentiation can be represented as convolution!

Image derivatives

- How can we differentiate a *digital* image F[x,y]?
  - Option 1: reconstruct a continuous image, *f*, then compute the derivative
  - Option 2: take discrete derivative (finite difference)

$$\frac{\partial f}{\partial x}[x,y] \approx F[x+1,y] - F[x,y]$$

How would you implement this as a linear filter?

$\frac{\partial f}{\partial x}$:

H_x

$\frac{\partial f}{\partial y}$:

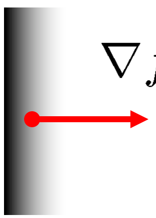
H_y

Source: S. Seitz

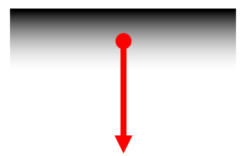
Image gradient

- The *gradient* of an image: $$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

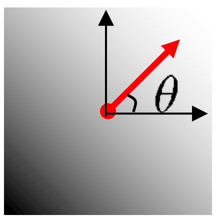
The gradient points in the direction of most rapid increase in intensity



$$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$$



$$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$$



$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

The *edge strength* is given by the gradient magnitude:

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

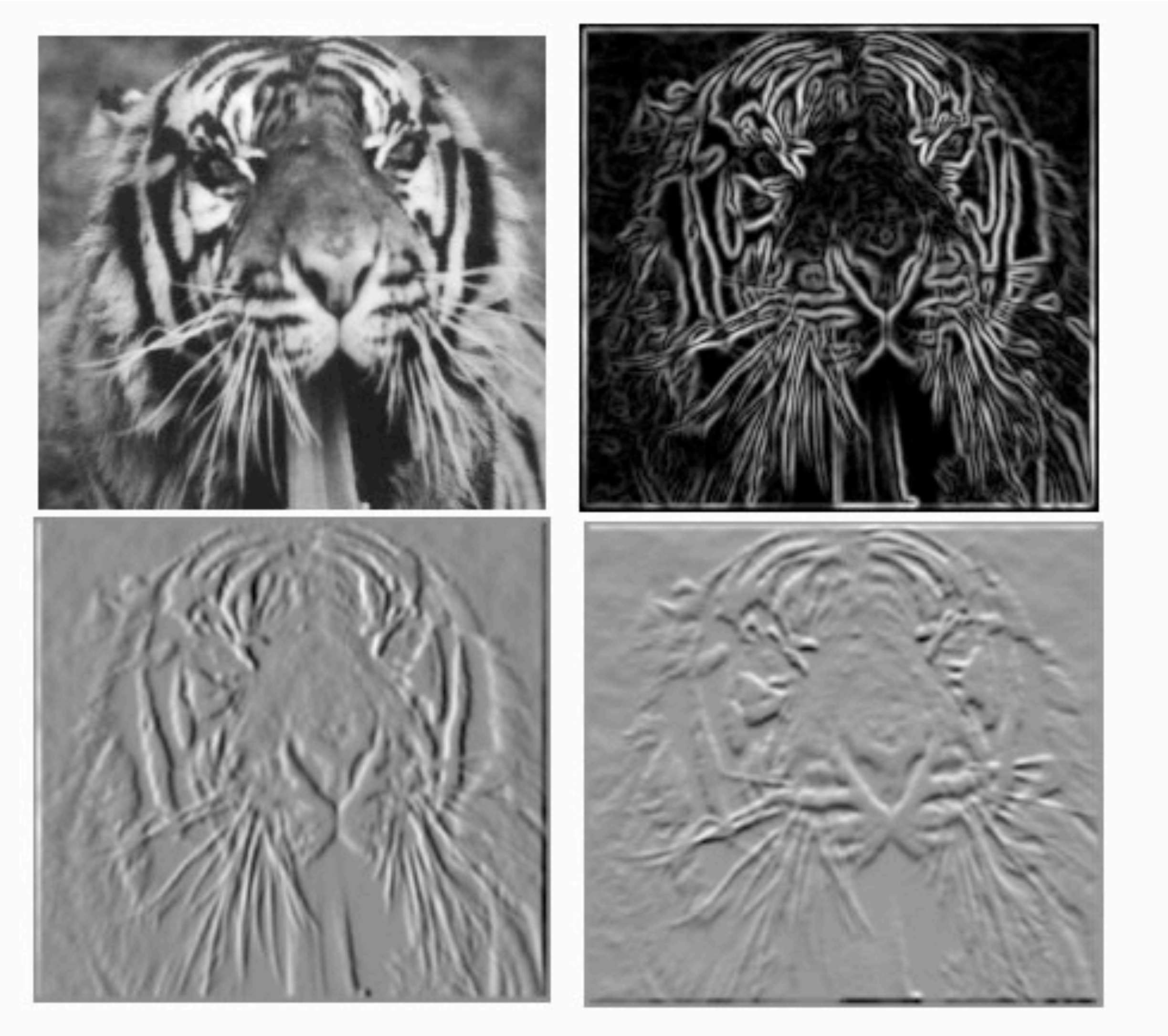
The gradient direction is given by:

$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$$

- how does this relate to the direction of the edge?

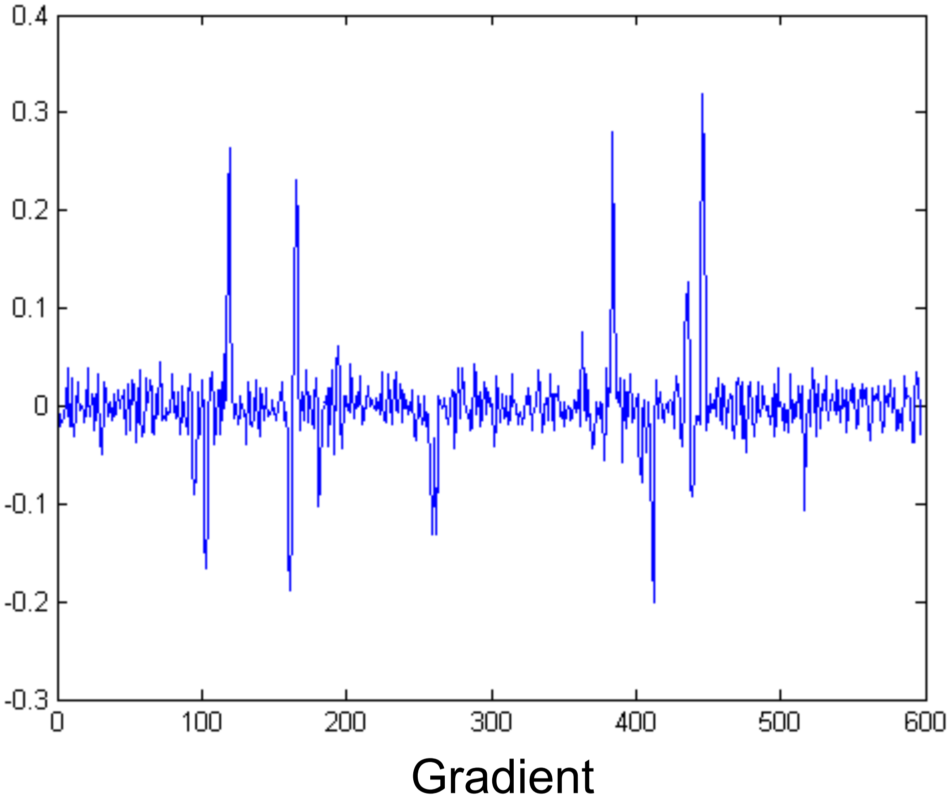
Source: Steve Seitz

Image gradient



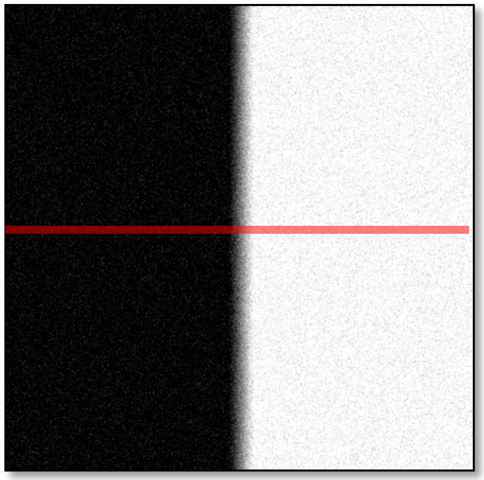
Source: L. Lazebnik

With a little Gaussian noise



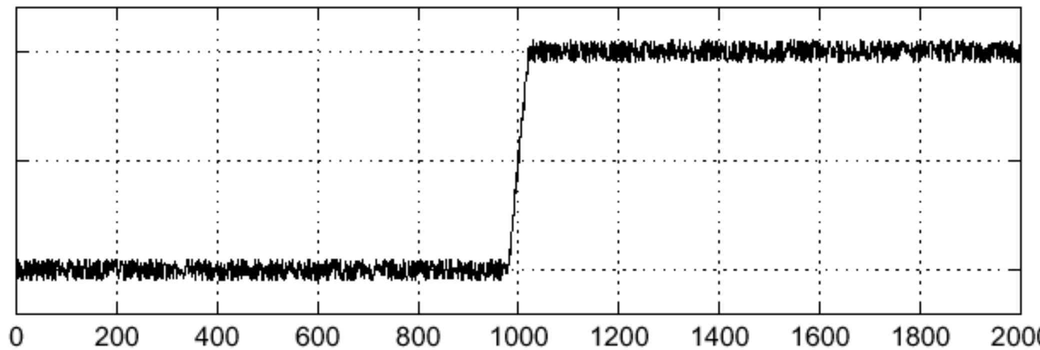
Source: D. Hoiem

Effects of noise

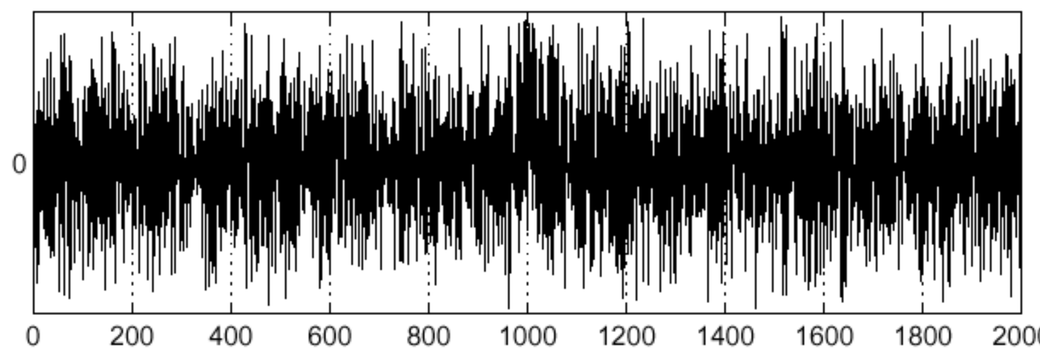


Noisy input image

$f(x)$



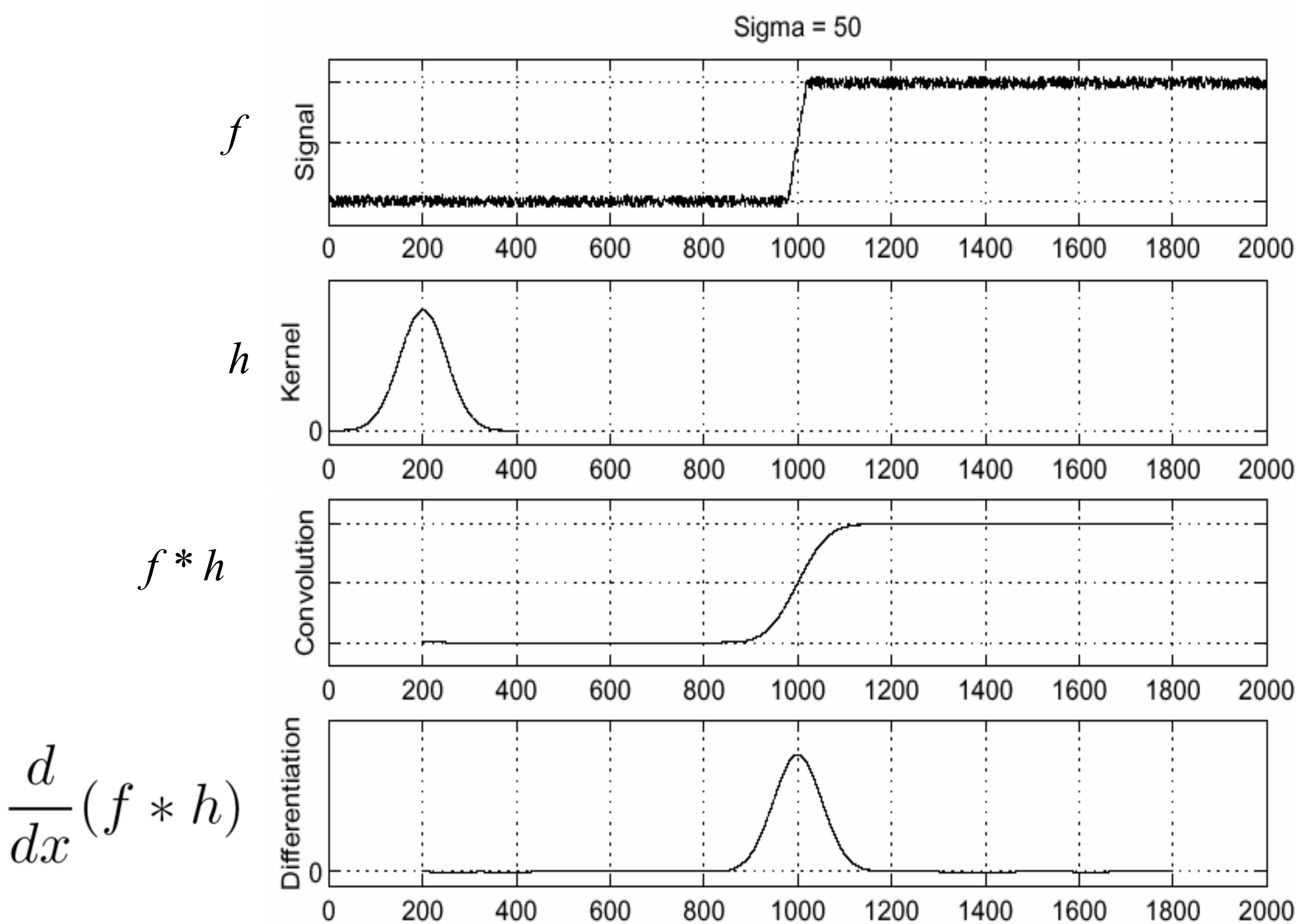
$\frac{d}{dx}f(x)$



Where is the edge?

Source: S. Seitz

Solution: smooth first



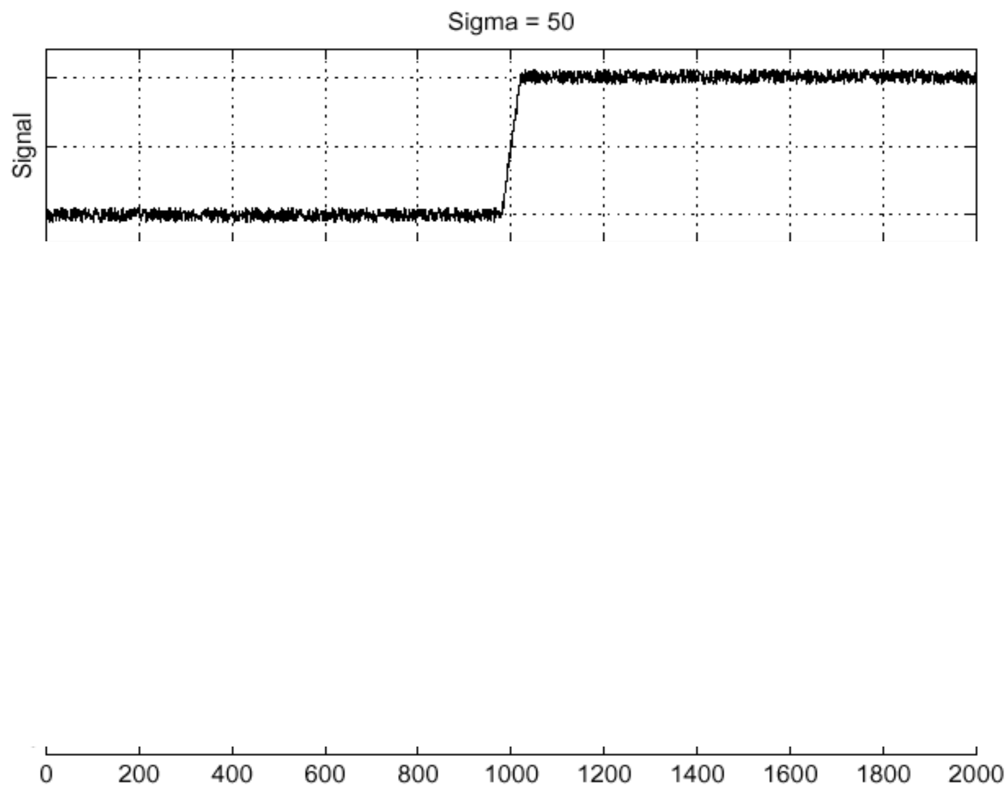
To find edges, look for peaks in $\frac{d}{dx}(f * h)$

Source: S. Seitz

Associative property of convolution

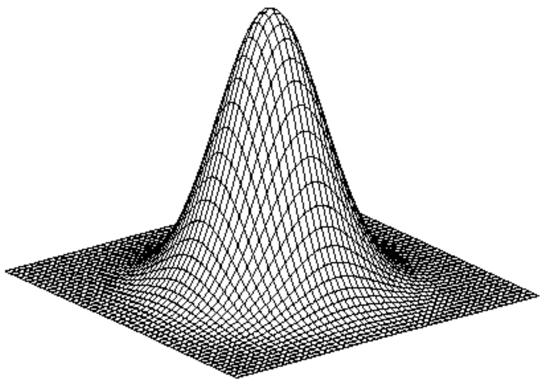
- Differentiation is a convolution
- Convolution is associative: $$\frac{d}{dx}(f * h) = f * \frac{d}{dx}h$$
- This saves us one operation:

f



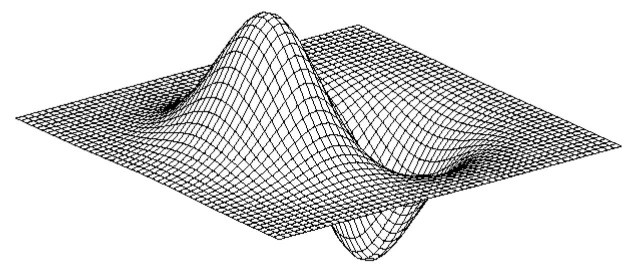
Source: S. Seitz

2D edge detection filters



Gaussian

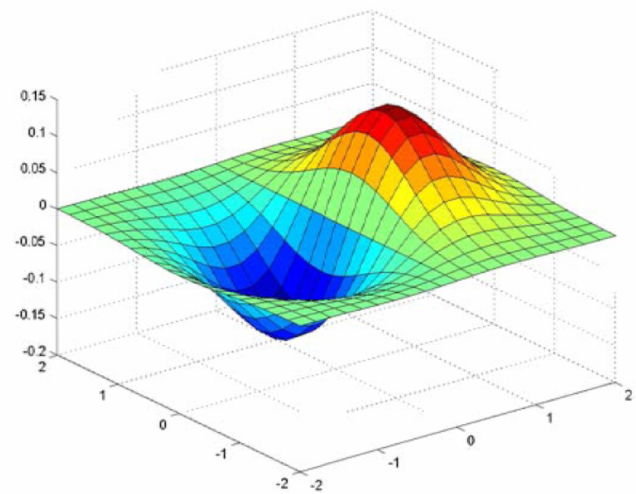
$$h_\sigma(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$



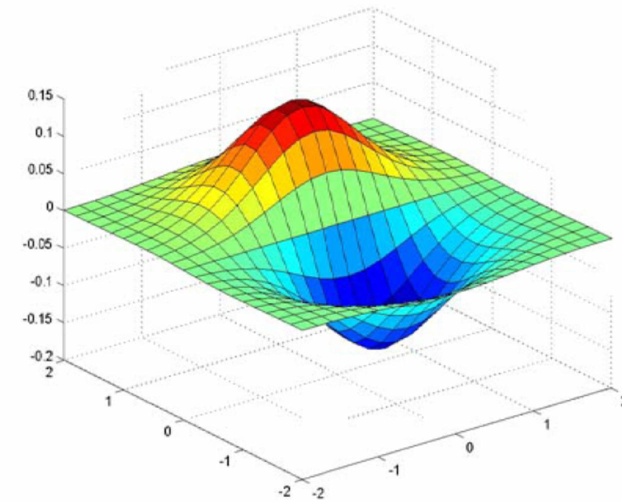
derivative of Gaussian (x)

$$\frac{\partial}{\partial x} h_\sigma(u,v)$$

Derivative of Gaussian filter



x-direction



y-direction