A Signal-Processing Framework for Inverse Rendering

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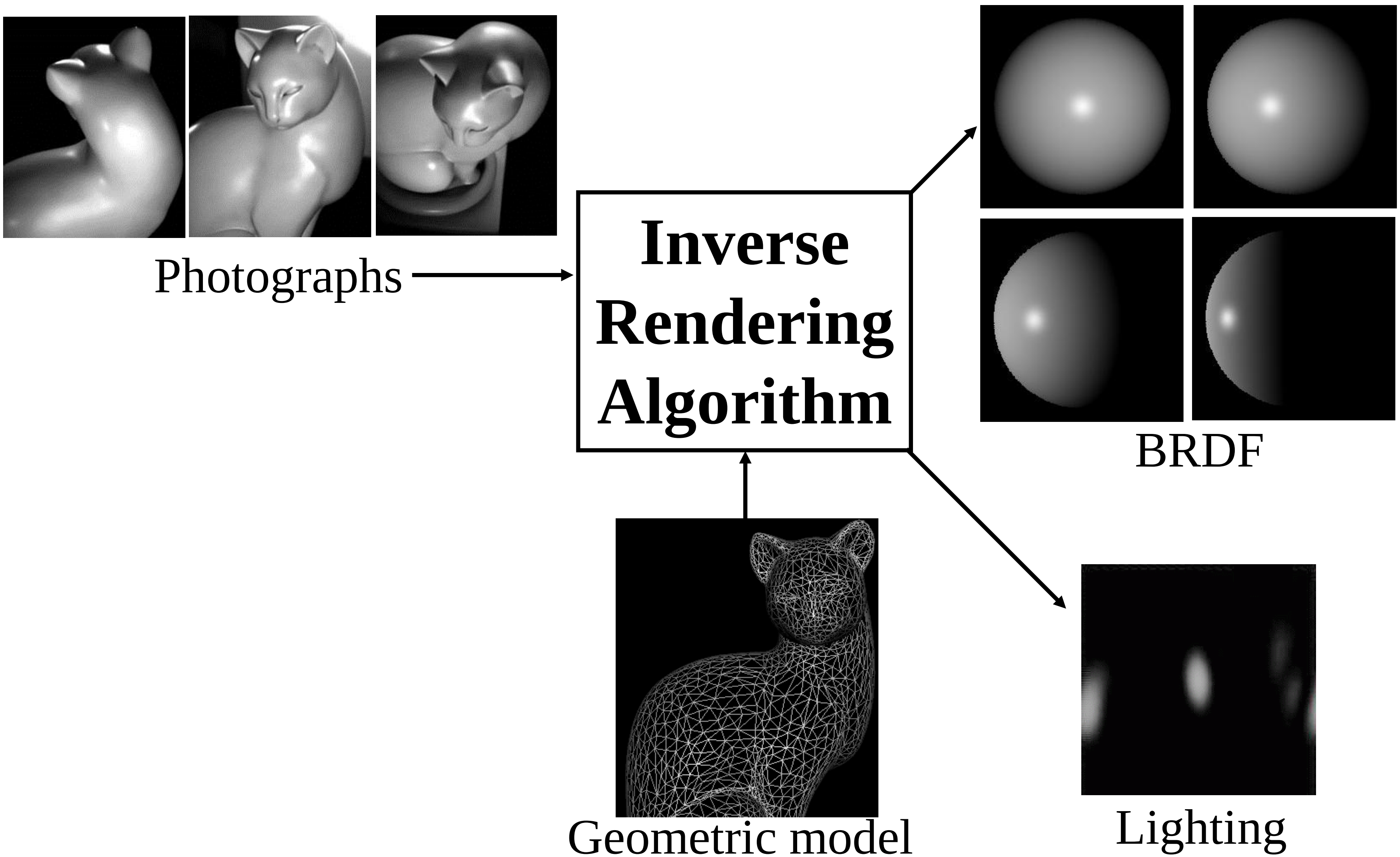
CS 6630 Realistic Image Synthesis

Inverse Rendering



Why inverse rendering

Precision measurement

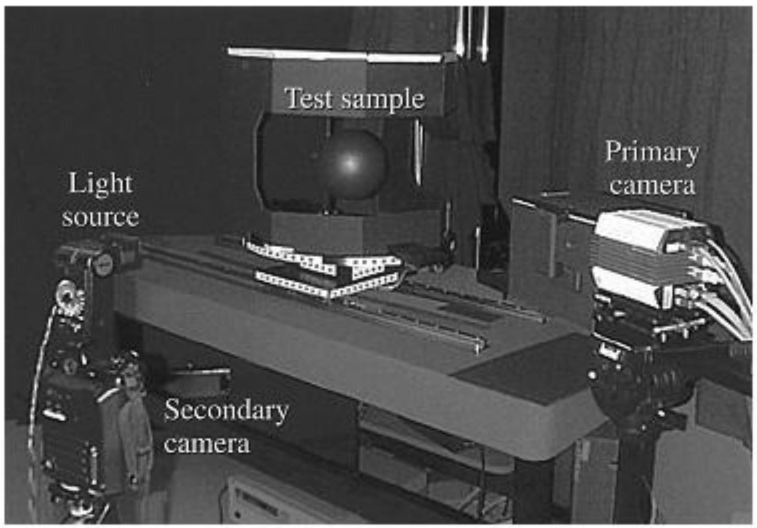
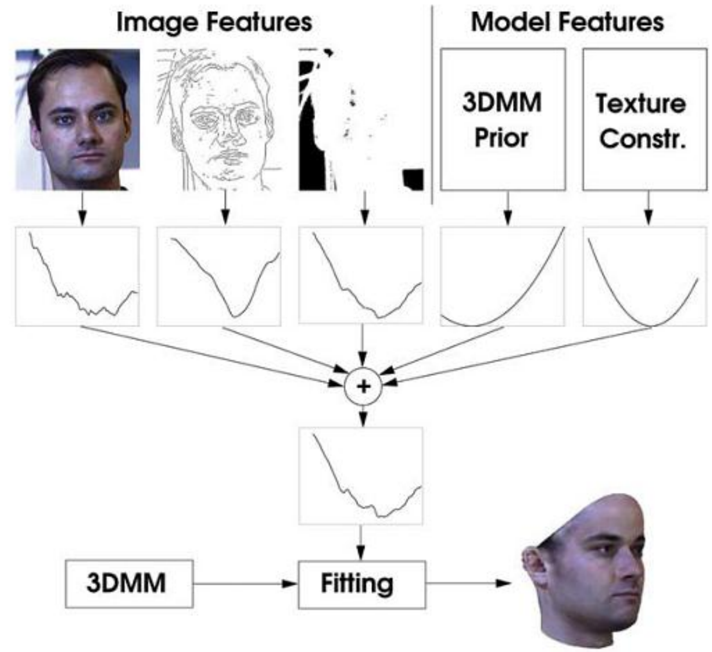


Image-based bidirectional reflectance distribution function measurement

Stephen R. Marschner, Stephen H. Westin, Eric P. F. Lafortune, and Kenneth E. Torrance

APPLIED OPTICS / Vol. 39, No. 16 / 1 June 2000

Data extraction



Face Recognition Using 3-D Models: Pose and Illumination

Novel face recognition algorithms, based on three-dimensional information, promise to improve automated face recognition by dealing with different viewing and lighting conditions.

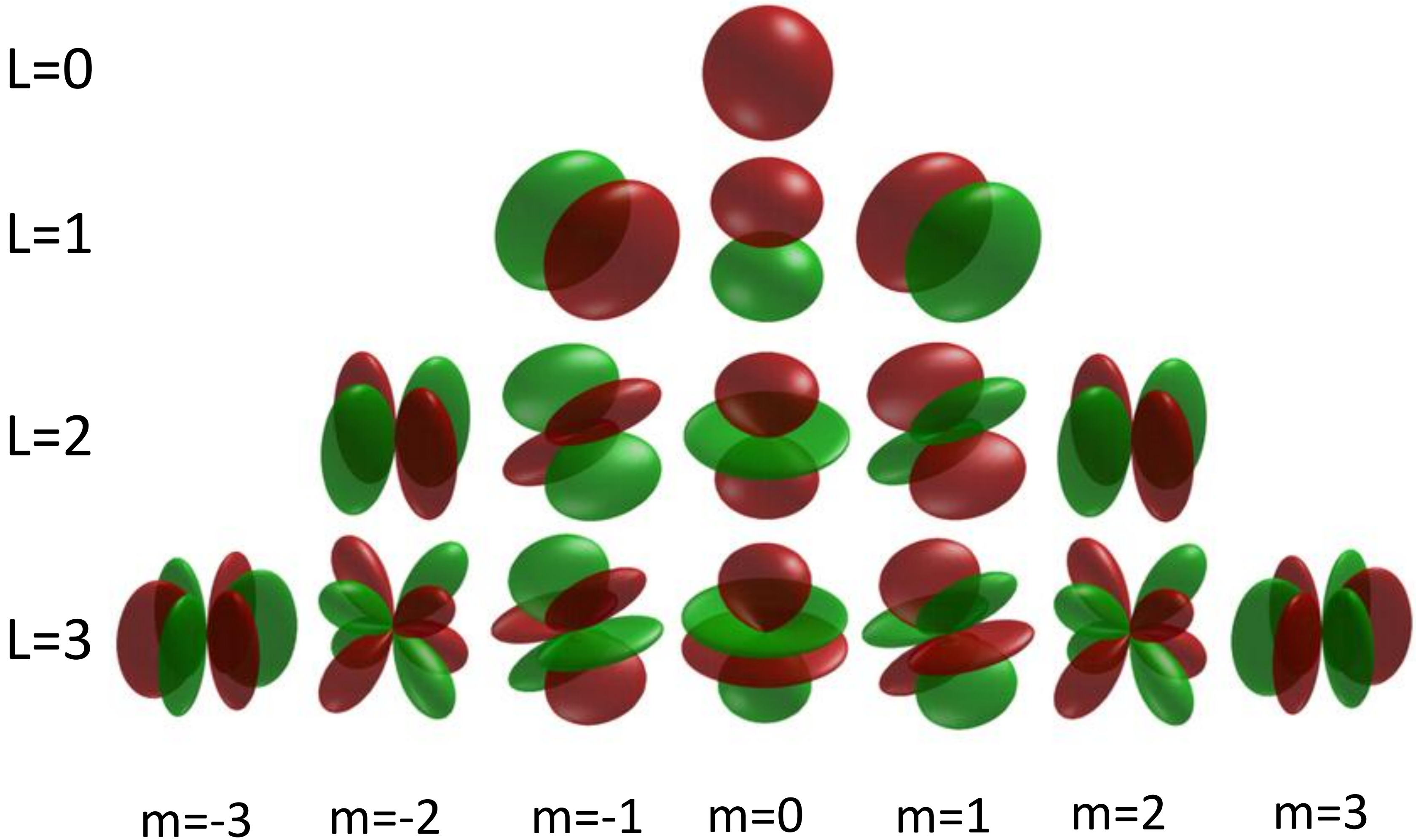
By Sami Romdhani, Jeffrey Ho, *Member IEEE*, Thomas Vetter, *Member IEEE*, and David J. Kriegman

Signal-Processing Framework

- How much information can I extract?
- The problem is well- or ill-posed?
- What is the best way to express the model?

Before only "handwaving" explanation

Spherical Harmonics



Spherical Harmonics

Analog to Fourier base for angles

$$Y_l^m(\theta, \phi) = \sqrt{\frac{(2l+1)}{(4\pi)}} \sqrt{\frac{(l-m)!}{(l+m)!}} \, P_l^m(\cos(\theta)) \, e^{im\phi}$$

Coefficients obtained via projection

$$C_l^m = \iint f(\theta, \phi) \, Y_l^m(\theta, \phi) \, d\theta \, d\phi$$

Orthonormality

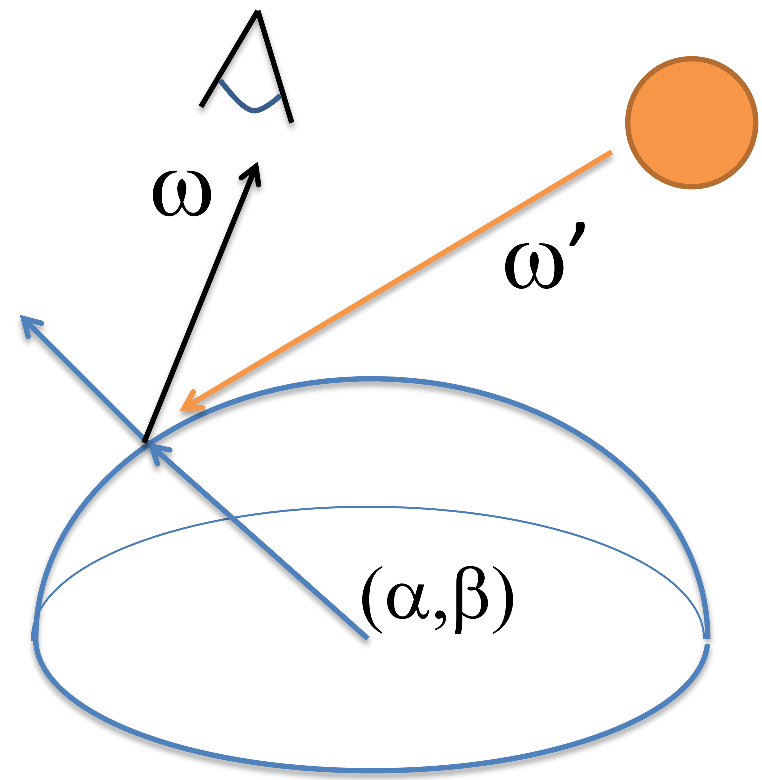
$$\int Y_l^m(\theta, \phi) \, Y_{l'}^{m'}(\theta, \phi)^* \, d\theta \, d\phi = \delta_{m,m'} \, \delta_{l,l'}$$

Inverse rendering

- Known geometry
- Fixed ‘far’ light
- Reflection

$$B(\omega) = \int L(\omega')\,\rho(\omega',\omega)\,d\omega'$$

$$B(\alpha,\beta,\omega) = \int L\left(R_{\alpha,\beta}(\omega')\right)\rho(\omega',\omega)\,d\omega'$$



Looks like convolution

(and convolution simple in Fourier’s space)

Inverse Rendering

- Plugging in SH

$$B(\ , \ , \ ) = L_l^m \ {}_{q,s}^{p,r} \int Y_l^m(R_{,}(\ ')) \, Y_q^p(\ ') \, Y_s^r(\ ) \, d$$

- Rotation in SH

$$Y_m^l(R_{,}(\ )) = D_{m,m'}^l(\ ) \, e^{m} \, Y_{m'}^l(\ )$$

- Symmetry in BRDF

$${}_{q,s}^{p,r} = {}_{q,s}^{p}$$

Inverse Rendering

- Reflection expanded in SH base

$$B_{p,q}^{l,m} \quad C_{p,q}^{l,m}(\;,\;,\;) : \quad {}_{l}L_{m}^{l} \; {}^{l}_{p,q} \quad C_{p,q}^{l,m}(\;,\;,\;)$$

$${}_{l}L_{m}^{l} \; {}^{l}_{p,q} : \quad B_{p,q}^{l,m}$$

BRDF

Normalization

Lightings

Reflection (measured)

Mirror BRDF

$$\hat{\rho}_{\ell pq} = (-1)^q \, \delta_{\ell p} \quad (1)$$

- The factor $(-1)^q$ is due to the change of π in azimuthal angle upon reflection.
- As is usual for delta functions, there is no dropoff with ℓ ("frequency"). This makes the inverse lighting problem well-conditioned.

Single directional source

$$L_{\ell m} = \delta_{m0} Y^{*}_{\ell 0}(0) = \delta_{m0} \Lambda_{l}^{-1} \quad (2)$$

- We define the light to be in the $+z$ direction—the "north pole". Spherical harmonics have the property that they have the value 1 (before normalization) at this point if $m = 0$, and the value 0 otherwise. Again the inverse lighting problem is well-conditioned.

Lambertian BRDF

$$\hat{\rho}_\ell = 2\pi \int_0^{\frac{\pi}{2}} \cos\theta_i' Y_{\ell 0}\left(\theta_i'\right) \sin\theta_i' \; d\theta_i' \qquad (3)$$

- Since there is no dependence on outgoing angle, p and q don't matter.
- This is zero for all odd values of ℓ, except $\ell = 1$ (due to the clamping at the horizon).
- The even terms fall off as $\ell^{-\frac{5}{2}}$. $> 99\%$ of the power is contained in the $\ell \leq 2$ terms (9 spherical harmonics). This means an approximate characterization easy, but the inverse lighting problem poorly conditioned.

Lambertian BRDF

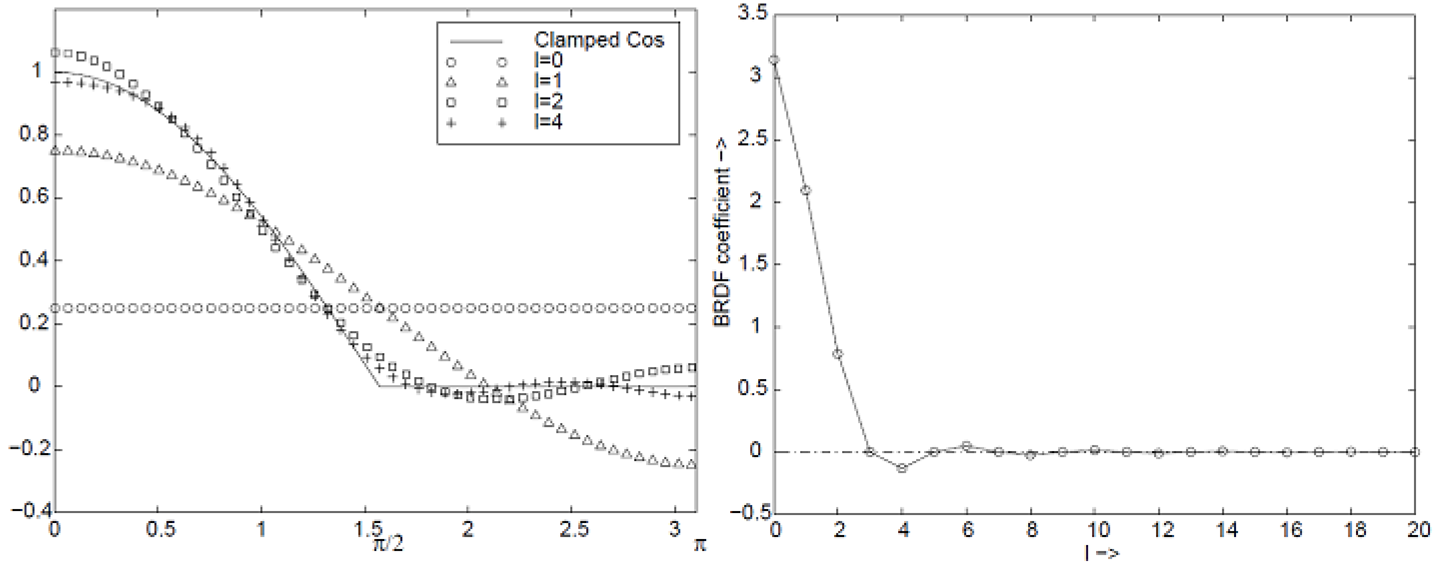


Figure 4: *Left: Successive approximations to the* clamped cosine *function by adding more spherical harmonic terms. For $l = 2$, we get a very good approximation. Right: The solid line is a plot of spherical harmonic coefficients $A_l = \Lambda_l \hat{\rho}_l$. For $l > 1$, odd terms vanish, and even terms decay rapidly.*

Phong BRDF

$$\hat{\rho} = \frac{s+1}{2\pi}\left(\vec{R}\cdot\vec{L}\right)^s \qquad (4)$$

$$\Lambda_\ell \hat{\rho}_\ell \approx \exp\left(-\frac{\ell^2}{2s}\right) \qquad s \gg 1, s \gg \ell \qquad (5)$$

- s is the specular exponent.
- This is approximately a Gaussian with width $\sim \sqrt{s}$. Beyond this, the inverse rendering problem begins to become ill-conditioned.
- As $s \to \infty$ we approach the mirror case.
- From the properties of convolution, we can represent the Phong BRDF by blurring the lighting and using a mirror BRDF instead.

Phong BRDF

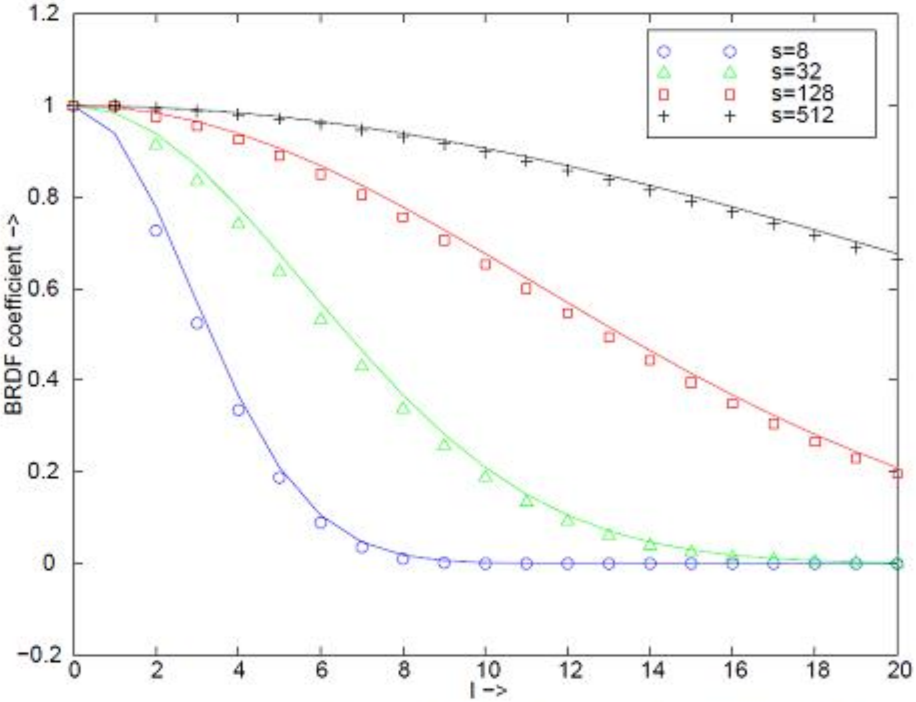


Figure 5: *Numerical plots of the Phong coefficients* $\Lambda_l \hat{\rho}_l$, *as defined by equation 18. The solid lines are the approximations in equation 19.*

Microfacet BRDF

$$\Lambda_\ell \hat{\rho}_\ell \approx \exp\left(-(\sigma\ell)^2\right) \quad (6)$$

- σ is the surface roughness.
- Add a Fresnel factor for non-normal exitance.
- Also approximately Gaussian; now the width is $\sim \sigma^{-1}$.
- The trick of blurring the lighting and using a mirror BRDF instead can work here too for $\ell \ll \sigma^{-1}$.

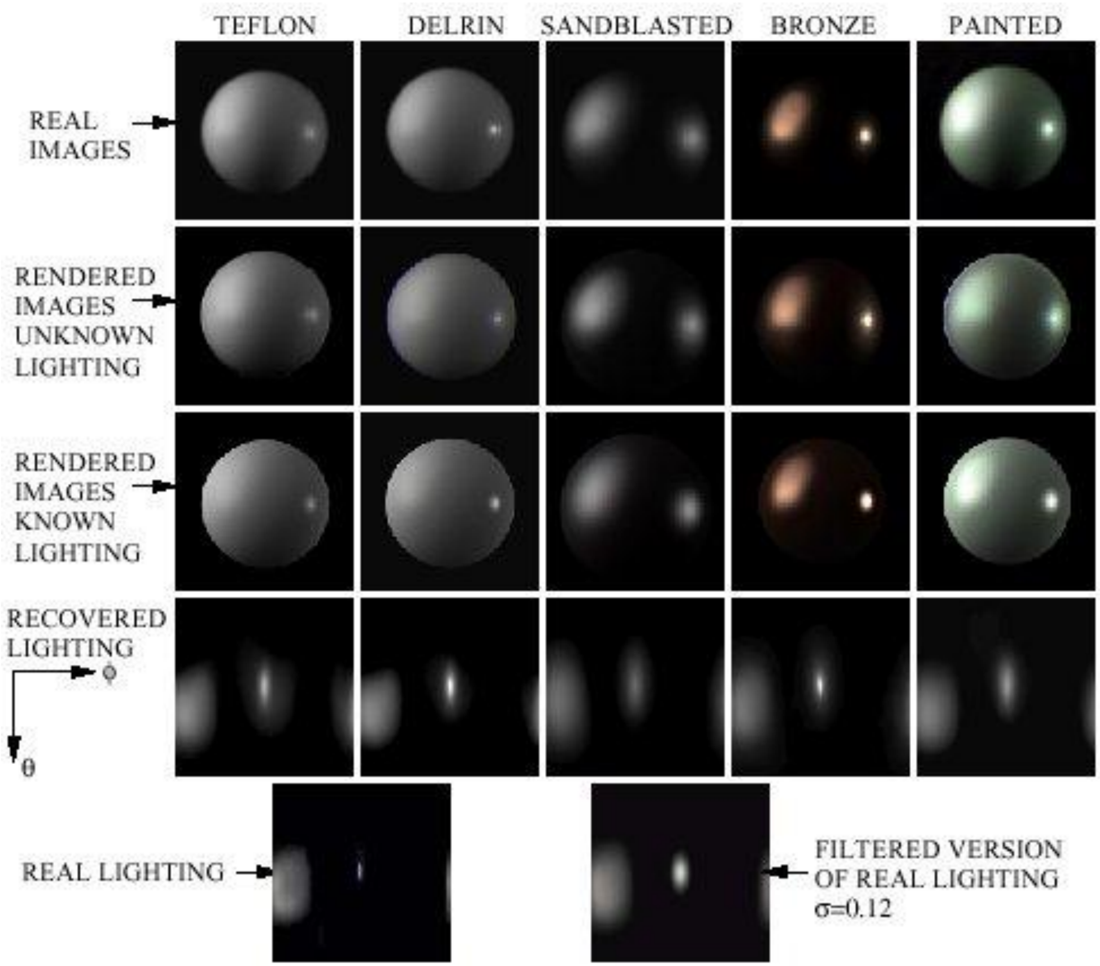
Decomposition of Lights for Microfacets

$$B = B_d + B_{\mathbf{s},\text{slow}} + B_{\mathbf{s},\text{fast}} \tag{7}$$

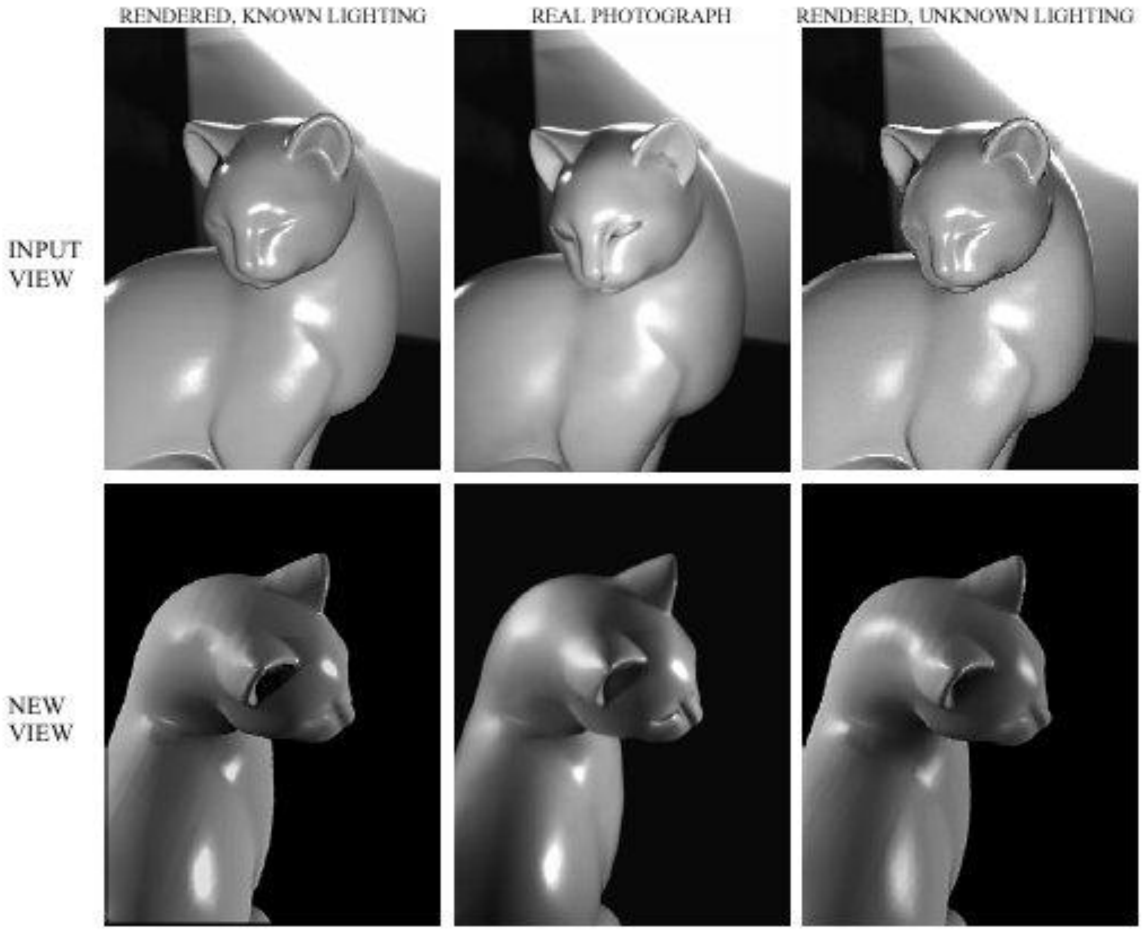
$$L = L_d + L_{\mathbf{s},\text{slow}} + L_{\mathbf{s},\text{fast}} \tag{8}$$

- B_d is the diffuse component. This can be represented using just the first 9 spherical harmonic terms of the lighting.
- $B_{\mathbf{s},\text{slow}}$ is the slow-varying lighting. Here we can blur the lighting and treat the BRDF as a mirror.
- $B_{\mathbf{s},\text{fast}}$ is the fast-varying lighting. Here we treat the lighting as a delta function. The reflection is approximated as a Gaussian.

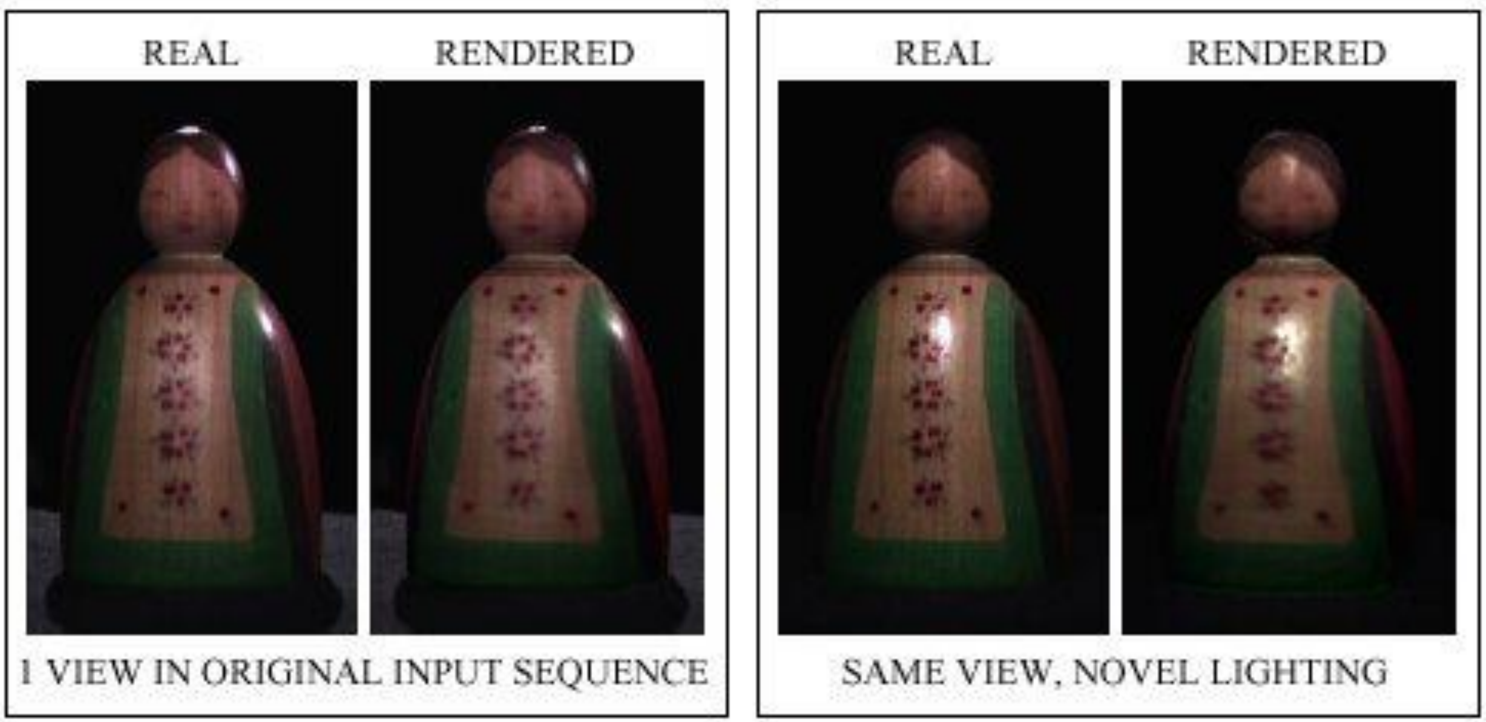
BRDF Recovery



From Complex Geometry



Questions?


REAL
RENDERED
1 VIEW IN ORIGINAL INPUT SEQUENCE
REAL
RENDERED
SAME VIEW, NOVEL LIGHTING