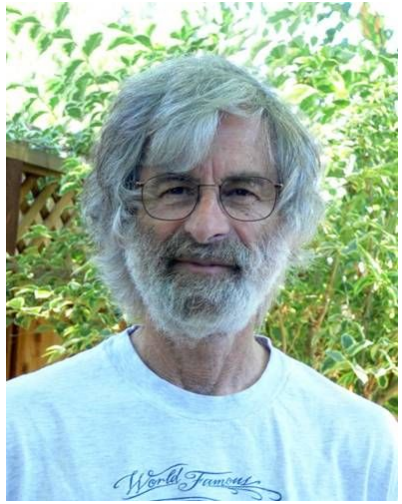
The Byzantine Generals Problem

Siqiu Yao

Authors

- Leslie Lamport
  - you again!
  - we all know him
- Robert Shostak
  - PhD in Applied Math, Harvard
  - SRI International
  - Founder, Ansa Software
  - Founder, Mira Tech
  - Borland Software
  - Founder Portera System
  - Founder Vocera
- Marshall Pease



Another story from Lamport?

Time, Clocks, and the Ordering of Events in a Distributed System 1978

The part-time parliament 1990

Another story from Lamport?

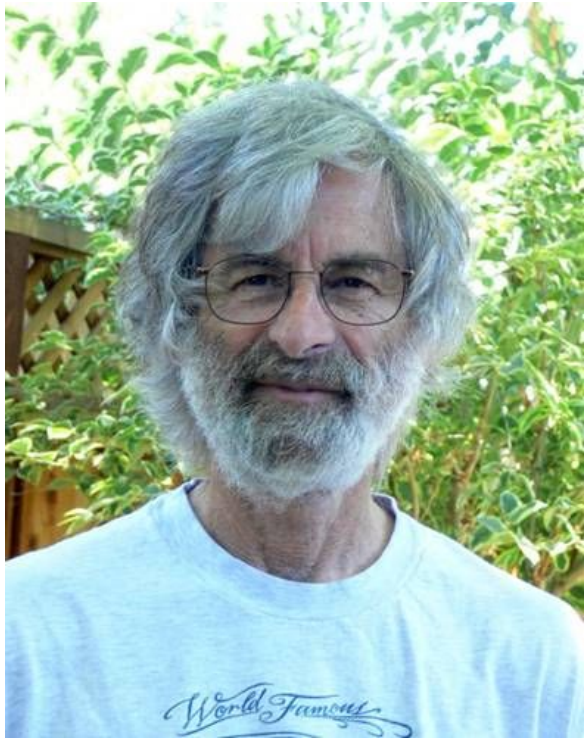
Time, Clocks, and the Ordering of Events in a Distributed System	1978
The Byzantine Generals Problem	1982
The part-time parliament	1990

How this story came

"I have long felt that, because it was posed as a cute problem about philosophers seated around a table, Dijkstra's dining philosopher's problem received much more attention than it deserves.

.....

The popularity of the dining philosophers problem taught me that the best way to attract attention to a problem is to present it in terms of a story."



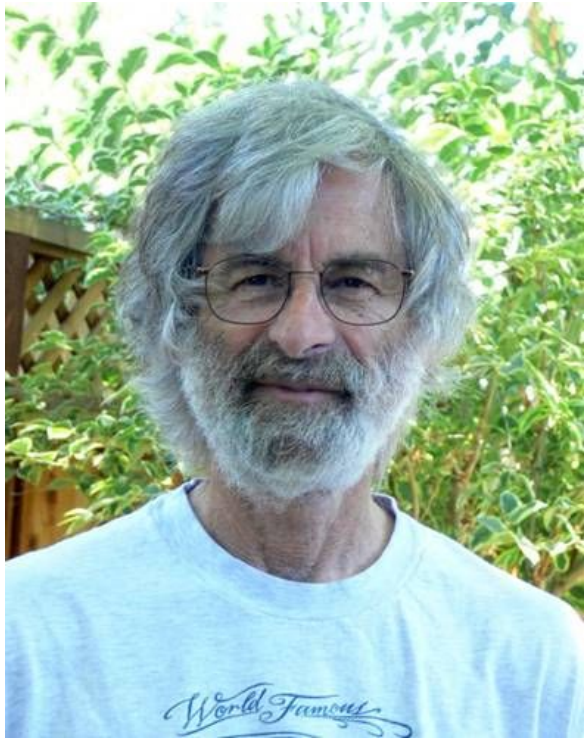
*http://lamport.azurewebsites.net/pubs/pubs.html#byz

How this story came

"

There is a problem in distributed computing that is sometimes called the Chinese Generals Problem, in which two generals have to come to a common agreement on whether to attack or retreat, but can communicate only by sending messengers who might never arrive.

"



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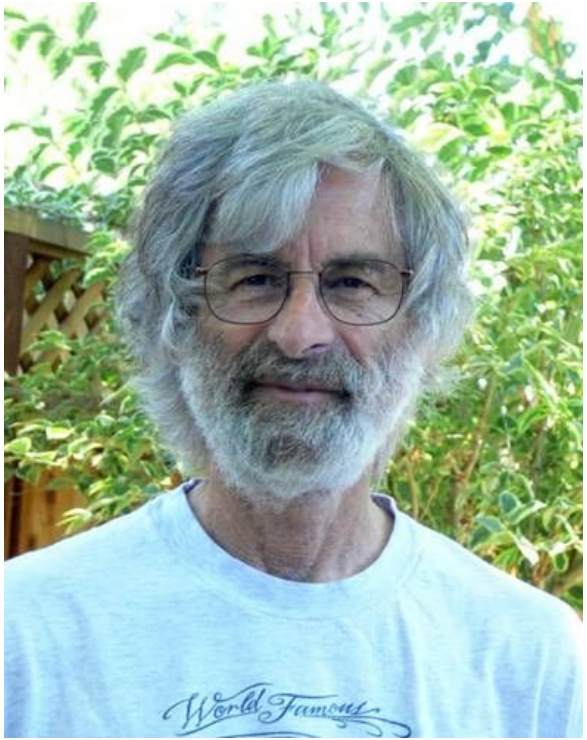
How this story came

"

I stole the idea of the generals and posed the problem in terms of a group of generals, some of whom may be traitors, who have to reach a common decision.

"



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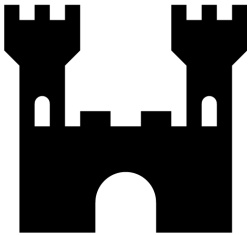
What is the Byzantine generals problem

Byzantine generals problem

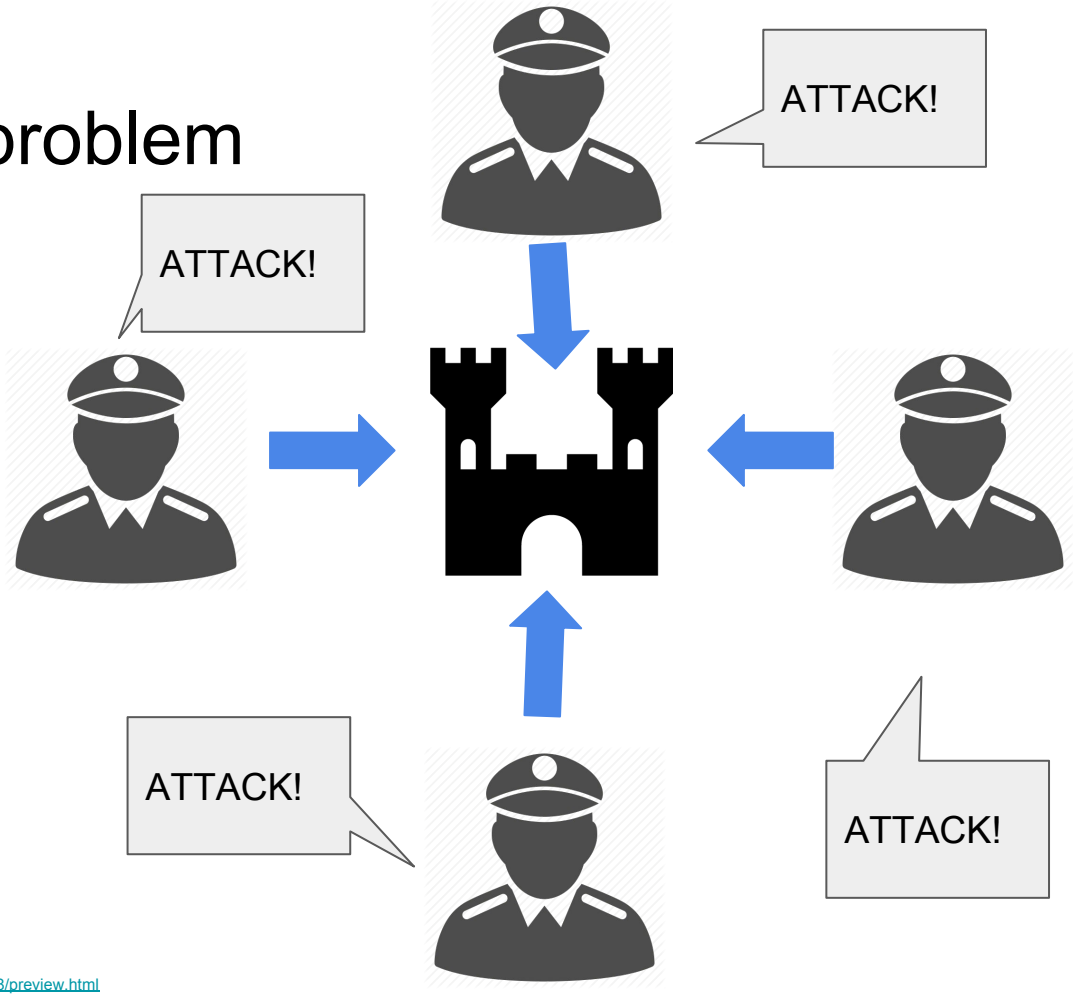
“several divisions of the Byzantine army are camped outside an enemy city, each division commanded by its own general. The generals can communicate with one another only by messenger. After observing the enemy, they must decide upon a common plan of action.”



*castle: http://simpleicon.com/castle.html
*general: https://www.kisspng.com/png-security-guard-police-officer-computer-icons-milit-609318/preview.html
*lieutenant: https://www.clipartmax.com/max/m2i8Z5i8b1H7N4H7/

Byzantine generals problem

- Generals should reach a consensus on the plan
- It could be ATTACK



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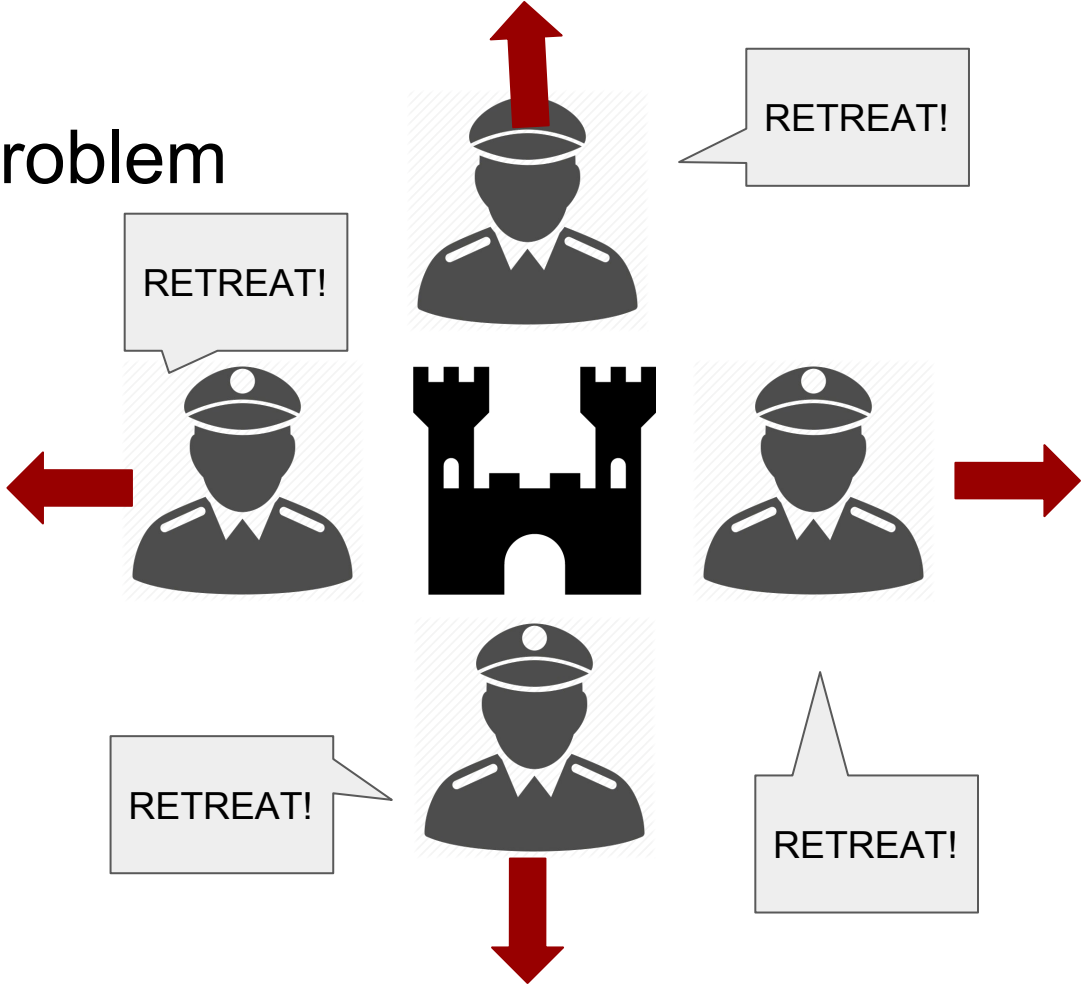
Byzantine generals problem

- Generals should reach a consensus on the plan
- Or RETREAT



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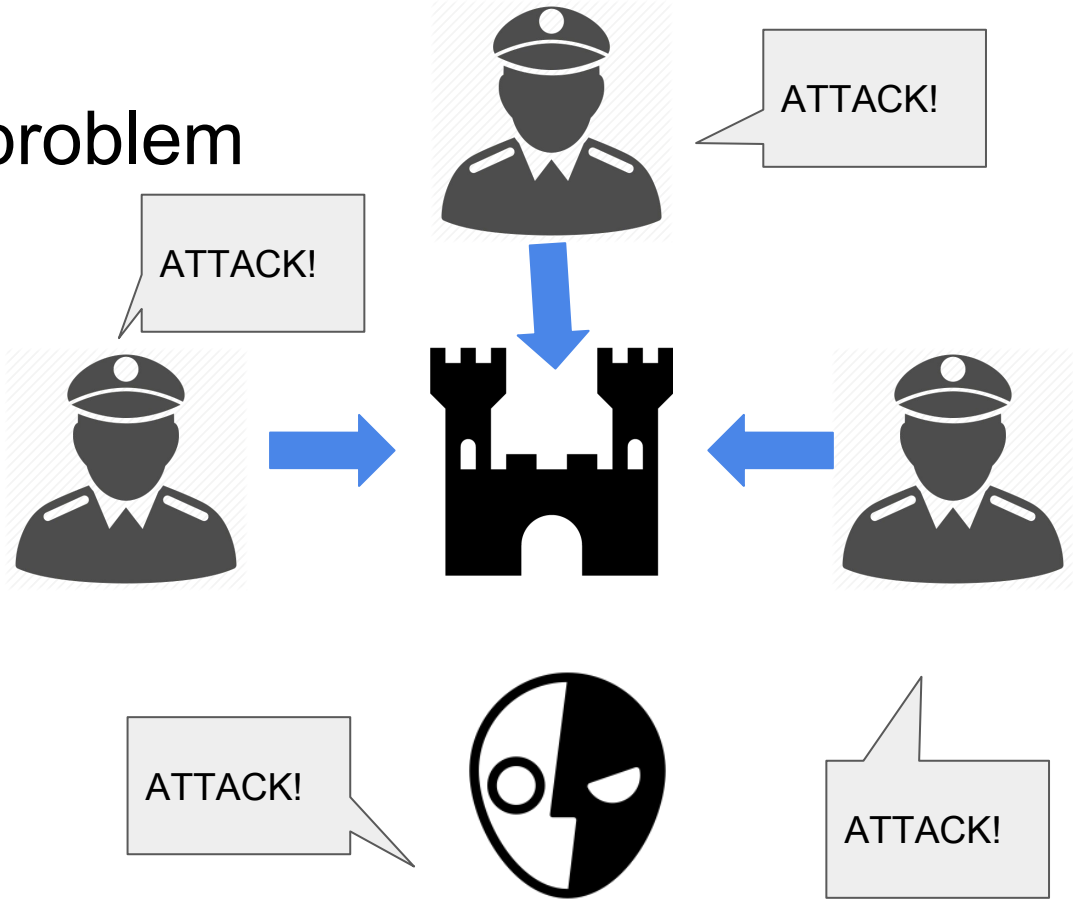
Byzantine generals problem

- But there might be traitors
- All loyal generals should reach a consensus



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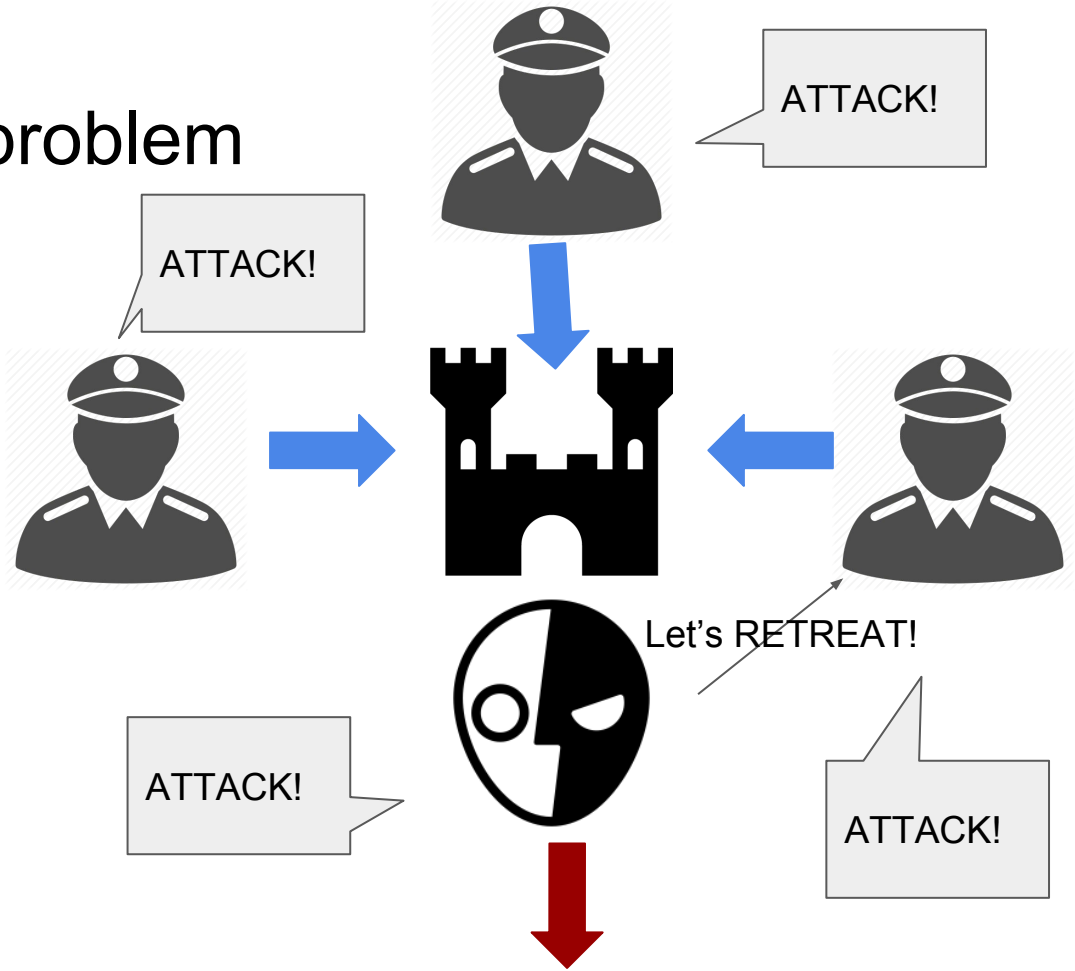
Byzantine generals problem

- But traitors can act arbitrarily
- All loyal generals should reach a consensus



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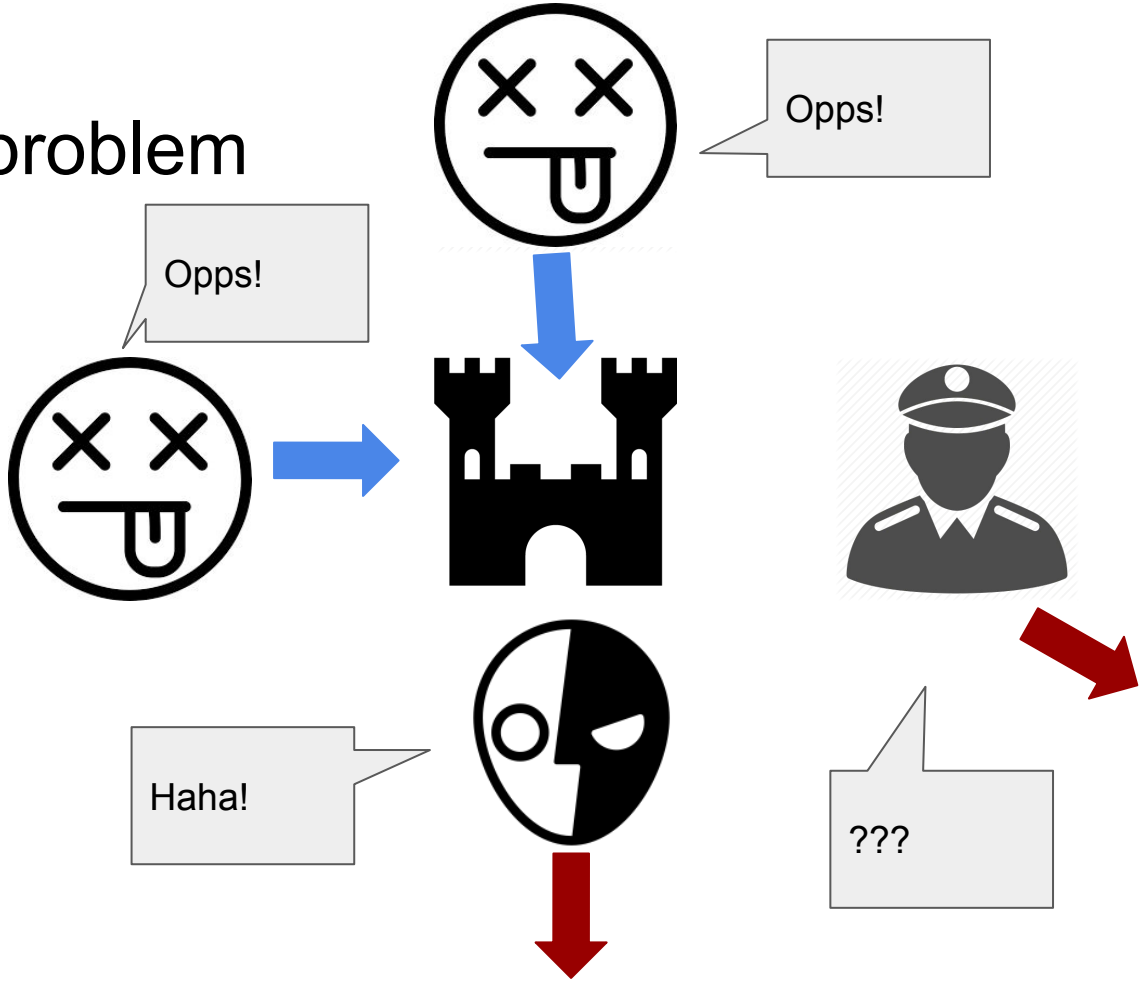

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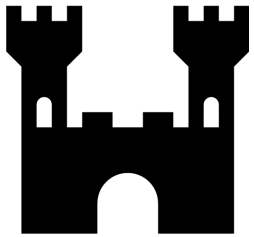
Byzantine generals problem

- A simplified version



“A commanding general sends an order to his n-1 lieutenant generals such that

IC1. All loyal lieutenants obey the same order.

IC2. If the commanding general is loyal, then every loyal lieutenant obeys the order he sends.”

What is the byzantine generals problem

- IC1. All loyal lieutenants obey the same order
- IC2. If the commanding general is loyal, then every loyal lieutenant obeys the order he sends.

What is the byzantine generals problem

- Consistency/Agreement
- IC2. If the commanding general is loyal, then every loyal lieutenant obeys the order he sends.

What is the byzantine generals problem

- Consistency/Agreement
- Validity

What is the byzantine generals problem

- Consistency/Agreement
- IC2. If the commanding general is loyal, then every loyal lieutenant obeys the order he sends.

What is the byzantine generals problem

- Consistency/Agreement
- Validity
- Liveness/Termination?

Impossibility Result

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what are oral messages?

Impossibility result

oral messages:

- every message that is sent is delivered correctly
- the receiver of a message knows who sent it
- the absence of a message can be detected

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oral messages:

- every message that is sent is delivered correctly
- authenticated channel
- the absence of a message can be detected

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Impossibility result

oral messages:

- every message that is sent is delivered correctly
- authenticated channel
- synchronous network

Impossibility result

“if the generals can send only oral messages, then no solution will work unless more than ⅔ of the generals are loyal.”

in a synchronous network, with authenticated channel, when m generals are traitors, no solution will work unless there are more than 3m generals

impossibility result - proof

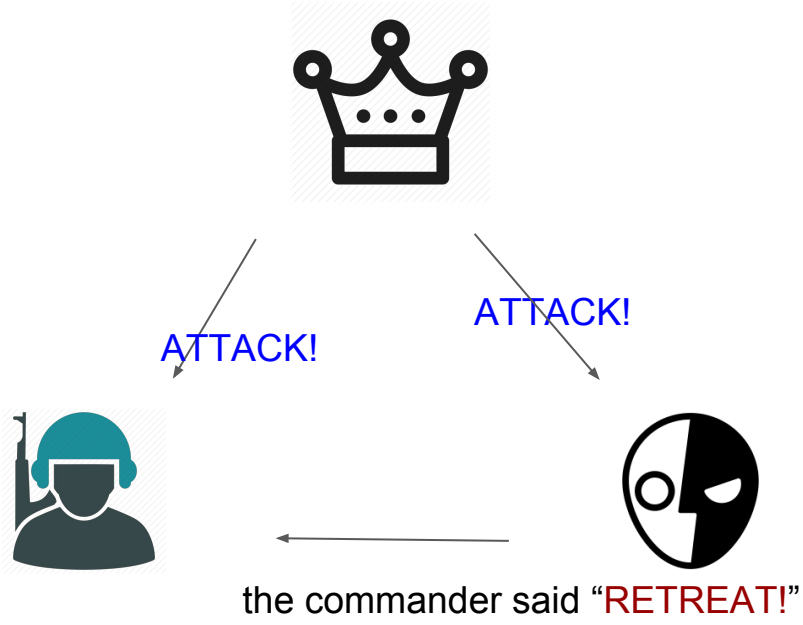
- case $m = 1$:

impossibility result - proof

- case m = 1:
  - scenario 1:
    - the commander is loyal
    - one lieutenant is a traitor

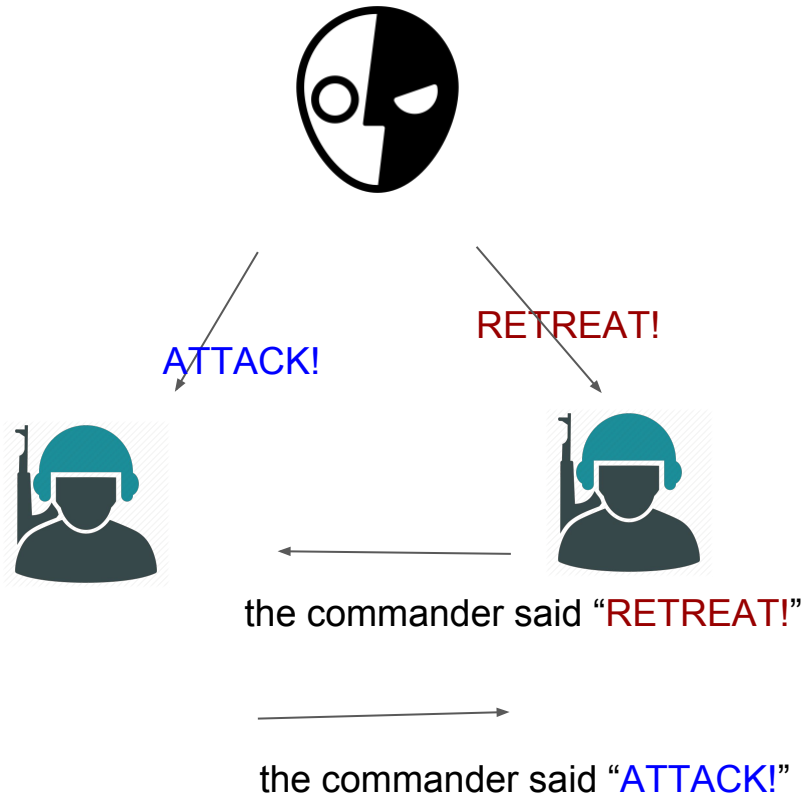
impossibility result - proof

- case m = 1:
  - scenario 1:
    - the commander is loyal
    - one lieutenant is a traitor
    - the left lieutenant should ATTACK

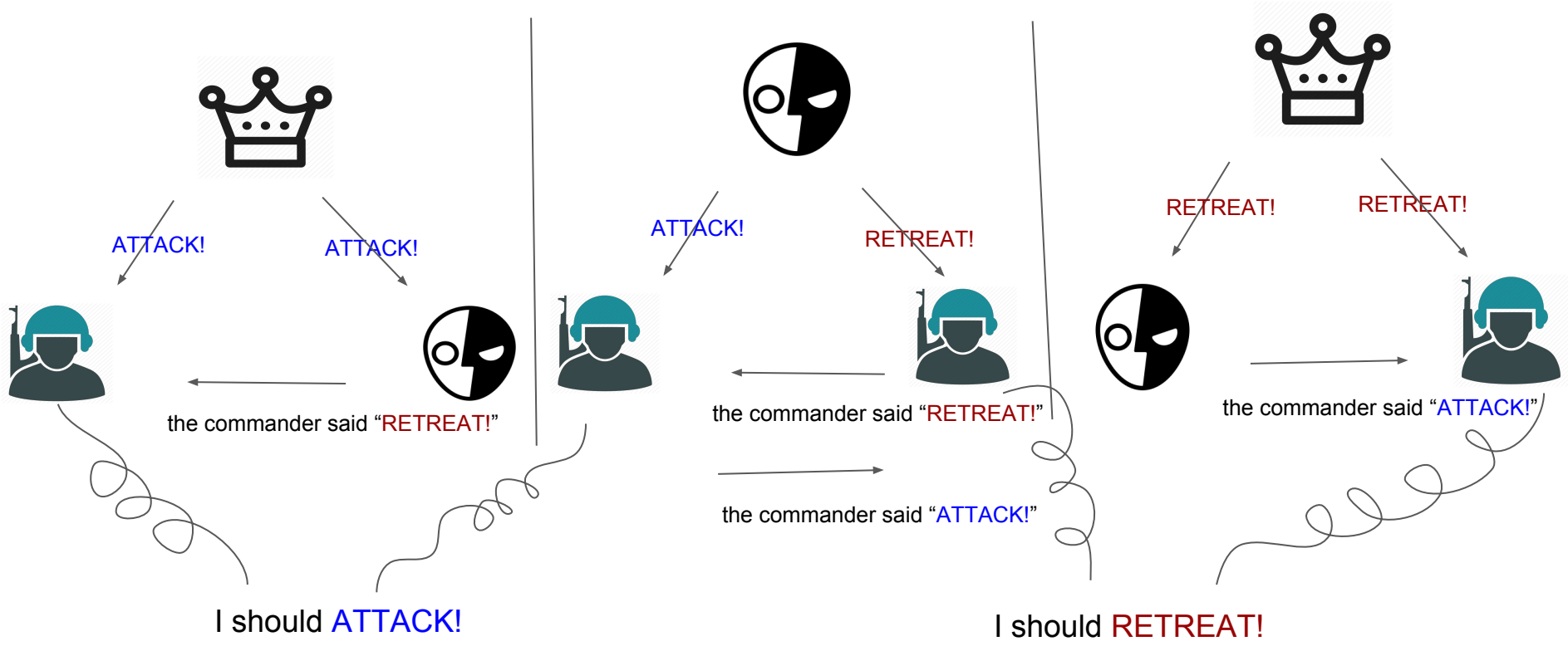


impossibility result - proof

- case m = 1:
  - scenario 2:
    - the commander is a traitor



Three scenarios



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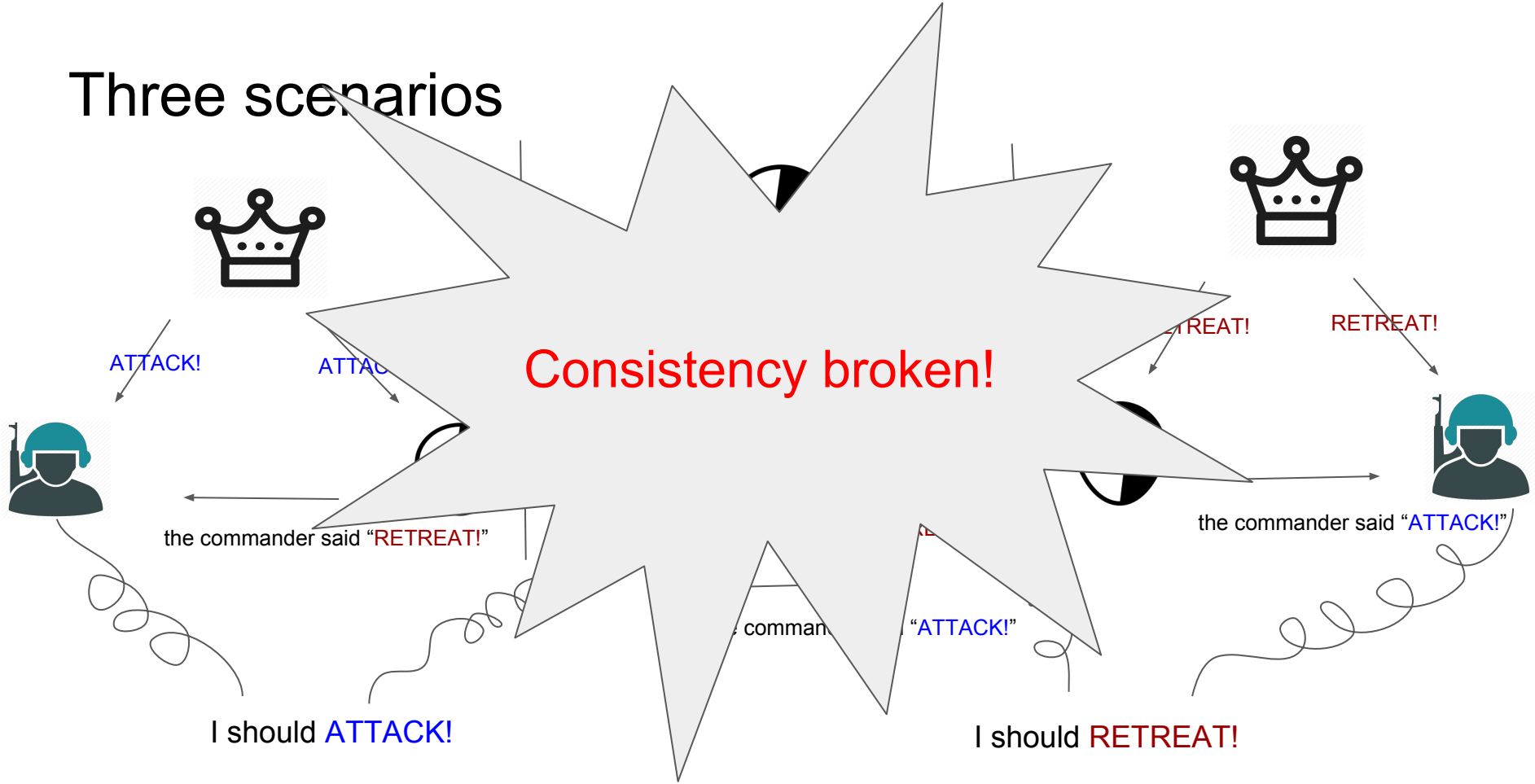


impossibility result

prove $m > 1$ by contradiction

- assume we have a solution protocol f for $3m$ generals when $m > 1$
- we can solve $m = 1$ case by leveraging f

impossibility result

prove m > 1 by contradiction

- assume the three generals are x, y, z, and x is the commander;
- according to protocol f
  - x simulates one commander and m-1 lieutenants
  - each of y and z simulates m lieutenants

impossibility result

prove m > 1 by contradiction

- assume the three generals are x, y, z, and x is the commander;
- according to protocol f
  - x simulates one commander and m-1 lieutenants
  - each of y and z simulates m lieutenants
- at most one of x, y, z is a traitor
  - at most m simulated traitors
  - protocol f can solve the case when there are at most m traitors

impossibility result

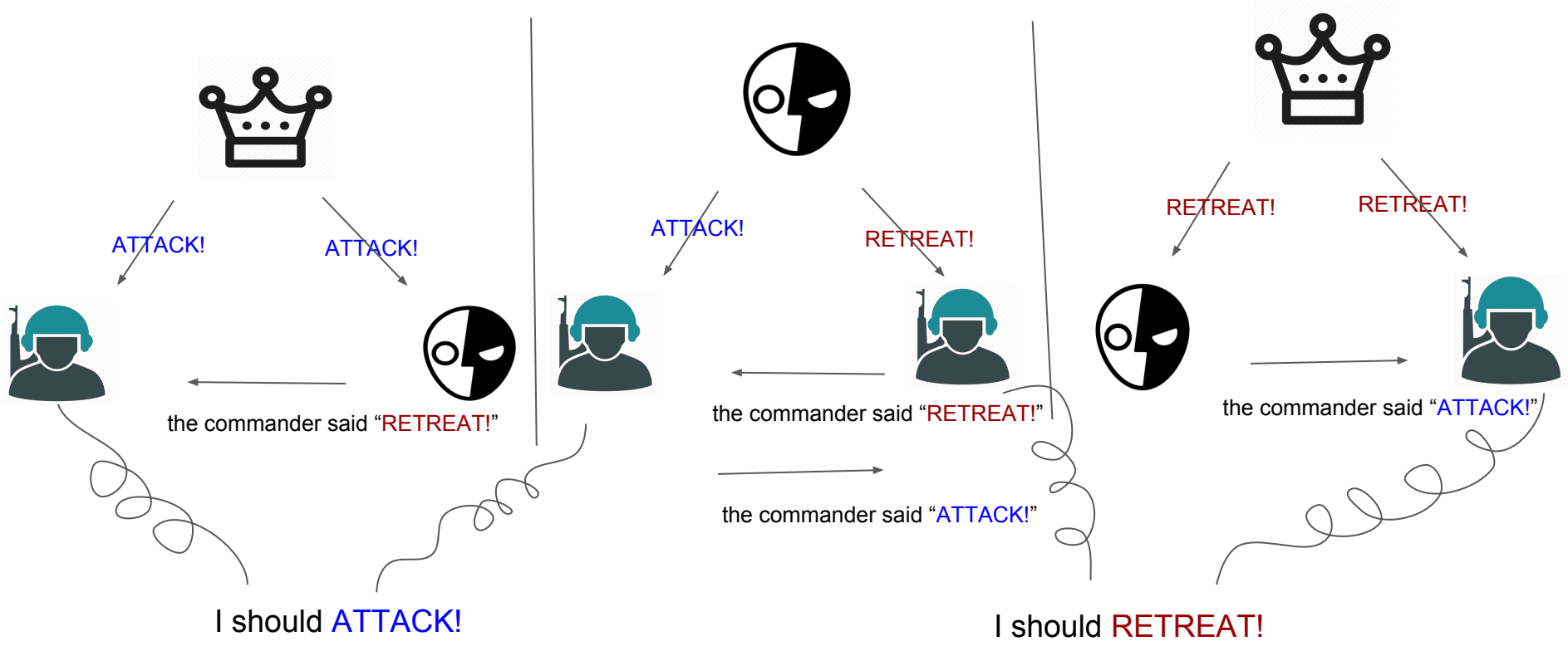
prove $m > 1$ by contradiction

- if we can solve case $m > 1$ then we can solve $m = 1$
- we proved case $m = 1$ cannot be solved
- contradiction!

Oral messages’ fault

- With only oral messages, traitors can lie by telling the wrong command they received

Three scenarios



Signed message

- With only oral messages, traitors can lie by telling the wrong command they received
- Signed messages
  - cannot be forged
  - anyone can verify the authenticity

Solutions:
oral messages and signed messages

Solutions - with oral messages

- OM(k)
  - k == 0
    - commander sends the value to every one
    - everyone return the value they received

Solutions - with oral messages

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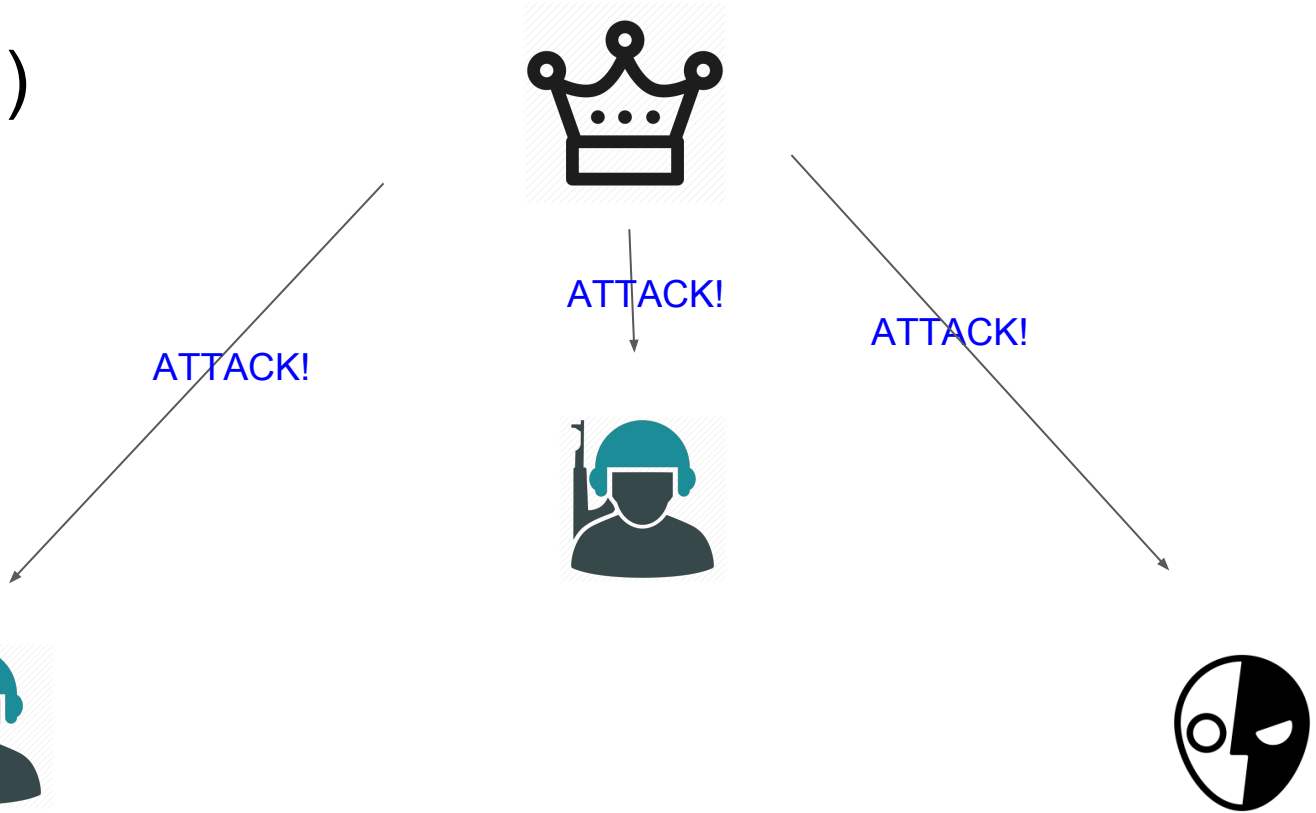
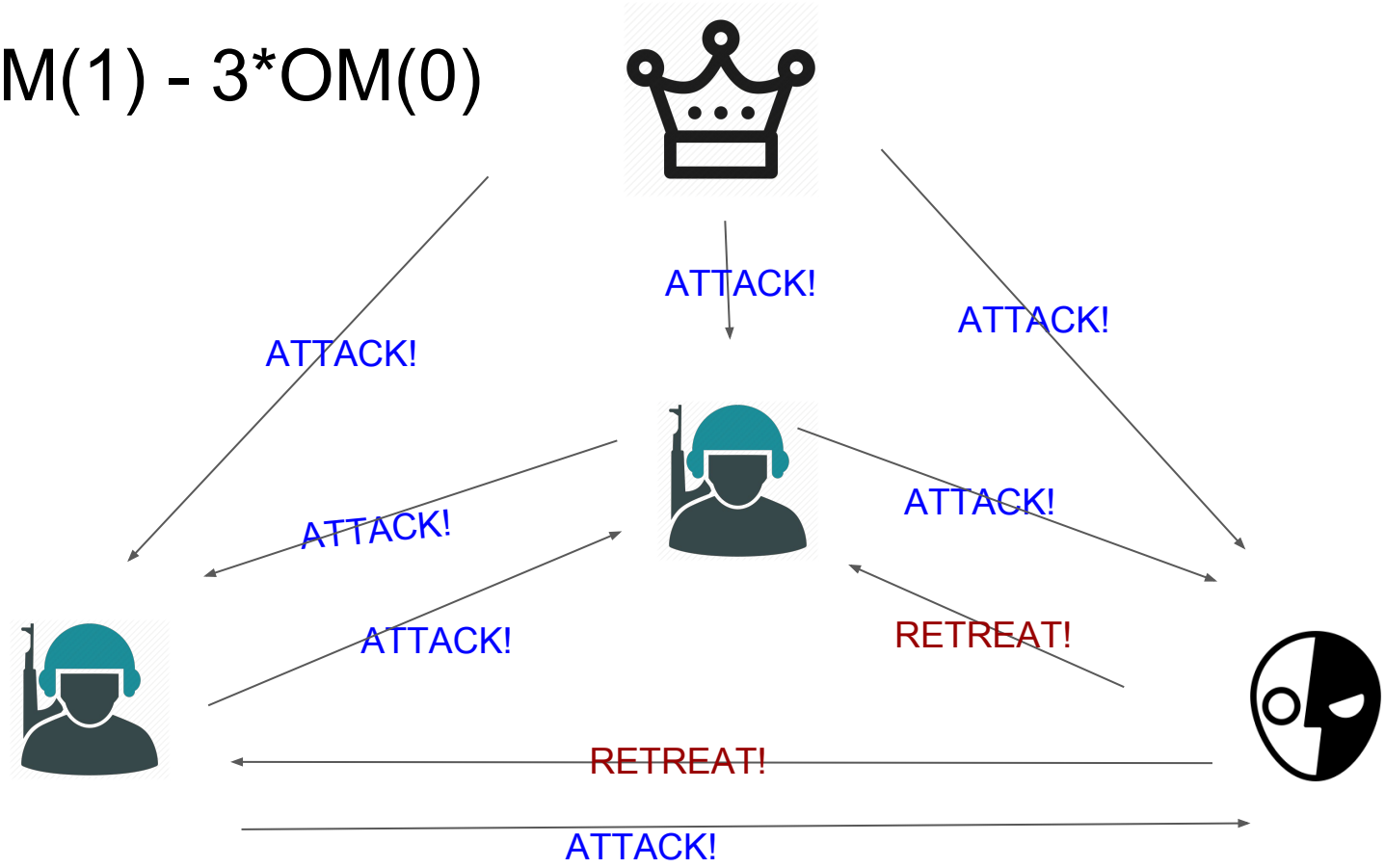
    - everyone return the value they received
  - k > 0
    - commander sends the value to every one
    - everyone start a smaller bgp OM(k-1) containing all ones but the current commander and become the new commander
    - everyone participated n-1 OM(k-1) and get n-1 values, return the majority

OM(1)



OM(1) - 3*OM(0)


ATTACK!
ATTACK!
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ATTACK!
RETREAT!
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ATTACK!

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- Intuition: for every message M received, solve a smaller bgp containing all but the current commander to tell others you received M

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- OM(m) for m traitors when 3m < n

Solutions - with oral messages

- OM(k) - Message complexity: (n-1)*MC(OM(k-1)) + n-1 = O(n^m)
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  - when there will be no more messages, return choice(V(i))
  - choice(V)
    - return v then V = {v}
    - return RETREAT when |V| = 0

SM(1)

0

ATTACK!:
0

RETREAT!:0

1

2

SM(1)

0

ATTACK!:
0

RETREAT!:0

RETREAT!:0:2

ATTACK!:0:1

1

2

SM(1)

0

ATTACK!:
0

RETREAT!:0

RETREAT!:0:2

ATTACK!:0:1

1

2

V(1) = V(2)

SM(1)

0

ATTACK!:
0

RETREAT!:0

RETREAT!:0:2

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1

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Choice(V(1)) = Choice(V(2))

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- SM(m+1) for m traitors

Solutions - with signed messages

- SM(k) - message complexity: O(n^2)
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Minimum number required for which an *f*-resilient consensus protocol exists

	synchrony	asynchrony	partial synchrony
fail-stop	f+1	inf	2f+1
crash	f+1	inf	2f+1 (Paxos)
byzantine with digital signature	f+1 (SM(f+1))	inf	
byzantine with authenticated channel	3f+1 (OM(f))	inf	

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byzantine with digital signature	f+1 (SM(f+1))	inf	???
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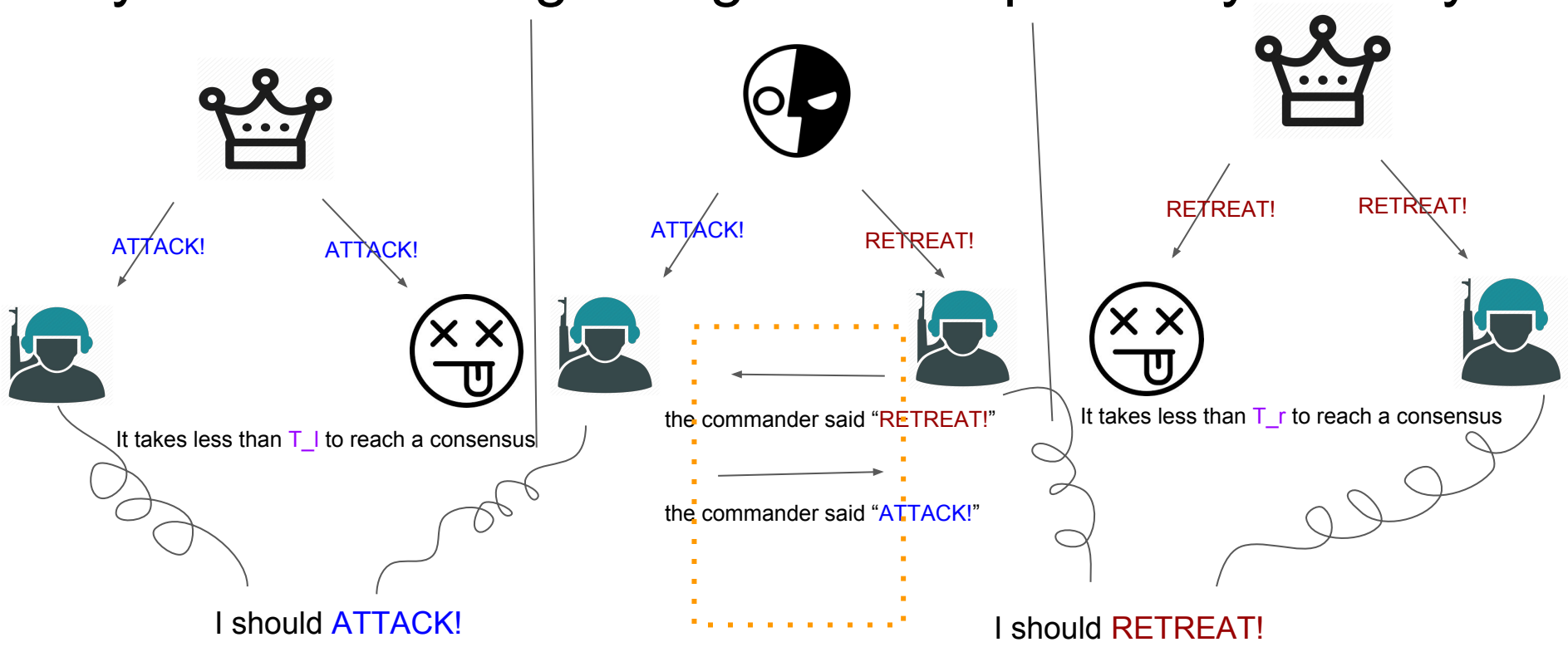
Byzantine with digital signature in partial synchrony

- No partial synchronous protocols can tolerate ⅓ faults.
- Sound familiar?
- Assume there exist a protocol that can solve it.

Byzantine with digital signature in partial synchrony



It takes more than max(T_l, T_r) for messaged to be delivered

Practical Byzantine Fault Tolerance

- Commander sends the value to every lieutenant
- Every lieutenant
  - if it receives a new value v, broadcast (prepare, v)
  - if it receives 2f+1 (prepare, v), broadcast (commit, v)
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- Ensure agreement
- Ensure liveness under an loyal commander

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- Ensure agreement
- Ensure liveness under an loyal commander
- What if the commander is faulty?
  - we need view change

Minimum number required for which an *f*-resilient consensus protocol exists

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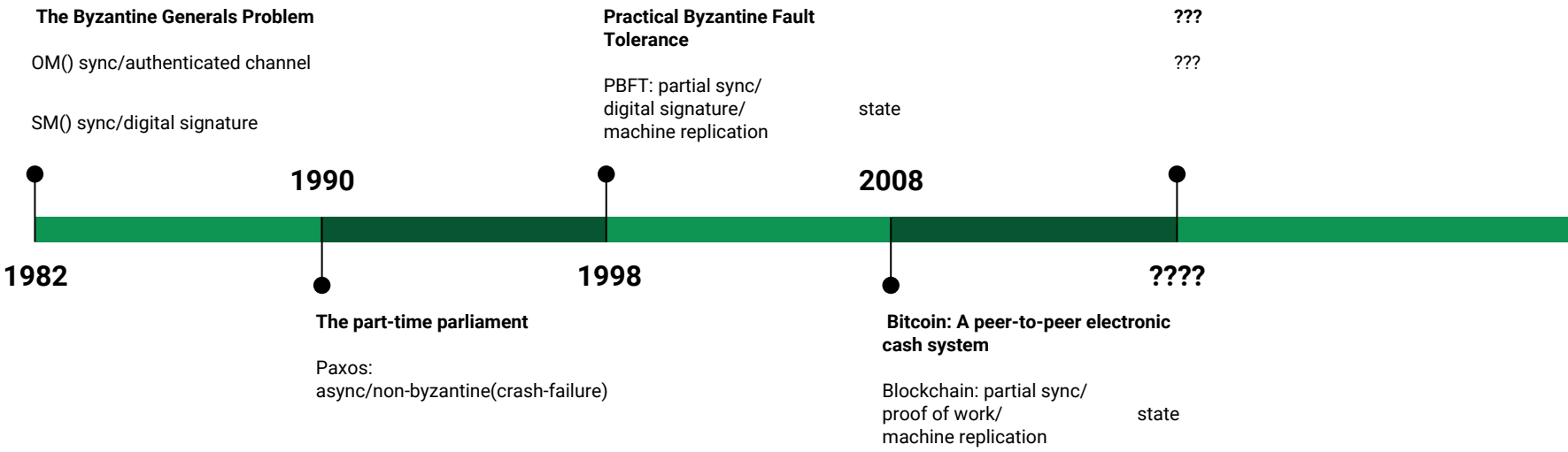
fail-stop	f+1	inf	2f+1
crash	f+1	inf	2f+1 (Paxos)
byzantine with digital signature	f+1 (SM(f+1))	inf	3f+1(PBFT)
byzantine with authenticated channel	3f+1 (OM(f))	inf	

Thoughts

- Defined Byzantine generals problem
- Proved lower bound in synchronous environment with authenticated channel
- Introduced solutions in synchronous environment with authenticated channel and with digital signature
- But today we usually discuss about the case when in partial synchronous/asynchronous environment with digital signature where PBFT works

Timeline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OM() sync/authenticated channel
SM() sync/digital signature
1982
1990
The part-time parliament
Paxos:
async/non-byzantine(crash-failure)
Practical Byzantine Fault Tolerance
PBFT: partial sync/
digital signature/
machine replication
state
1998
2008
Bitcoin: A peer-to-peer electronic cash system
Blockchain: partial sync/
proof of work/
machine replication
state
???
???
????