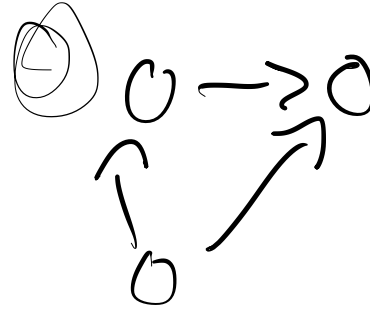
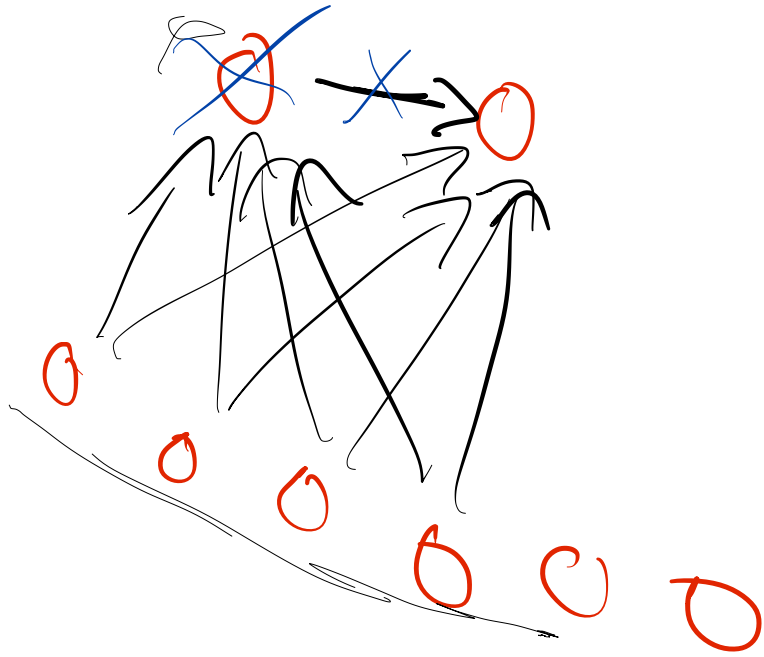
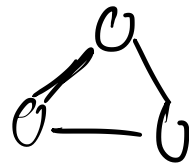


April 23, 2020



Counting triangles



Model:  $O(1)$  check if  $(i, j) \in G$

$O(d_i)$  to get neighbors of  $i$

① Brute force

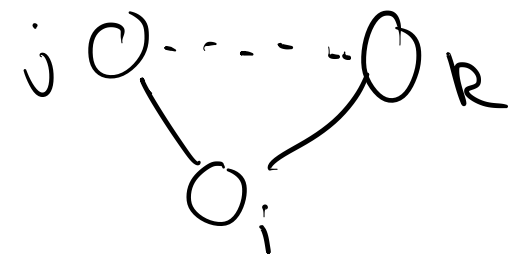
for  $i, j, k \in V$

check:  $(i, j), (i, k), (j, k)$

$\Theta(|V|^3)$

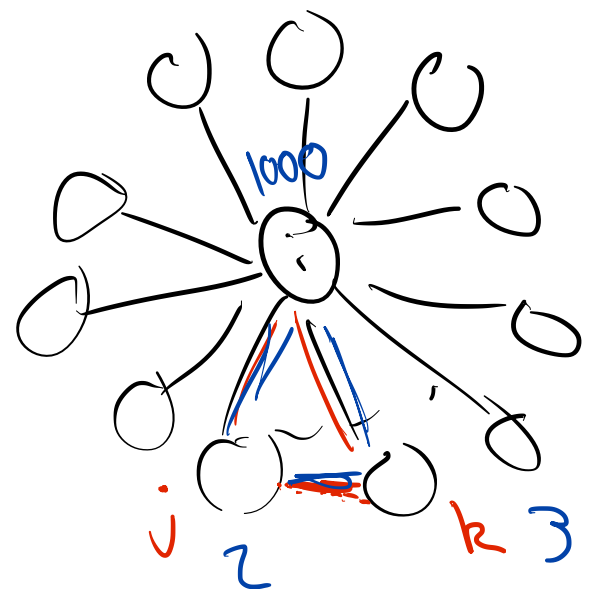
## ② Neighbors pairs

for  $i \in V$



for  $j, k$  neighbor of  $i$   
check  $(j, k)$

$$\Theta\left(\sum_i \binom{d_i}{2}\right)$$



$$c|V| \quad \Omega(|V|^2)$$

$$O(|V|) \text{ edges} \\ \Rightarrow \Omega(|V|^2)$$

③ How-degree center neighbor pairs

(A) sort nodes by degree  $\sigma$

$$\sigma(i) < \sigma(j) \Rightarrow d_i \leq d_j$$

(B) for  $i \in V$

for neighbors  $j, k$

$$\sigma(i) < \sigma(j), \sigma(k)$$

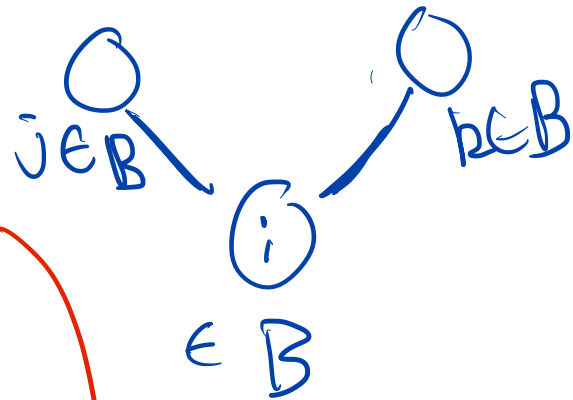
check  $(j, k)$

Claim:  $O(m^{3/2})$   $m = |E|$

Proof:  $B = \{i \in V \mid d_i > \sqrt{m}\}$

$S = \{j \in V \mid d_j \leq \sqrt{m}\}$

$$O\left(\sum_{i \in S} d_i^2 + \sum_{j \in B} d_j^2\right)$$



$$\bar{d}_i = |\{j \mid (i,j) \text{ or } (j,i) \text{ or } (i,j) < \text{or } (j,i)\}| \leftarrow \bar{d}_i \leq d_i$$

Small nodes:  $\sum_{i \in S} \bar{d}_i^2 \leq \sqrt{m} \sum_{i \in S} \bar{d}_i$

$$\leq \sqrt{m} \sum_{i \in V} d_i = \sqrt{m} \cdot 2m = O(m^{3/2})$$

Big nodes:

$$|B| \sqrt{m} < \sum_{i \in B} d_i \leq 2m$$

$$\Rightarrow |B| \leq \frac{2m}{\sqrt{m}} = O(\sqrt{m})$$

work:  $O\left(\binom{|B|}{3}\right) = O(m^{3/2})$