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THE STRUCTURE OF POSITIVE INTERPERSONAL RELATIONS IN SMALL GROUPS*

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Abstract

The authors sought to test Homans's proposition that small groups inevitably generate a social structure which combines subgroups (cliques) and a ranking system. We present a graph theoretical model of such a structure and prove that a necessary and sufficient condition for its existence is the absence of seven particular triad types. Expected frequencies of the seven triad types in random graphs are deduced from elementary probability theory, and we suggest that a reasonable operational statement of Homans's theory is that in most groups, the seven key triads are less frequent than the random model would predict. A data pool of sociograms and sociomatrices from 427 groups was collected from diverse published and unpublished studies. Random samples of 30 school and 30 adult groups were drawn from the pool and analysed. Significant majorities of both samples showed deviations from chance in the directions predicted. As a check, 60 simulated groups with truly random relationships were analysed and found to be close to the chance expectations and quite different from the real data samples. Overall, we claim support for Homans's theory.

In The Human Group[1] George Homans presents a set of closely linked propositions about subgroup formation and ranking. In paraphrase, his argument is this:

1) In any group the external system (loosely, the group's environment) makes it inevitable that frequencies of interaction will be unevenly distributed among the member pairs (p. 86).

2) Because differential frequencies of interaction, interpersonal liking, and similarity in other sentiments and activities go together, pairs and larger subsets with initially higher rates of ineraction come to be increasingly differentiated from the rest of the group, forming subgroups (cliques) characterized by high rates of voluntary interaction, positive interpersonal sentiments, and normative consensus (pp. 112, 118, 120).

3. Nevertheless, the members are more nearly alike in the norms they hold than in their conformity to these norms (p. 126) and since the closer a person's activities come to the norm, the higher his rank will be (p. 141), all groups develop systems of ranking.

It is hard to avoid the inference that if we examine voluntary interaction and sentiments in small groups, Homans expects us to find two sorts of structures, differentiation into cliques and elaboration into ranks, and he expects us to find them in group after group after group. Furthermore, he expects us to find both structures with the same variables. Not only do subgroup members have higher rates of interaction, but so do higher ranking members (p. 182). Not only do subgroup members have higher frequencies of liking, but higher ranking persons are better liked (p. 148).

These propositions are as well known as any in sociology, yet we have little systematic evidence for them. Homans himself says:

> ...let us be clear that it (the association between interaction and liking) is only a hypothesis, not a theorem. We have offered no proof, except what is provided by the behavior of the Bank Wiremen, and a statistician would say that a single instance is not nearly enough. Plenty of confirmatory evidence could be found in anthropological and sociological studies of small groups (p. 114).

This paper aims to test Homans's structural propositions using simple statistical models developed from graph theory and applying them to a data pool of interpersonal relations measures (sociograms and sociomatrices) for 427 groups. To the extent that our model is plausible, our probabilistic reasoning is valid, and our 427 groups are representative, the results provide favorable evidence for the propositions.

A Graph Theoretical Model

While Homans's definitions are notoriously crisp, he nowhere defines the total structure which is implied by his twin principles. We take the liberty of sketching such a model, hoping that it does justice to the original.

We begin with the notion of a "positive relation" and say that person _i_ has a positive relation to person _j_ if he: 1) frequently interacts with _j_ on a voluntary basis (formal authority is excluded from the hypotheses, pp. 244-248),

2) expresses a positive sentiment about j, 3) would prefer to interact with j on a voluntary basis, or 4) claims that j is his friend.

Note that the opposite of a positive relation, non-positive, may be neutral (indifferent) or negative (dislikes, avoids, etc.). Note further that positive relationships are not defined as symmetrical. If i has a positive relation to j, j may or may not reciprocate it. There are three logical possibilities which we will call M for mutual positive relations, A for asymmetric relations in which there is a positive relation from i to j or j to i but not both, and N for mutual non-positive relations. In graphs, we may draw them as follows ($i \longleftrightarrow j$ = M) ($i \longrightarrow j$ or $i \longleftarrow j$ = A) ($i \leftarrow - - \rightarrow j$ = N).

Having granted that pair relations may be symmetric or asymmetric, we should now turn to the triads produced by all possible combinations of members taken three at a time. The logical heart of our model will consist of a set of propositions about these triads. However, our discussion will be clearer, though less rigorous, if we skip ahead to the sort of group structure which is implied by our yet unstated triad propositions.

We begin by treating a group's ranking structure as a series of ordered levels, which is another way of saying that there may be more people than status distinctions in the group. It is useful to think of the levels as stories in a building, in the sense that people on a given floor do not differ in level, any two persons on different floors are unambiguously ordered by level, and the stories form a complete order.

The building analogy is useful, but misleading in one important sense. While floors and ceilings mark the levels in a building, in structural theory we seek to generate such features from the pattern of pair relations themselves. Indeed one may think of social structure as those characteristics of a group which may be deduced from the characteristics of pair relations within the group. This leads us to one of the main ideas of the model: Relations of the sort we have called A are assumed to connect persons in different levels, while M and N relations are assumed to connect persons in the same level. Further, we assume that in pairs connected by A relations, the recipient of the positive relationship is in the higher level.

We are claiming that if you and I like each other or if neither of us likes the other we are probably in the same status level in our group, but if I like you and you do not like me, you are probably in a different and higher level. We may think of such A relations as "admiration" and summarize the whole business with the slogan, "admiration flows up levels."

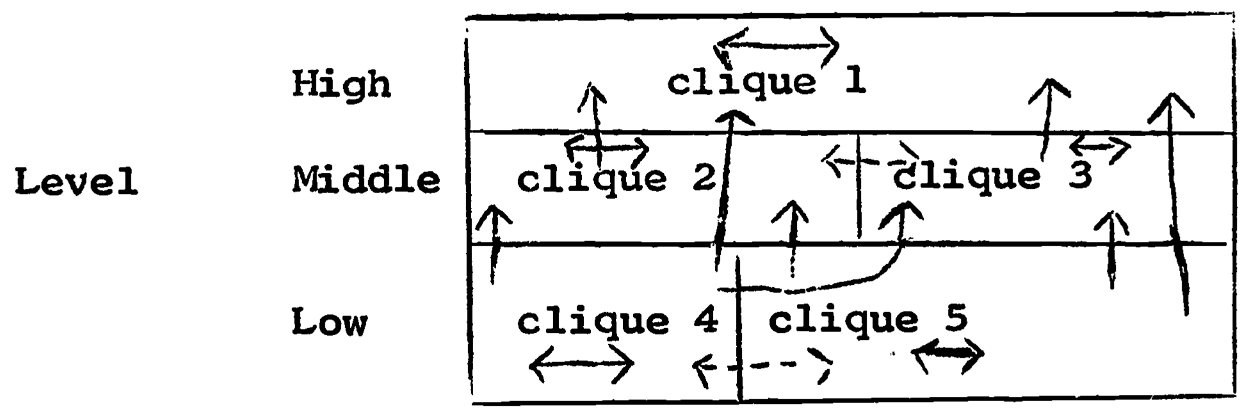
The second major idea of the model is that within a level there may be disjoint subsets of people (cliques or sub-groups) analogous to people in different rooms on a floor of a building. Again, these must be defined relationally. All intra-level relationships, of course, must be type M or type N if all A relations lie between levels, which leads us to the second main idea of the model: M relations are assumed to

<u>connect persons in the same clique within a level. N relations are assumed to connect persons in different cliques with a level.</u>[2] Using our building analogy we advance the slogan "N relations make partitions."

Putting both ideas together, the heart of the model is the notion that in small groups the members tend to be divided into levels by the pattern of their A relations and within levels they tend to be divided into cliques by the pattern of their M and N relations. Figure 1 puts the idea in rough schematic form:

Figure 1.

Levels, Cliques and Relations



Remembering that we have not yet stated any principles which guarantee that such a structure must emerge and be consistent, let us examine Figure 1. We see that it has three levels and five cliques, though the number of cliques and levels is not fixed by the model. Within each clique all relationships are of the M type, where cliques differ in level there are always A relationships with the arrows pointing up, and between cliques at the same level there are N relationships. We further note that the top level has only one clique, which is to say that all cliques are assumed to lie within a level but it is not assumed that every level has a clique. None of the levels in figure 1 has more than two cliques, but this is simply because the diagram would be too cluttered.

Let us now ask whether Figure 1 is a plausible translation of Homans's ideas.

Considering cliques (subgroups) first, Homans defines them as follows:

> ...If we say that individuals A,B,C,D,E... form a group, this will mean that at least in the following circumstances hold. Within a given period of time, A interacts more often with B,C,D,E,...than he does with M,N,L,O,P...whom we choose to consider outsiders or members of other groups. B also interacts more often with A,C,D,E,... than he does with outsiders, and so on for the other members of the group. (p. 84)

In other words cliques are subsets of individuals with higher rates of positive relationships among themselves than with outsiders. This, of course, is the definition of cliques in the theory of structural balance and clusterability.[3] Our cliques have this property since each pair within a clique has two positive relationships, while each inter-clique pair has either one positive relationship (if _i_ and _j_ are in different levels) or none (if _i_ and _j_ are in the same level).

Because Homans nowhere gives a formal definition of ranking it is harder to say that our version fits his second principle. However, the model has two properties which seem natural for a ranking.

First, people in higher levels receive more positive relations. Consider person *i* in level *i* and person *j* in some lower level, *j*. Mr. *j* receives positive relations from everyone in levels 1....*j-1* plus anyone in his own clique in level *j*. Lofty Mr. *i*, however, receives all of these plus relations from everyone in level *j* whether or not they are in *j*'s own clique, plus some from everyone in levels which might occur above *j* and below *i*, plus those from anyone in *i*'s own clique. In the limiting case where *i* and *j* are the only members of adjacent levels, *i* still receives one more positive relation than *j*, the one from *j* to *i*, which is not, by definition, reciprocated. Thus, in general, if two persons differ in level, the one in the higher level will receive more positive relations. This property is not only consistent with Homans's statement that leaders are more popular, but when interpreted in terms of interaction, it squares with his proposition, "the higher a man's social rank, the larger will be the number of persons that originate interaction for him. Men that are not highly valued must seek others rather than be sought by them (p. 182)."

Second, positive relations are transitive. If *i* has a positive relation to *j* and *j* has a psoitive relation to *k*, then *i* will always have a positive relationship to *k*. The proof is simple, but tedious, and will not be presented in detail. One takes all the possible triads permitted under the triad propositions to be stated later, examines the six possible three-step paths in each (e.g., _i_ to _j_ to _k_, _i_ to _k_ to _j_, _j_ to _i_ to _k_....etc.) and sees that there are none in which the first two are positive and the third non-positive.

Since differential popularity and transitivity are the two most common definitions of ranking systems, we feel that the model has some plausibility.

We have not, however, successfully reflected every structural proposition in The Human Group. There is at least one near miss and one clear difference of opinion.

The near miss is the claim that all cliques are ranked (p. 139). In our model, the cliques are partially ordered (e.g. in Figure 1, the partial ordering is 1 > (2,3) > (4,5) but cliques within a level are not ordered.

The difference of opinion is worth some discussion. Homans states:

> ...the more nearly equal in social rank a number of men are, the more frequently they will interact with one another...if a person does originate interaction for a person of higher rank, a tendency will exist for him to do so with the member of his own subgroup who is nearest him in rank (p. 184).

In a group with three or more levels, this proposition implies N relationships between persons whose levels are not adjacent (Mr. Low seeks out Mr. Middle who seeks out Mr. High, but Messrs. High and Low do not seek out each other). Our model does not allow this. It does imply that the very highest rates of positive relations will be in the same level (intra-clique relations), but so will the lowest (inter-clique relations).

Similarly, Homans implies that if _i_ directs a positive relation to someone above him in rank, it will not go to everyone in that rank, while our model implies that everyone directs positive relations to everyone in every rank above him. The issue is of some interest because Homans uses it to argue, in effect, that interaction (and thus presumably sentiments) tend toward the structure graph theorists call a "tree from a point,"[4] the most common structural model for formal organizations. Homans does go on to soften his proposition by saying that it is less true of smaller groups and those in less severe environments (p. 184). The question is an empirical one and the reader who is concerned about it should watch carefully in the data analysis for the results on what will be called "0-2-1-b" triads. For now, we merely note that our model assumes groups so small or in such benign environments that positive relations do not "go through channels."

We end our preliminary discussion of the model by noting its logical ties to some other models. It can be shown that by varying our assumptions about the presence of M, A, and N relations, the model can be changed into other well known structures.

1) If all pair relations are symmetrical, M or N, the structure consists of a single level and is equivalent to clusterability or structural balance.

2) If all pair relations are antisymmetrical, A, the structure consists of as many levels as people, all cliques are size one, and it is a transitive tournament.[5]

3) If all pair relations are M or A, there is only one clique at each level and the structure is what Hempel calls a "quasi-series."[6]

Triads

Having described the model in a non-rigorous fashion, it is time to state it more precisely. The procedure is this: we will list all the possible triads that could occur in a graph with M, A, and N pair relations, postulate that some of them do not exist, and then show that the structure discussed above is implied by the postulates.

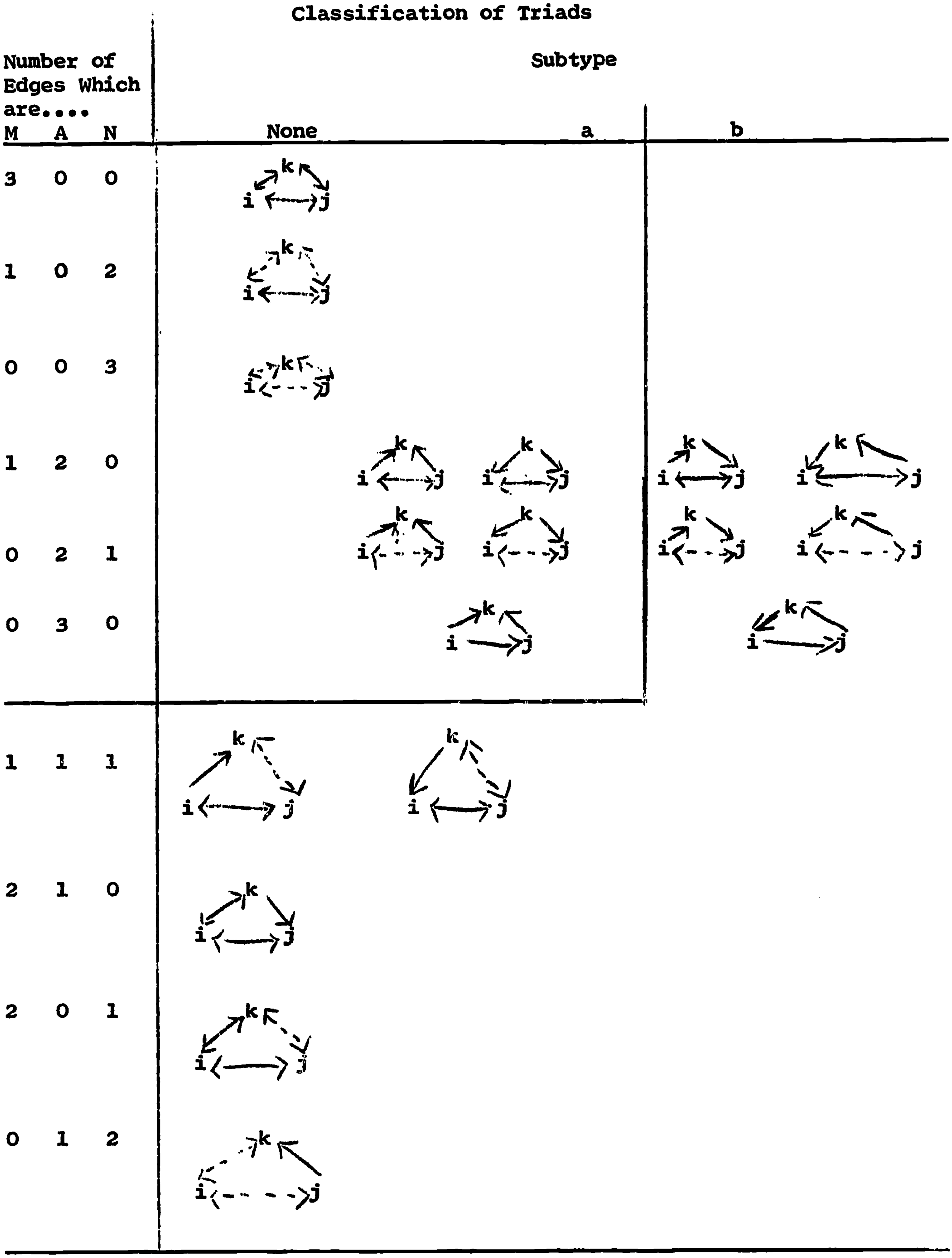
We begin by counting the number of M, A, and N sides of a triad, using a three digit code in which the first digit is the number of M edges, the second is the number of A edges, and the third is the number of N edges. Thus, a 3-0-0 triad has three M edges: a 1-1-1 triad has one M edge, one A edge, and one N edge. There are ten possibilities: 3-0-0, 2-0-1, 1-2-0. 1-1-1, 1-0-2, 0-1-2, 0-3-0, 0-2-1, and 0-0-3. Within such types, triads may vary structurally if there are A relationships, depending on the "directions of the arrows." These sub-types will be defined later and identified by letters following the numerical code, e.g. 0-3-0-a, 0-3-0-b.

Figure 2 is a catalogue of the possible triads in this classification.

(Figure 2 here)

Figure 2

Classification of Triads



Down the vertical axis we see the 10 possible triad types when direction of A relations is ignored. In the middle of the list we see the three types (1-2-0, 0-2-1, 0-3-0, 1-1-1) where direction of the A lines makes a difference, and we further note that these have been subdivided into subtypes "a" and "b". We also see a horizontal line below 0-3-0 and a vertical line between "a" and "b".

In a nutshell, the model states that <u>triads below the horizontal line and to the right of the vertical line never exist</u>, or rather than when they are absent, the total structure will have all of the marvelous properties discussed above.

Let us examine each triad type, beginning with the permissable cases. Again, we are not giving formal proofs but we will soon.

Triads of type 3-0-0 are certainly permissable as they must be three persons in the same clique at the same level.

Triads of type 1-0-2 consist of two persons, <u>i</u> and <u>j</u>, in the same clique at one level, and a third, <u>k</u>, in a different clique at that level.

Triads of type 0-0-3 consist of persons from three different cliques at the same level.

Triads of type 1-2-0-a consist of two persons, <u>i</u> and <u>j</u>, in the same clique at the same level, and a third person, <u>k</u>, in a higher or lower level.

Triads of type 0-2-1-a consist of two persons, <u>i</u> and <u>j</u>, in different cliques at the same level, and a third person, <u>k</u>, in a higher or lower level.

Insert, page 14 between 7th and 8th lines from bottom:

In 2-1 0 triads, i and j are in the same clique at the same level. The M relation between i and k implies that k is also in that clique, but the A relation between k and j implies that k is in a higher level.

In 2-0-1 triads, i and j are in the same clique. The M relation between i and k implies that k is also in that clique, but the N relation between k and j implies that k is in a different clique. (These are the well-known nonclusterable triads in balance theory.)

In 0-1-2 triads, the two N relations imply that i, j, and k are all in the same level, although in different cliques; but the A relation between j and k implies that k is in a higher level.

Triads of type 0-3-0-a consist of persons from three different levels such that k is the highest, i is the lowest, and j is intermediate.

We now explain why none of the remaining triads can be assigned to cliques and levels without some contradiction.

In 1-2-0-b triads i and j are in the same clique at the same level, but k is above one and below the other, a contradiction.

In 0-2-1-b triads, i and j are from different cliques at the same level, but k is above one and below the other, a contradiction.

In 0-3-0-b triads, we see the notorious "cyclic triads"[7] that can not be ordered. The arrow from k to i, for example, implies that i is above k, but the directed path i to j to k implies the opposite.

In 1-1-1 triads (regardless of the direction of the A relation) i and j must be placed in the same clique at the same level, but the directed line between i and k implies that k is in a different level while the N relationship implies that k is in the same level, a contradiction.

Having seen that any of the permissable triads can be assigned to levels and cliques without a contradiction but none of the other triads can, we are ready to show that if all the triads are consistent, the entire graph must be consistent.

We want to prove the following:

In a graph with M, A, and N relationships, the points can be arranged simultaneously into disjoint subsets called levels and disjoint sub-subsets called cliques, such that: a) points are in different levels if and only if they are connected by A relationships (and consequently in the same level if they are connected by M or N relations) b) points are in the same clique (and at the same level as a consequence of "a") if and only if they are connected by M relations (and consequently in different cliques at the same level if they are connected by N relations) and c) the levels form a complete order......if and only if the graph has no triads of types 2-1-0, 0-1-2, 1-1-1, 2-0-1, 1-2-0-b, 0-2-1-b, or 0-3-0-b.

The argument draws heavily upon the theorem of clusterability[8] and is influenced by the notion of duo-balance.[9]

We begin by altering the notation of the lines (edges) so that M and N relations are "positive" and A relations are "negative." Inspection of Figure 2 reveals that there are no permissable triads with two "positive" and one "negative" line (i.e. 2-1-0, 0-1-2, and 1-1-1 triads are not permitted). From the clusterability theorem it follows that the points can be arranged in unique disjoint subsets such that all lines within subsets are "positive" (M or N) and all lines between subsets are "negative" (A). We call these subsets levels and note that we have satisfied "a" above.

Next we consider points and lines within a level. Each level is a graph consisting of points connected By M or N lines. The clusterability theorem tell us that unique disjoint subsets will emerge if and only if there are no triads with two "positive" and one "negative" line. If we call M positive and N negative, the fact that 2-0-1 triads are not permissable implies that levels are internally clusterable, which satisfies "b" above.

Finally, we note in Figure 2 that any pair of points connected by an M or N line (i.e. points within the same level) and connected to a third point by an A relation have A relations identical in direction (see 1-2-0-a and 0-2-1-a in Figure 2). This enables us to condense the graph so that each level becomes a single point. That is, anything we show for the condensed graph must be true for each point within a given level. The condensed graph is complete, directed, and a-cyclic, i.e. a transitive tournament. This satisfies "c" above and completes the proof.

A Probabilistic Model

The discussion so far is of some interest as a logical exegesis of Homans's propositions and because it reveals a bridge between tournaments and structural balance. Nevertheless, it is of little scientific use because it is stated in a strong deterministic fashion. The validity of the propositions requires that each and every triad meet the assumptions. Thus, a graph of, say, 30 people, which has 4060 triads, only one of which was not permissable, is just plain "wrong" by the arguments above; yet intuition tells us that such a graph is "pretty near" the model.

In order to make these ideas useful in empirical research it is necessary to develop a probabilistic version that claims forbidden triads are relatively rare, rather than totally absent.

As a standard for "relatively rare" we will use the triad frequencies to be expected in a "random graph" - a graph with the same frequencies of M, A, and N pair relations but where particular pair relations are assigned by some chance mechanism.

The gain is enormous. In the relatively large collection of data to be discussed later there are few if any sociograms which meet the graph theoretical conditions, but quite a number which show the predicted probabilistic trends. We must remember, though, to qualify our interpretations; since the results, while fairly consistent, are statistical. Putting it another way, while we can seldom demonstrate the unequivocal existence of these structures, we can often demonstrate tendencies in the direction of these structures.

The probability argument is elementary. We simply say that if m proportion of the pair relations are of type M, a proportion are of type A, and n proportion of type N and lines are assigned at random, then the expected proportions for triads of various types is obtained by multiplying these independent probabilities.

We give as an example, type 2-1-0. Consider an arbitrary triad with edges I, II, and III. If I and II are M and III is A, then it is type 2-1-0 and the expectation for this event is (m) (m) (a) = m^2a. There are, however, two other ways a 2-1-0 triad might occur (I = A, II = M, III = M and also I = M, II = A, and III = M) each of which has the same probability m^2a. The three expectations sum to 3 m^2a, the expected proportion of 2-1-0 triads in a random graph.

Expectations for each of the remaining nine triad types in the rows of Figure 2 are easily calculated in a similar fashion.

For those triads with both "a" and "b" subtypes (1-2-0, 0-2-1, 0-3-0) it is necessary to take one further step. We see in Figure 2 that for 1-2-0 and 0-2-1 there are four equiprobable outcomes within each, two of which are permissable and two of which are not. Thus, for 1-2-0-b and 0-2-1-b we halve the expectations for the general type. In the case of 0-3-0-b, it is well known[10] that the expected proportion of cyclic triads in a random tournament is .250. Thus the expectation for 0-3-0 triads is multiplied by .250 to give the expectation for 0-3-0-b.

Table 1 gives the results.

It is straightforward but tedious to count all the triads in a graph (sociomatrix or sociogram) and to calculate the expectations from the observed frequencies of M, A, and N pair relations. It is easier and much more accurate to have the work done by an electronic computer. The junior author has written a program which calculates the number of observed and expected triad types and the necessary pair data so that a complete analysis of a graph emerges in a few seconds on a single sheet of paper.[11]

In the next section we will report results from a number of groups, but it may be useful to present a few detailed examples now to make the procedure clear.

Table 1.

Expected Triad Prcportions in a Random Graph with m, a, and n proportions of M, A, and N pair relations

Triad Type M	A	N		Expectation
Not Permissable				
2	1	0		$3 m^2 a$
0	1	2		$3 a n^2$
1	1	1		$6 m a n$
2	0	1		$3 m^2 n$
1	2	0	b	$1.5 m a^2$
0	2	1	b	$1.5 a^2 n$
0	3	0	b	$.25 a^3$
Permissable				
3	0	0		m^3
1	0	2		$3 m n^2$
0	0	3		n^3
1	2	0	a	$1.5 m a^2$
0	2	1	a	$1.5 a^2 n$
0	3	0	a	$.75 a^3$

Let us examine data from Theodore Newcomb's study, The Acquaintance Process.[12] Newcomb established an experimental dormitory at the University of Michigan where new transfer students participated in two studies testing his "ABX" theory, a set of principles about interpersonal relations and attitudes closely related to balance theory. Newcomb's theory gives special attention to the effects of similarity in values on pair relations. However, in Chapters 8, 9, and 10 he provides a richly detailed analysis of sociometric structures, that, taken as a whole, suggests his data might fit our model. We shall not attempt a detailed re-analysis of his week-by-week data, but merely report the results for the 15th week of each year. The raw sociometric data are complete rankings of the 17 men in the study, the first year's criterion being "how much you like each man," the second year's being, "favorableness of feeling." We dichotomized ranks at the median.

The model requires that non-permissable triads be rare, not that each type of permissable triad be disproportionately common. Therefore, we will report the results only for the seven types at issue even though the program prints results for all triad types. Table 2 gives the results for the two experimental groups.

Table 2

Triad Results in The Acquaintance Process

Triad M	A	N	Year I Week 15 expected	observed	difference	Year II Week 15 expected	observed	difference
2	1	0	75.4	68	-7.4	72.7	66	- 6.7
0	1	2	75.4	76	+0.6	72.7	62	-10.7
1	1	1	150.7	163	+12.3	145.3	157	+11.7
2	0	1	69.1	54	-15.1	51.9	25	-26.9
1	2	0b	41.1	39	- 2.1	50.9	27	-23.9
0	2	1b	41.1	37	- 4.1	50.9	31	-19.9
0	3	0b	7.5	3	- 4.5	11.9	4	- 7.9
Total			460.3	440	-20.3	456.3	372	-84.3

The results may be viewed in three ways.

First, we may ask how many predictions are successful. In year I five out of seven triads show the predicted negative value and in year II six out of seven. In both groups a majority of the predictions are correct.

Second, we may ask whether there are fewer non-permissable triads in total than one would expect in a random graph. The bottom row of the table shows that in Year I there is a cumulative deficit of -20.3 and in Year II -84.3. As a rough index we will divide these cumulated differences by the cumulated expectations (460.3, 456.3), giving values of -.044 in Year I and -.185 in Year II. Both results support the hypothesis: in Year I there are 4 per cent fewer non-permissable triads than in a random graph with the same pair relationship frequencies, while in Year II there are 18 per cent fewer.

Third, treating the two groups as a sample, we may ask the fates of particular hypotheses. For what it is worth, we note that both groups support the predictions for 2-1-0, 2-0-1, 1-2-0-b, 0-2-1-b, and 0-3-0-b; neither group supports the hypothesis for 1-1-1; while we get one confirmation and one disconfirmation for 0-1-2. In a sample of two groups, this approach is not very revealing, but we present it to set the stage for later analysis of larger samples.

In general we conclude that the two Acquaintance Process groups tend toward Homans's clique and level theory of social structure.

The data in Table 2, like most sociometric data contain both A relations and mutual M and N relations. However, the same approach may be taken where the data are perfectly symmetrical, in which case we are making a statistical test for clusterability alone. The men in the Bank Writing Room (p. 69) are a good example, as well they might be, since they were the impetus for this whole business. Data on friendship among these men are reported as perfectly symmetrical, and since 2-0-1 triads are the only non-permissable ones with no A relations, to test for clusterability one merely examines the results for this triad type. Computer analysis gives a total of 19.1 expected, 10 observed, a difference of -9.1, and an index value of -.476. There are 48 per cent fewer 2-0-1 triads in the Bank Wiring Room than backers of a chance model would anticipate. We conclude, as we already knew from reading The Human Group, that the men in this famous work group tended to form cliques.

When dealing with prefectly symmetrical data, we can ask a question which is irrelevant in the larger model - does the group tend toward balance? The clusterability theorem states that a group that can be divided into cliques (one with no 2-0-1 triads) will have exactly two cliques when there are no 0-0-3 triads, but will have three or more cliques when 0-0-3 triads are present. This suggests that we use the observed and expected proportions of 0-0-3 triads as a probabilistic measure of tendencies toward structural balance (division into exactly two cliques) in data which have been shown to be relatively clusterable. In the Bank Wiring Room data we observe 230 0-0-3 triads, expect 229.2, find a difference of + 0.8, compute an index of + .004, and infer no tendency toward balance.

Our third example is the opposite case, completely anti-symmetrical pair relationships, known in graph theory as a tournament. Anti-symmetric pair relationships are rare in human data[13] but in the form of "dominance relationships" they are common among other animals.[14] A typical example appears in an essay by Phyllis Jay on the Indian langur, a small monkey.[15]

She counted dominance interactions in a group of six adult and two subadult males and reported 138 such interactions in an eight-by-eight matrix. Coding a frequency of one or more as positive, the data can be handled by our program. The results appear in Table 3.

Table 3

Triad Results for Dominance Relations in Indian Langur Data

Triad M A N	expected	observed	difference
2 1 0	0.7	0.0	- 0.7
0 1 2	1.6	1.0	- 0.6
1 1 1	2.1	2.0	- 0.1
2 0 1	0.1	0.0	- 0.1
1 2 0 b	4.0	0.0	- 4.0
0 2 1 b	6.1	3.0	- 3.1
0 3 0 b	7.8	0.0	- 7.8
Total	22.4	6.0	-16.4

All seven deviations are negative and the index (-16.4/22.4) of -.732 is healthily negative. Indeed this "inhuman group" fits the model better than the vast majority of the human groups we have examined. Furthermore, because more than half of the negative deviations comes from type 0-3-0-b we see that these data, while not perfectly anti-symmetrical, tend toward the tournament model.

Examples which tend to support the model are encouraging, but we must not confuse "for instance" with "generally." Furthermore, while we have no frame of reference for making the judgment, intuition tells us that some of the discrepancies (e.g. the index value of -.044 for Year I in Table 2) seem tiny. We have not dared to enter the combinatorial thicket to answer the question of whether a particular graph is significantly different from the chance expectations. We believe that evidence for our hypotheses is best obtained by analysis of a large and heterogeneous set of sociomatrices. If we find that more than half of them depart from the chance model in the directions we predicted, we will consider our results "significant," regardless of the size of the particular discrepancies. We would be delighted to find social science generalizations which always produced whopping effects, but they are in short supply. We must settle for an attempt to show that the hypotheses hold in a large number of groups, although in many of the groups they may be holding by the skin of their teeth.

In the next section we will seek evidence on the model by analysing two random samples of 30 groups each from a data bank of 427 groups.

The Data

We propose to essay our hypotheses by examining the results in a large set of sociograms. Strictly speaking, a test is impossible because a truly representative sample of groups would consist of a probability sample from all possible subsets of humanity. Practically speaking, it is possible to collect and analyse data from a reasonable number of diverse groups through the secondary analysis of a data pool of sociograms. If we obtain consistent results in diverse groups with assorted sociometric items collected by numerous independent investigators, we believe that a prima facie case will be established.

The authors spent most of a calendar year collecting sociograms and sociomatrices to form a data pool. Our aim was simply to collect as many matrices as we could from as many different studies as possible. No a priori standards of quality or content were imposed save that we excluded a handful of matrices where no information was available on the group or the content of the items, e.g. where an author of a methods text wrote, "here is a sociogram" and gave no further information.

The final pool consists of 1092 sociograms from 549 groups, collected from 162 sources.

In terms of sources, 71 were journal articles, of which, as one might expect, 47 were from Sociometry or Sociometry Monographs, the next highest count being a mere three from The American Sociological Review. An additional 31 books provided sources along with 13 pamphlets, yearbooks, agricultural experiment station bulletins, etc., and four unpublished theses. Most important, perhaps, are the 43 individual investigators who graciously provided us with raw, unpublished data in response to personal inquiries and letters to the editor in selected social science journals. Many of thse people went to

considerable trouble to help us and we regret that space limitations preclude detailed acknowledgments beyond listing their names:

William Bezdek, University of Chicago
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Richard Boyle, University of California, Los Angeles
Julia S. Brown, University of Iowa
Edson Caldwell, Sacremento State College, California
Donald Campbell, Northwestern University
Theodore Caplow, Columbia University
Katheleen Evans, University College of South Wales and Monmouthshire
Fred Fiedler, University of Illinois
Bruce Frisbie, University of Chicago
John Gagnon, University of Indiana
Richard Gilman, University of Chicago
Donald Goldhammer, University of Chicago
Robert Graebler, Niles Township High School, Skokie, Ill.
Eleanor Hall, University of Chicago
Paul Hurewitz, Hunter College
John James, Portland State University
Richard Jessor, University of Colorado
Donald L. Lantz, Educatinnal Testing Service
Robert A. LeVine, University of Chicago
William H. Lyle, United States Bureau of Prisons
Raymond Maurice, Columbia University
David Moment, Harvard University
Nicholas Mullins, Vanderbilt University
Theodore M. Newcomb, University of Michigan
Mary L. Northway, University of Toronto
Gordon O'Brien, University of Illinois
James Peterson, University of Chicago
Charles H. Proctor, North Carolina State University
Kullervo Rainio, Helsinki University
Anatol Rapoport, University of Michigan
Jack Sawyer, University of Chicago
Maria D. Simon, Institute for Advanced Studies, Vienna
Ralph M. Stogdill, The Ohio State University
Hilda Taba, San Francisco State College
Eugene Talbot, Austen Riggs Hospital
Herbert Thelen, University of Chicago
Michael G. Weinstein, Harvard University
Milo E. Whitson, California State Polytechnic College
Thomas Wilson, Dartmouth College
Leslie D. Zeleny, The American University in Cairo

The 162 sources provided data on 549 groups. The majority are students, but there are also work groups, military units, and neighborhoods and there are a few esoteric tidbits such as employees of the Costa Rican Census Bureau,[16] German Olympic rowing teams,[17] woodcutters in a Bulgarian village,[18] and all the employees of a large woolen mill;[19] along with such old favorites as the Bank Wiring Room, the girls at Hudson,[21] MIT's Westgate student housing project,[22] and the Rattlers, Eagles, Bulldogs and Red Devils.[23]

Table 4 gives a summary of selected group characteristics in the pool.

Table 4

Characteristics of Groups in the Data Pool

(a) Group Type

1. Students

Level	Setting: Class Room	Dormitory	Summer Camp	Institution	Voluntary Association	Total
Pre-School	17					17
Grades 1-6	137				5	142
Grades 7-12	104		10	8*	17	139
College	17	11				28
Total	275	11	10	8	22	326

(* mostly correctional institutions)

2. Adults

Neighbors, villages	54
Civilian employees	54
Military units	34
Prisoners, mental patients	24
Voluntary associations	11
"Classes"*	15
Other	12
	204

(* defined as persons falling into a logical class, not necessarily members of an interacting group..e.g. all the doctors in a community or all truck farmers in a county)

(b) Number of Persons

40 or more	93
30-39	80
20-29	168
10-19	150
3-9	58
	549

(c) Location

United States and Canada	478
Latin America	31
England and Europe	19
All other	19
Unknown	2
	549

(d) Sex Composition

All male	155
Mixed	191
All female	124
Not applicable*	68
	549

(* e.g. whole families)

The 549 groups provided 1092 different matrices, roughly two apiece. It is quite difficult to classify them in terms of content, but Table 5 will perhaps convey the flavor.

Table 5

Content of 1092 sociomatrices in the Data Pool

Content	N
Prefer for task interaction (e.g. prefer to serve with on committee..)	164
Prefer for socio-emotional interaction (e.g., invite to a party, play with at recess)	160
"Friend," actual or preferred	160
Prefer for generalized interaction (e.g., prefer to sit next to in school, prefer as room mate)	103
Valued trait (e.g. named as leader, rated as a good dancer, preferred as class president)	99
"Like" or "Like best"	81
High rate of voluntary interaction (e.g. visit frequently, see frequently)	56
All other, including combinations of the above	269
Total	1092

While every matrix in the pool was catalogued and an IBM card describing its major characteristics was punched, financial limitations made it impossible to code, punch, and analyse each one. Instead, we drew two random samples of groups, one of 30 school age youngsters, the other of 30 adult groups. Each is a simple random sample (where more than one matrix was available on a group, one was chosen through a random number) of groups so that the findings may be used to estimate characteristics of the total data pool.

Certain restrictions were placed on the sample so that not all groups or matrices were eligible. The four restrictions are: 1) size was limited to eight through 80, the upper limit being chosen so that the data could be punched on a single IBM card, the lower set arbitrarily so that every group would have more than 50 triads, 2) data where the distribution of M, A, and N pair relations was forced by the investigator (e.g., where only mutual positive choices were presented) were excluded,[23] 3) a handful of cases where the respondents were not people or families (e.g. nations, tribes, Indian castes) were excluded, and 4) three content codes were excluded as outside the scope of Homans's hypotheses: a) sheer kinship, b) formal authority, and c) "relational analysis" items where individuals are asked to guess the choices of others in the group.

The result of these restrictions is that the generalizations from the sample apply to 279 out of 326 school groups and 148 out of 204 adult groups, a total universe of 427 groups.

The statistical procedure used is sequential analysis,[24] i.e. we treated each sample as a cumulative set of random samples steadily increasing in size from 1 to 30. Since each hypothesis tested has an unequivocal answer in each matrix[25] we used tables for the sign test[26] to assess the results. That is, we made a sequential test of the null hypothesis that 50 per cent of the groups show negative deviations from the expectations of the random graph model.

We also possess results on some 200 matrices which had been coded as the data were acquisitioned and before financial exigencies forced us to shift to sampling. They are not technically representative of the data pool; but for what it is worth, the results in this convenience sample are essentially the same as those in the two random samples.

Table 6 and Table 7 describe the 60 groups in the two samples.

(Table 6 here)

(Table 7 here)

Results

We will first report the global results for the model as a whole and then turn to the results for the seven specific hypotheses. Tables 8 and 9 give the global results for the two samples.

(Table 8 here)

(Table 9 here)

The seven columns in each table may be read as follows, using group 5 in the school sample as an example. The entry in column one, "-14.3," tells us that there is a total of 14.3 fewer triads of the seven-types-predicted-to-be-rare than the chance model would lead us to expect; the entry in the second column, "250.3," is the total number of "bad" triads expected under chance; the entry in the third column, "-.051," is our index of the degree of discrepancy (-14.3/250.3) and it tells us that there are roughly five per cent fewer "bad" triads than we would find in a random graph with the same frequencies

Table 6

SAMPLE OF SCHOOL GROUPS

Group	Size	Sex Composition	Type of Group	Criterion	Source
1.	14	all male	boys in 7th grade class, Greendale School, Greendale, Wisconsin	three best friends in this group	Arthur Singer, "Certain Aspects of Personality and Their Relation to Certain Group Modes and Constancy of Friendship Choices," *Journ. Educ. Res.*, 1951, 45: 33-42
2.	24	mixed	6th grade class, Univ. of Chicago Laboratory school	three best friends	Herbert Thelen, unpublished data.
3.	24	mixed	6th grade class "in a small city"	best friend	Beverly Grossman and Joyce Wrighter, "The Relationship Between Selection-Rejection and Intelligence, Social Status and Personality Amongst Sixth Grade Children," *Sociometry*, 1948, 11: 346-55.
4.	17	all male	boys in all Negro 5th grade in Tampa, Florida	"with whom you would like to work"	Donald Lantz, unpublished data.
5.	16	mixed	3rd grade in Univ. of Michigan Laboratory school	"I get along best with these children in doing work in school"	Willard C. Olson, "The Improvement of Human Relations in the Class Room," Childhood Education, 1945-46, 22: 317-25
6.	16	unknown	graduate student class at American University in Cairo	preferred to join small discussion group	Leslie D. Zeleny, unpublished data.
7.	18	mixed	2d grade in Univ. of Toronto Laboratory school	Who would you like best to have sitting near you in the classroom?"	Mary L. Northway, unpublished data

Table 6 (continued)

Group	Size	Sex Composition	Type of Group	Criterion	Source
8.	25	all female	Inmates in Park Cottage at New York Training School for Girls, Hudson, N.Y.	preference to eat at same table	Helen Jennings, "Structure of Leadership - Development and Sphere of Influence," Sociometry, 1937, 1: 99-143.
9.	16	all female	girls from 5th grade class in Tampa, Florida	"..with whom you would like to play"	same as group 4.
10.	35	mixed	"a second grade"	"Who would you like to work with on our science project?"	Merl E. Bonney, unpublished data.
11.	22	mixed	6th grade class, Univ. of Chicago Laboratory school	same as group 2	same as group 2
12.	17	all male	boys from a 5th grade class in Tampa, Florida	same as group 4	same as group 4
13.	15	all female	girls from "a fourth grade class room"	best friends	Commission on Teacher Education, Helping Teachers Understand Children, American Council on Education, 1945.
14.	27	all female	inmates in cottage C3 at New York Training School	preference for sharing the same house	J. L. Moreno, Who Shall Survive, Beacon House, Beacon, N. Y., 1953
15.	24	mixed	5th grade class, Univ. of Chicago Laboratory School	same as group 2	same as group 2
16.	25	mixed	"a third grade"	prefer to work with on the Christmas play	same as group 10
17.	14	all male	8 year olds attending a settlement house in Boston	prefer to sit next to	Sumner Cohen, "Group Structural Changes as Affected by Age of Members and Passage of Time," unpublished undergraduate honors thesis. Harvard University, 1952.

Table 6 (continued)

Group	Size	Sex Composition	Type of Group	Criterion	Source
18.	25	mixed	"a first grade"	prefer to sit next to	Milo E. Whitson, unpublished data
19.	27	all female	inmates in cottage C15 at New York Training School for Girls, Hudson, N.Y.	same as group 14	same as group 14
20.	18	all male	boys from a 5th grade in New York City	..whom you would like to sit beside you	Joan Henning Criswell, "A Sociometric Study of Race Cleavage in the Classroom" Archives of Psychol. 1939, No. 235
21.	10	mixed	class at experimental nursery school for welfare families	good friend	Eleanor Hall, unpublished data
22.	26	all female	inmates in cottage C4 at New York Training School for Girls, Hudson, N.Y.	same as group 14	same as group 14
23.	15	all male	boys from a 5th grade in Tampa, Florida	same as group 8	same as group 4
24.	25	mixed	"a third grade class"	prefer to sit next to	Paul Hurewitz, unpublished data
25.	22	all female	students in a teacher preparation college in Wales	preference as companion for practice teaching	Kathleen Evans, unpublished data
26.	18	all female	junior high school home economics class	preference for working on the same committee	Helen H. Jennings, "Sociometric Grouping in Relation to Child Development," in Caroline Tryon, ed., Fostering Mental Health in our Schools, National Education Association, 1950
27.	40	mixed	7th-8th-9th grades in a Protestant church Sunday school	like to have on committee to plan party	same as group 10
28.	28	mixed	12th grade class in a Texas high school	...whom would you like to work with on research for our next oral discussion	same as group 10
29.	29	mixed	5th & 6th grade class in Wilmington, Delaware	prefer to sit with	Hilda Taba, unpublished data
30.	16	all male	boys from a 5th grade in Tampa, Florida	same as group 23	same as group 4

Table 7

Sample of Adult Groups

Group	Size	Sex Composition	Type of Group	Criterion	Source
1.	32	all male (?)	physicians in a mid-western city	"Who are the three or four physicians with whom you most often find yourself discussing cases or therapy.."	James S. Coleman, Elihu Katz and Herbert Menzel, Medical Innovation, Bobbs-Merrill, 1966.
2.	20	mixed	random sample of names in North Carolina State faculty directory	"Have you ever spoken with this person?"	C. H. Proctor, unpublished data
3.	16	all male	executives in a manufacturing firm	"Do any of your personal friends work (here)?"	Harrison White, Research & Development as a Pattern in Industrial Management, unpublished Ph.D. thesis, Princeton Univ., 1960
4.	20	families	families in a neighborhood in San Juan, Puerto Rico Barrio #25	report mutual aid or mutual visiting and entertaining	Theodore Caplow, unpublished data from study reported in Caplow, et. al., The Urban Ambience, The Bedminster Press, 1964
5.	41	mixed	medical corps trainees at a naval base	"like"	William Bezdek, unpublished data.
6.	56	all male	squadron of Naval aviators	"What two men would you most like to fly wing with in combat?"	John G. Jenkins, "The Nominating Technique as a Method of Evaluating Air Group Morale," Journ. Aviation Med., (1954) 19: 12-19
7.	20	families	same as Group 4 - Barrio #11	(same as Group 4)	(same as group 4)
8.	13	all male	maintenance department workers in a woolen mill	"Who are your friends here - the persons you like best?	John James, unpublished data from study reported in "Clique Organization in a Small Industrial Plant," Research Studies State College of Washington; 1951 19: 125-130.

Table 7 (continued)

Group	Size	Sex Composition	Type of Group	Criterion	Source
9.	13	all female	night shift in weaving department of woolen mill	(same as group 8)	(same as group 8)
10.	11	all male	salesmen in a steel corporation	"friends"	Abraham Zalesnik and David Moment, Casebook on Interpersonal Behavior in Organizations, Wiley, 1964
11.	28	all male	truck farmers	"from whom they secured advice and information about vegetable growing innovations"	Everett Rogers, Diffusion of Innovations, Free Press, 1962.
12.	20	families	same as group 4 - Barrio #12	(same as group 4)	same as group 4
13.	23	all male	night shifting in finishing department of woolen mill	(same as group 8)	same as group 8
14.	8	all male	warehouse workers in a woolen mill	(same as group 8)	same as group 8
15.	37	mixed	engineering department in a large firm	named as "being supportive" (in interpersonal relations)	David Moment, unpublished data from David Moment and Abraham Zaleznik, Role Development and Interpersonal Competence, Harvard University Graduate School of Business, 1963
16.	13	men	day shift in picking room of woolen mill	(same as group 8)	same as group 8
17.	30	mixed	teachers at a guidance and counseling institute	preference for working together	Merl E. Bonney, unpublished data
18.	13	all male	executives in a woolen mill	(same as group 8)	same as group 8
19.	8	all male	German 8 man amateur rowing team	"With which two rowers would you like best to sit in the same boat?"	Hans Lenk, "Conflict and Achievement in High Performance Sports Teams," Soz. Welt. 1964, 15: 307-343
20.	42	all male	same as group 5	same as group 5	same as group 5

Table 7 (continued)

Group	Size	Sex Composition	Type of Group	Criterion	Source
21.	14	all male	workers in the bank wiring room at the Western Electric company	observed to help one another	F. J. Roethlisberger & W. J. Dickson, Management and the Worker, Harvard University Press, 1946
22.	14	all female	night shift in spinning department of a woolen mill	same as group 8	same as group 8
23.	45	families	rural neighborhood in Peru	visiting relationship	Charles P. Loomis and J. Allan Beegle, Rural Social Systems Prentice Hall, 1950
24.	8	all male	same as group 19	like best to have as a room mate during a trip	same as group 19
25.	20	families	same as group 4 - Barrio #14	same as group 4	same as group 4
26.	50	all male	same as group 5	same as group 5	same as group 5
27.	14	all male	night shift in weaving department of a woolen mill	same as group 8	same as group 8
28.	45	all female	"women in an adult education center in a poverty stricken district"	"With whom would you like to work on a committee?"	Rose Cologne, "Experimentation with Sociometric Procedures in a Self-Help Comm. Center," Sociometry, 1943 6:27-67.
29.	17	all male	American naval aviation squadron	with whom they would wish to go on a mission	Paul H. Maucorps, Psychologie des Mouvements Sociaux, Presses Universitaires de France, 1950
30.	45	mixed	employees in research & development department of a large firm	frequently spend free time (such as coffee breaks) together	Richard Gilman, unpublished data.

Table 8

Global Results for School Room Sample

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Sum of	Sum of			Predictions		
Group	Deviations	Expectations	(1)/(2)	Cum.	Right	Wrong	Cum.
1.	-6.1	278.0	-.022	1	4	3	1
2.	+3.6	570.3	+.006	1	5	2	2
3.	-8.6	529.6	-.016	2	4	3	3
4.	-122.5	398.5	-.307	3	5	2	4
5.	-14.3	250.3	-.057	4	5	2	5
6.	-53.6	363.4	-.147	5	6	1	6
7.	-16.4	323.5	-.051	6	4	3	7*
8.	-31.6	723.6	-.044	7	6	1	8*
9.	-150.8	357.9	-.421	8	6	1	9*
10.	-181.9	3140.8	-.057	9	5	2	10*
11.	-11.8	469.9	-.025	10*	5	2	11*
12.	-54.7	407.6	-.134	11*	5	2	12*
13.	-12.4	230.5	-.054	12*	6	1	13*
14.	-68.4	765.4	-.089	13*	5	2	14*
15.	+12.4	610.5	+.020	13*	4	3	15*
16.	-72.0	1322.9	-.054	14*	5	2	16*
17.	-24.0	183.8	-.131	15*	6	1	17*
18.	-33.3	1230.3	-.027	16*	6	1	18*
19.	-55.9	998.0	-.056	17*	6	1	19*
20.	+ 6.5	284.5	+.023	17*	4	3	20*
21.	+ 5.2	45.9	+.113	17*	4	3	21*
22.	-124.1	1128.0	-.110	18*	7	0	22*
23.	-67.0	284.0	-.236	19*	5	2	23*
24.	-11.4	925.5	-.012	20*	4	3	24*
25.	- 5.5	597.6	-.009	21*	5	2	25*
26.	-24.3	300.4	-.081	22*	4	3	26*
27.	+ 6.3	2839.6	+.002	22*	4	3	27*
28.	-116.7	1714.6	-.068	23*	5	2	28*
29.	-14.1	1011.2	-.014	24*	4	3	29*
30.	-49.8	317.8	-.157	25*	5	2	30
Per Cent Correct				83%			100%

* = cumulative frequency of correct predictions is significant at .01 level for sign test.

Table 9

Results for Adult Sample

Group	(1) Sum of Deviations	(2) Sum of Expectations	(3) (1)/(2)	(4) Cum.	(5) Predictions Right	(6) Predictions Wrong	(7) Cum.
1.	-56.5	1357.5	-.042	1	5	2	1
2.	+ 0.3	225.7	+.001	1	2	5	1
3.	- 5.4	129.4	-.042	2	5	2	2
4.	-405.9	698.9	-.581	3	6	1	3
5.	+ 15.1	1073.0	+ .014	3	6	1	4
6.	-236.6	3433.5	-.069	4	5	2	5
7.	-108.2	584.1	-.185	5	5	2	6
8.	-22.1	138.1	-.160	6	5	2	7
9.	-17.2	139.1	-.123	7	6	1	8
10.	-10.1	82.1	-.123	8	6	1	9
11.	-10.1	458.1	-.022	9	6	1	10*
12.	-59.0	556.0	-.106	10	5	2	11*
13.	-25.6	392.6	-.065	11	5	2	12*
14.	+ 1.8	23.2	+.078	11	5	2	13*
15.	-456.2	3884.1	-.117	12	6	1	14*
16.	- 2.8	76.9	-.036	13	5	2	15*
17.	-229.5	2256.5	-.102	14*	7	0	16*
18.	-47.8	178.8	-.267	15*	6	1	17*
19.	-7.6	33.6	-.226	16*	6	1	18*
20.	-47.7	920.7	-.052	17*	6	1	19*
21.	+ 3.5	117.6	+ .030	17*	3	4	19*
22.	+ 0.7	168.3	+ .004	17*	4	3	20*
23.	-54.1	2257.1	-.024	18*	4	3	21*
24.	+ 2.8	30.3	+.092	18	3	4	21*
25.	-21.3	357.3	-.060	19*	2	5	21*
26.	-53.7	1432.7	-.037	20*	6	1	22*
27.	+1.7	94.7	+.018	20*	5	2	23*
28.	-266.3	2668.5	-.100	21*	2	5	23*
29.	+0.1	24.9	+.004	21	3	1+	24*
30.	-1524.1	9442.1	-.161	22*	5	2	25*
Per Cent Correct				73%			83%

* = cumulative frequency of correct predictions is significant at .01 level for sign test.

+ = only 4 predictions possible because extreme symmetry of data made expectations of 0.0000 for three triad types.

of pair relations; the entry in column four, "4," tells us that of the first five groups drawn in the random sample, four had negative deviations in column 1 (and hence, column 3); the entries in columns five and six, "5" and "2," tell us that of the seven separate predictions about triad types, five were correct (the types were rarer than the random graph prediction) and two were incorrect; and the entry in column seven, "5," indicates that of the first five groups, all five showed majorities of correct predictions. The absence of asterisks in columns four and seven indicates that these cumulative frequencies are not significant at the .01 level against the null hypothesis that half the predictions are correct.

The simplest prediction is that in most groups, most of the specific predictions will be correct. Column 7 of Tables 8 and 9 reveals that the claim has some merit. All 30 school room groups and 25 out of 30 adult groups (8 3%) show majorities of correct predictions, although there is only one case, group 22 in Table 8, where all seven are correct. This deviation from the 50 per cent success expected in random data becomes significant on the seventh group for the school sample and the tenth group among the adults. We infer that a significant majority of the 427 groups in the data pool have mostly correct predictions.

A second way to assess the model as a whole is in terms of the total deficit or surplus of non-permissable triads, column 3 in the tables. In 83 per cent of the school groups and 73 per cent of the adult groups there are deficits, as predicted. The former becomes significant at the .01 level on the tenth group and the latter becomes significant at the 14th group, although it wanders around the border of the critical region during the later groups in the series. We infer that a significant majority of the 427 groups in the data pool have deficits of non-permissable triads.

Both forms of the global hypothesis are confirmed, and in this limited operational sense we are led to agree with Homans that group after group will tend to form cliques and ranked levels on the basis of positive interpersonal relations.

We now know that the model, taken as a whole, does pretty well in its empirical tests. Next we ask whether each of its seven parts fares equally well. Tables 10 and 11 provide the necessary information.

(Table 10 here)

(Table 11 here)

A glance at the tables reveals enough asterisks to convey the impression that most of the individual predictions are successful, although we note one case (type 0-1-2 in the school sample) where the hypothesis is rejected significantly. Table 12 summarizes the results in Tables 10 and 11.

Table 10

Results for Specific Hypotheses in the School Sample

Triad Type

Group	210	Cum.	012	Cum.	111	Cum.	201	Cum.	120-b	Cum.	021-b	Cum	030-b	Cum
1.	+	0	+	0	-	1	-	1	-	1	-	1	+	0
2.	+	0	+	0	-	2	-	2	-	2	-	2	-	1
3.	+	0	+	0	-	3	-	3	-	3	-	3	+	1
4.	-	1	+	0	+	3	-	4	-	4	-	4	-	2
5.	-	2	-	1	+	3	+	4	-	5	-	5	-	3
6.	-	3	+	1	-	4	-	5	-	6	-	6	-	4
7.	+	3	+	1	-	5	-	6	+	6	-	7*	-	5
8.	-	4	+	1	-	6	-	7	-	7	-	8*	-	6
9.	-	5	-	2	+	6	-	8	-	8	-	9*	-	7
10.	+	5	+	2	-	7	-	9	-	9	-	10*	-	8
11.	+	5	+	2	-	8	-	10*	-	10*	-	11*	-	9
12.	-	6	+	2	-	9	+	10	-	11*	-	12*	-	10
13.	-	7	+	2	-	10	-	11*	-	12*	-	13*	-	11*
14.	+	7	-	3	-	11	+	11	-	13*	-	14*	-	12*
15.	+	7	+	3	-	12	-	12	+	13*	-	15*	-	13*
16.	+	7	+	3	-	13	-	13	-	14*	-	16*	-	14*
17.	+	7	-	4	-	14*	-	14*	-	15*	-	17*	-	15*
18.	+	7	-	5	-	15*	-	15*	-	16*	-	18*	-	16*
19.	+	7	-	6	-	16*	-	16*	-	17*	-	19*	-	17*
20.	+	7	+	6	-	17*	-	17*	-	18*	-	20*	+	17*
21.	-	8	+	6	+	17*	-	18*	-	19*	+	20*	-	18*
22.	-	9	-	7	-	18*	-	19*	-	20*	-	21*	-	19*
23.	-	10	+	7	-	19*	+	19*	-	21*	-	22*	-	20*
24.	+	10	+	7	-	20*	-	20*	+	21*	-	23*	-	21*
25.	+	10	+	7	-	21*	-	21*	-	22*	-	24*	-	22*
26.	+	10	+	7	-	22*	-	22*	-	23*	+	24*	-	23*
27.	+	10	+	7n	-	23*	-	23*	+	23*	-	25*	-	24*
28.	+	10	+	7n	-	24*	-	24*	-	24*	-	26*	-	25*
29.	+	10	+	7n	-	25*	-	25*	+	24*	-	27*	-	26*
30.	-	11	+	7n	+	25*	-	26*	-	25*	-	28*	-	27*
Per Cent Negative		37%		23%n		83%*		87%*		83%*		93%*		90%*

+ = frequency of triad type is equal or greater than chance expectation

- = frequency of triad type is less than chance expectation

* = cumulative frequency of correct predictions is significant at .01 level for sign test

n = cumulative frequency of <u>incorrect</u> predictions is significant at .01 level for sign test.

Table 11

Results for Specific Hypotheses in the Adult Sample

Triad Type

Group	210	Cum.	012	Cum.	111	Cum.	201	Cum.	120-b	Cum.	021-b	Cum.	030-b	Cum.
1.	+	0	-	1	-	1	-	1	+	0	-	1	-	1
2.	+	0	-	2	+	1	+	1	+	0	+	1	-	2
3.	+	0	+	2	-	2	-	2	-	1	-	2	-	3
4.	+	0	-	3	-	3	-	3	-	2	-	3	-	4
5.	-	1	+	3	-	4	-	4	-	3	-	4	-	5
6.	-	2	-	4	+	4	+	4	-	4	-	5	-	6
7.	+	2	-	5	-	5	-	5	-	5	-	6	+	6
8.	+	2	-	6	+	5	-	6	-	6	-	7	-	7
9.	+	2	-	7	-	6	-	7	-	7	-	8	-	8
10.	+	2	-	8	--	7	-	8	-	8	-	9	-	9
11.	-	3	-	9	+	7	-	9	-	9	-	10*	-	10*
12.	+	3	-	10	-	8	-	10	+	9	-	11*	-	11*
13.	+	3	-	11	-	9	-	11	-	10	-	12*	+	11
14.	+	3	+	11	-	10	-	12	-	11	-	13*	-	12*
15.	+	3	-	12	-	11	-	13*	-	12	-	14*	-	13*
16.	-	4	-	13	-	12	+	13	+	12	-	15*	-	14*
17.	-	5	-	14*	-	13	-	14*	-	13	-	16*	-	15*
18.	+	5	-	15*	-	14	-	15*	-	14*	-	17*	-	16*
19.	-	6	-	16*	+	14	-	16*	-	15*	-	18*	-	17*
20.	-	7	-	17*	+	14	-	17*	-	16*	-	19*	-	18*
21.	-	8	+	17*	+	14	-	18*	+	16	-	20*	+	18*
22.	+	8	+	17	+	14	-	19*	-	17*	-	21*	-	19*
23.	+	8	-	18*	-	15	+	19*	+	17	-	22*	-	20*
24.	-	9	+	18	-	16	+	19*	+	17	-	23*	-	21*
25.	+	9	-	19*	+	16	+	19*	+	17	-	24*	+	21*
26.	-	10	-	20*	-	17	-	20*	-	18	+	24*	-	22*
27.	+	10	+	20*	-	18	-	21*	-	19	-	25*	-	23*
28.	+	10	-	21*	+	18	+	21*	+	19	-	26*	+	23*
29.	-	11	+	21	-	19	-	22*	x	x	x	x	x	x
30.	+	11	+	21	-	20	-	23*	-	20	-	27*	-	24*
Per Cent Negative		37%		70%		67%		77%*		69%		93%*		80%*

+ = frequency of triad type is equal or greater than chance expectation

- = frequency of triad type is less than chance expectation

* = cumulative frequency of correct predictions is significant at .01 level for sign test.

x = both observed and expected frequencies are 0.0000 because of high symmetry of relationship.

Table 12

Summary of Outcomes in Tables 10 and 11*

			Outcome				
			Disconfirmed		Neither	Confirmed	
Significance			.01	.05		.05	.01
M	A	N					
2	1	0			(A 37%) (S 37%)		
0	1	2	(S 23%)			(A 67%)	
1	1	1				(A 67%)	(S 83%)
2	0	1					(A 77%) (S 87%)
1	2	0 b				(A 69%)	(S 83%)
0	2	1 b					(A 93%) (S 93%)
0	3	0 b					(A 83%) (S 90%)

* A = adult sample, S = school sample, figures in parentheses are the percentages of groups within the sample in which the hypothesis is confirmed.

We observe the following:

a) Three hypotheses (0-2-1-b, 0-3-0-b, 2-0-1) are confirmed at the .01 level in both samples.

b) Two hypotheses (1-2-0-b and 1-1-1) are confirmed at the .01 level in one sample and the .05 level in the other.

c) One hypothesis (0-1-2) is confirmed at the .05 level in one sample and disconfirmed at the .01 level in the other.

d) One hypothesis (2-1-0) tends toward disconfirmation in both samples, though it is not significant in either.

Beyond the fact that the over-all success of the model does not come from a minority of the triad types (five out of the seven hypotheses are supported at the .05 level in both samples) it is difficult to interpret the pattern in Table 12. The question is whether the less successful hypotheses (2-1-0 and 0-1-2) suggest meaningful defects in the model or whether we have merely been chastised by those gods who have decreed that sociological data shall never come out cleanly.

More soberly, since each specific hypothesis uses fewer observations than the global hypothesis, the specific predictions may be expected to show more internal variability than the over-all tests. Such a probabilistic view would lead us to stress the distribution of the results - eight tests significant at the .01 level, three at the .05 level, and only three less successful, rather than to examine each prediction in isolation.

On the other hand, granted that it is _ex post facto_, there is a faint pattern in the results. Referring back to the proof of the graph theoretical model above, we remember that it is divided into three parts, part (a) about levels, part (b) about cliques within levels, and part (c) about order between levels. The three hypotheses about order between levels (1-2-0-b, 0-2-1-b, and 0-3-0-b) are all relatively successful; the single hypothesis about clique structures within levels (2-0-1) is nicely confirmed; and it is the three hypotheses dealing with levels _per se_ which include our less fortunate results. We can not infer that the level notion is to be rejected, for three of the six tests about it are significant at the .05 level, but it does seem to be a weaker part of the model.

Putting it another way, we may say that we have had some success in showing that A relationships tend toward a rank structure and some success in showing that M and N relations tend toward clusterability, but we have had more limited success in showing how these two structures are integrated to make a coherent whole.

In summary:

1) In both samples, the over-all hypotheses of the model are supported so frequently that we infer most groups in the data pool show the predicted structural trends.

2) In both samples, three of the seven specific hypotheses are supported at the .01 level of significance and two at the .05 level.

3) Two of the seven specific hypotheses have equivocal or negative outcomes. Whether this is random variation or a substantive flaw in the model is unclear. If it is a substantive flaw, the weakness seems to lie not in the idea of ranking or the idea of cliques, but in the assumptions about how the ranking system and the clique system are articulated.

Random Data

We have concluded that a fairly large and heterogeneous collection of groups exhibits the structural trends we derived from The Human Group. Nevertheless, the confirmation is probabilistic; while most groups tend toward the structures implied by the model, none fitted the model perfectly. Hence, the validity of our probabilistic argument is crucial. Since the probability distribution of graph properties is not well studied, except for some properties of tournaments, we felt that it would be useful to buttress our case by a Monte Carlo analysis which might reveal any serious flaws in the reasoning. A thorough Monte Carlo analysis is a formidable task because one should run a large number of random groups varying in size and in M-A-N values. This was impractical for our small project, but for what it is worth, we did run two sets of 30 "groups" with data generated by random numbers. We set each at size 20, the median size for groups in the two samples, and set 15 per cent of the random "choices" as positive, again the median in the two data samples.

Table 12 summarizes the results:

Table 12

Results in Data Samples and Two Samples of Thirty 20 Person Groups Simulated with Random Numbers

		Data Samples		Simulated	
		School	Adult	I	II
No. of Groups		30	30	30	30
Percentage with negative indices		83%	73%	30%	40%
Percentage of groups with 4 or more hypotheses confirmed		100%	83%	50%	53%
Individual Hypotheses	level				
	:01	5	3	1	
Confirmed	:05		3		
	.05			3	3
	:05	1	1	3	3
Disconfirmed	:05				
	.01	1			1

Beginning with the index defined in Table 8, Column 3, we see that less than half of the indices are negative in the simulated data, while 83 percent and 73 per cent are negative in the actual data. (Incidentally, in the combined random number samples 55 out of 60 groups have indices with values between ≠.049 and -.049, and only three are less than -.049. This suggests that index values of -.050 or lower are worth considering seriously even though they appear small intuitively.)

Turning next to the number of hypotheses confirmed per group, we find the random data confirm four or more hypotheses in 50 per cent of the groups in sample I and 53 per cent of the groups in sample II compared with 100 per cent and 83 per cent in the two actual data samples.

Finally, considering the individual hypotheses, we find seven confirmations and seven disconfirmations in the random data, one confirmation and one disconfirmation significant at the .01 level; in contrast to 11 confirmations, all significant at the .05 level and three disconfirmations, one significant at the .01 level, for the real data. That in 14 tests we get two differences significant at the .01 level in the random ddta gives some support to our previous notion that individual hypotheses have a lot more random fluctuation than the global hypotheses.

In sum, two sets of truly random data are a lot closer to our deduced expectations than are the real data we analysed. The result is a double comfort. On the one hand, it suggests but does not prove that our probabilistic argument and the computer program are not grossly incorrect. On the other, it confirms our conclusion that the real data differ from those obtained from random graphs.

We conclude by repeating from our introduction: to the extent that our model is a plausible interpretation of Homans's ideas, our probabilistic reasoning is valid, and our 427 groups are representative, we believe our study provides favorable evidence for Homans's claim that if we examine voluntary interaction and sentiments in small groups, we will find two structures, differentation into cliques and elaboration into ranks - and we will find them in group after group after group.

Footnotes

1. George C. Homans, The Human Group, New York, Harcourt, Brace and Company, 1950. All references in this paper not otherwise cited are to the same source. For a very similar view, see Roger Brown, Social Psychology, New York, Free Press, 1965, pp. 51-100.

2. The technical reader will note that this is the same definition of cliques used in the theory of structural balance.

3. Dorwin Cartwright and Frank Harary, "Structural Balance: A Generalization of Heider's Theory," The Psychological Review, (1956) 63: 277-93, James A. Davis, "Clustering and Structural Balance in Graphs," Human Relations, (1967) 20: 181-7.

4. Frank Harary, Robert Z. Norman, and Dorwin Cartwright, Structural Models, New York, John Wiley & Sons, Inc., 1965, p. 283.

5. Ibid., pp. 289-317.

6. Carl G. Hempel, Fundamentals of Concept Formation in Empirical Science, International Encyclopedia of Unified Science, Vol. II., No. 7, University of Chicago Press, 1952, pp. 58-62.

7. Harary, Norman and Cartwright, op. cit., pp. 296-304.

8. Davis, op. cit.

9. John G. Kemeny and J. Laurie Snell, Mathematical Models in the Social Sciences, Ginn and Company, 1962, p. 107.

10. Harary, Norman and Cartwright, op. cit., p. 303.

11. The program has an additional feature, not discussed here. If the members (points) can be dichotomized, e.g. by sex, it will present a sub-analysis within each half of the dichotomy and for all triads which are heterogeneous in terms of the dichotomy.

12. Theodore M. Newcomb, The Acquaintance Process, New York, Holt, Rinehart and Winston, 1961. Professor Newcomb was extraordinarily helpful in providing us with the new data from his study, in a set of circumstances in which most men would have been considerably less cooperative.

13. In the two samples to be analysed below, 55 out of 60 groups show a greater tendency toward reciprocation of positive relations than chance would predict. Cf. Renato Tagiuri, "Social Preference and its Perception," in R. Tagiuri and L. Petrullo, eds., Person Perception and Interpersonal Behavior, Stanford University Press, 1958, pp. 316-336.

14. Cf. William Etkin, Social Behavior and Organization Among Vertebrates, University of Chicago Press, 1964 pp. 256-295. For a more detailed review of dominance per se, cf. N. E. Collias, "Aggressive Behavior Among Vertebrate Animals," Physiol. Zool., 1944, 17: 83-123 or V. C. Wynne-Edwards, Animal Disperson in Relation to Social Behavior, Hafner Publishing Company, 1962, pp. 127-144. Two older essays of considerable interest to Sociologists are, Thorleif Schjelderup-Ebbe, "Social Behavior of Birds," in Carl Murchison, ed., A Handbook of Social Psychology, Clark University Press, 1935, pp. 947-972 and W. C. Allee, The Social Life of Animals, W. W. Norton & Co., Inc., 1938, pp. 175-208. For a verbal description, but alas no data, of a system with cliques and levels, cf. Konrad Z. Lorenz, King Solomon's Ring, Thomas Y. Crowell, Co., 1952, p. 151.

15. Phyllis Jay, "The Common Langur of North India," in Irven DeVore, ed., Primate Behavior, New York, Holt, Rinehart and Winston, 1965, p. 245.

16. Charles H. Proctor, unpublished data.

17. Hans Lenk, "Conflict and Achievement in High Performance Sports Teams," Soziale Welt (1964) 15: 307-343.

18. Irwin T. Sanders, "Sociometric Work with a Bulgarian Woodcutting Group," Sociometry (1939) 2: 58-68.

19. John James, unpublished data.

20. Jacob L. Moreno, Who Shall Survive? New York, Beacon House, 1953.

21. Leon Festinger, Stanley Schachter and Kurt Back, Social Pressures in Informal Groups, Stanford University Press, 1950.

22. Muzafer Sherif, O. J. Harvey, B. Jack White, William R. Hood, and Carolyn W. Sherif, Intergroup Conflict and Cooperation, Norman, Oklahoma, Institute of Group Relations, University of Oklahoma, 1961; Muzafer Sherif, "Experiments in Group Conflict," Scientific American, November, 1956, pp. 2-6.

23. For the Bank Wiring Room, Adult Group Number 21, the data already presented were not eligible. This explains why the results for that group differ from those we have already seen.

24. For an introductory discussion, see Irwin D. J. Bross, Design for Decision, New York, The MacMillan Company, 1953, pp. 130-1444.

25. Since computers calculate expectations to a large number of decimals, it is very rare for observed and expected values to be exactly equal. Such rare cases are counted against the hypothesis.

26. William J. MacKinnon, "Table for Both the Sign Test and Distribution Free Confidence Intervals of the Median for Sample Sizes to 1,000," Journ. Am. Stat. Assoc. (1964) 59: 935-960.