Dec 16, 2020

$Ax = b$

$PA = LU$

$A = LL^T$

$\kappa_2(A) = \sigma_{max}/\sigma_{min}$

$\min \|Ax - b\|_2$

$A = QR$

$A = U\Sigma V^T$

$\kappa_2(A)$ b, range(A)

orthog. matrices

$Ax = \lambda x$

$A = V\Lambda V^T$

$A = QTQ^H$

$A = UHU^T$

$A = U\Sigma V^T$

orthog matrices

condition number $\Rightarrow$ sensitivity to perturbations

$AQ_k = Q_{k+1}\bar{T}_k$

$AQ_k = Q_{k+1}\bar{H}_k$

$AV_k = U_{k+1}\bar{B}_k$

$A^T U_k = V_{k+1} B_{k+1}^T$

condition number $\Rightarrow$ convergence

structure matters!!...

sparse

symm

blocked

fast operators

diag dominant

orthog

symm

orthog

Other tools

- optimization/analysis: CG (Krylov), $2x^T A^T A x = 2A^T b$

  Gauss-Seidel $\Leftrightarrow$ coord desc. $\Rightarrow A^T A x = A^T b$

  $$r(x) = \frac{x^T A x}{x^T x}$$

- graph theory: sparse direct
- hardware: blocking/BLAS 3, layouts, floating point
- statistics: trunc. SVD $\Leftrightarrow$ reg LS
- approx theory: convergence of iterative methods

Where to go from here

CS 6220: Data-sparse matrix computations

structure ⇒ super fast

- rank-structured mats $Ax=b$ in $O(n)$ time
- Rand NLA
- FMM, FFT

CS 6241: Matrix comps in data science

- NMF, other facts
- GPs / kernel methods
- Graph data

Thank you!