HW for 2019-11-18

(due: 2019-11-25)

You may (and should) talk about problems with each other and with me, providing attribution for any good ideas you might get. Your final write-up should be your own.

For this assignment, we will consider the one-dimensional model system on $[0, 1]$ with a grid of $n = 100$ interior points. The interior points have coordinates $x_j = jh$ for $h = 1/(n+1)$. You should complete the code framework in the class repository as outlined below (Julia or MATLAB/Octave) and comment on what you see.

1: Gauss-Seidel sweeps Implement a simple block Gauss-Seidel sweep in the solve_bgs function. What do you observe about the plots of the discrete sine transform of the error?

2: Bubnov-Galerkin Implement a simple Bubnov-Galerkin approximation to the correction equation

$$T(\Delta u) = h^2 f - T\hat{u}$$

in the function solve_coarse. The tester uses a space of polynomials (represented in the Chebyshev basis for stability); how does the error vary depending on the polynomial degree? Also compute the optimal error in the space and the quasi-optimality constant. How tight is the standard quasi-optimality bound in this case?

3: Two-grid iteration The final loop in the tester alternates between Galerkin projections with a polynomial space and Gauss-Seidel sweeps. Discuss why the combined iteration works better than either approach alone.