HW for 2019-10-28

(due: 2019-11-04)

You may (and should) talk about problems with each other and with me, providing attribution for any good ideas you might get. Your final write-up should be your own.

1: On the border Suppose the bordered matrix

$$M(s) = \begin{bmatrix} A - sI & b \\ c^T & 0 \end{bmatrix}$$

is nonsingular, and consider the linear system

$$\begin{bmatrix} A - sI & b \\ c^T & 0 \end{bmatrix} \begin{bmatrix} f(s) \\ g(s) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

- Show that $g(\lambda) = 0$ iff λ is an eigenvalue of A.
- Modify the hw7newton code (Julia or MATLAB) to implement the Newton iteration
  $$\sigma_{k+1} = \sigma_k - g(\sigma_k)/g'(\sigma_k).$$
  You should see quadratic convergence in the tester, as indicated by the $g(\sigma_{k+1})$ having roughly the order of magnitude of $g(\sigma_k)^2$.

2: Real rotations Suppose $A \in \mathbb{R}^{n \times n}$ has a unique (algebraic multiplicity 1) complex conjugate pair of eigenvalues $\mu \exp(\pm i\theta) = \alpha + \beta i$ with maximal modulus ($\mu > |\lambda|$ for all other eigenvalues λ) and corresponding eigenvectors $u \pm v i$. Show that power iteration from a random starting vector in $\mathbb{R}^n$ gives the sequence

$$v_k \approx u \cos(k\theta + \gamma) - v \sin(k\theta + \gamma)$$

for large k.

3: Shifted solver Suppose $H \in \mathbb{R}^{n \times n}$ is given upper Hessenberg matrix. Write a QR-based solver that runs in $O(n^2)$ time to solve linear systems of the form $(H - \sigma I)x = b$. Your code should satisfy the interface in the class repository.