

HW for 2019-10-21

(due: 2019-10-28)

You may (and should) talk about problems with each other and with me, providing attribution for any good ideas you might get. Your final write-up should be your own.

1: Contrary conditioning Given a scalar $C > 1$, find $A \in \mathbb{R}^{2 \times 2}$ for which the eigenvalues are both one but $\kappa_2(A) \geq C$.

2: Interesting identity Suppose $X, Y \in \mathbb{R}^{n \times k}$. Show that if $\lambda \neq 0$ is an eigenvalue of XY^T then

$$\begin{bmatrix} -\lambda I & X \\ Y^T & -I \end{bmatrix}$$

is singular. Via this formulation, show λ is also an eigenvalue of $Y^T X$.

3: Vector variations Differentiate the eigenvalue equation $Ax = \lambda x$ subject to a constraint $l^*x = 1$ in order to write a linear system for the derivative of x and λ under a small change to A .