HW for 2019-09-30

(due: 2019-10-07)

You may (and should) talk about problems with each other and with me, providing attribution for any good ideas you might get. Your final write-up should be your own.

1: Tree falling The code tree_solve in the repository computes the solution to $Ax = b$ where A is a tree-structured symmetric positive definite matrix. Rewrite the reference code to run in $O(n)$ time.

2: Remove a row Suppose $A \in \mathbb{R}^{m \times n}$ is decomposed as

$$A = \begin{bmatrix} A_1 \\ a_2 \end{bmatrix}, \quad A_1 \in \mathbb{R}^{(m-1)\times n}, a_2 \in \mathbb{R}^{1 \times n}$$

and both A and A_1 are rank n. Suppose we have an economy QR of A, show how to compute $\hat{x} = A_1^{\dagger} b_1$ in $O(n^2)$ time (no code is needed for this problem, just give the idea).

3: Pesky probability Supposing Z is a standard normal random variable, find the degree 8 polynomial $p(z)$ that minimizes

$$\phi(p) = E_Z[(p(Z) - \cos(Z))^2].$$

What is the optimal value of ϕ? Please use the Gauss-Hermite quadrature code included in the repository (gausshq.m and gausshq.jl) to compute $E_Z[f(Z)]$ for any f appearing in your computations.