HW for 2019-09-16

(due: 2019-09-23)

You may (and should) talk about problems with each other and with me, providing attribution for any good ideas you might get. Your final write-up should be your own.

1: Gauss transformations Let $G = I - \tau e_k^T$ be a Gauss transformation matrix (so the only nonzeros in τ appear after entry k). Then

- Show that $G^{-1} = I + \tau e_k^T$.
- Argue that $\|G\|_\infty = 1 + \|\tau\|_\infty$ and $\|G\|_1 = 1 + \|\tau\|_1$.
- The singular values of G are all one, except for two of them that are the positive roots of the equation

$$p(\sigma^2) = 1 - (2 + \|\tau\|^2)\sigma^2 + \sigma^4 = 0.$$

  Using this fact (which you are not required to prove), write a code $\kappa_2(G) = \|G\|_2\|G^{-1}\|_2 = \sigma_1(G)/\sigma_n(G)$. Your code should give remain accurate (to within a few ulps) when $\|\tau\|$ is very large or small (including $\tau = 0$).

Bonus: Prove the fact about the singular values used in the second part.

2: Follow the arrow Consider the *arrow matrix*

$$A = \begin{bmatrix} D & b \\ c^T & f \end{bmatrix}$$

where D is diagonal.

- Show that A is invertible if the diagonal entries d_i are all nonzero and $f - \sum_i b_i c_i / d_i \neq 0$.
- Write an $O(n)$ time routine to compute $\det(A)$
- Write an $O(n)$ time routine to solve the system $Ax = y$.

Your codes should follow either the MATLAB or the Julia template in the class repository.