Final

(due: 2019-12-18)

You may (and should) use any references you wish: books, notes, MATLAB help, literature. But please do not consult with anyone outside the course staff. Templates for the required codes are provided in the class Github repository.

1: Stationary steps Consider the iteration

$$x^{(k+1)} = x^{(k)} + M^{-1}A^T(b - Ax^{(k)})$$

where M is positive definite and $A \in \mathbb{R}^{m \times n}$ with $m \geq n$ is full rank.

2 pts What is the fixed point of the iteration?

2 pts What is the matrix for the error iteration?

2: Eigenbounds Let $A \in \mathbb{R}^{n \times n}$ be symmetric and $E \in \mathbb{R}^{n \times n}$ be arbitrary, but with $\|E\|_2 \leq \epsilon$.

2 pts Argue that $(A - sI) + E$ must be nonsingular if $|s - \lambda| > \epsilon$ for every eigenvalue λ of A.

2 pts Argue that the eigenvalues of $A + E$ must lie in a union of disks of radius ϵ about the eigenvalues of A.

2 pts Argue that if all the eigenvalues of A are at least 2ϵ apart, then all the eigenvalues of $A + E$ must be real.

3: Eigenvector estimation Let the columns of $V \in \mathbb{R}^{n \times k}$ be an orthonormal basis for a k-dimensional subspace $\mathcal{V} \subset \mathbb{R}^n$. Suppose $A \in \mathbb{R}^{n \times n}$ has an approximate eigenvalue $\mu \in \mathbb{R}$ (given)

2 pts Write an efficient code to find a unit-length vector $v \in \mathcal{V}$ such that $\|(A - \mu I)v\|$ is minimal.

2 pts Give a backward error bound for the v computed in the previous part (i.e. give a bound on the norm of an E such that $(A + E)v = \mu v$).

4: Concatenated constraints Consider the problem

$$\text{minimize } x^T A x \text{ s.t. } x^T M x = 1 \text{ and } Cx = b$$

where $A, M \in \mathbb{R}^{n \times n}$ are symmetric, M is positive definite, and $C \in \mathbb{R}^{m \times n}$ with $m < n$.

2 pts Find $\bar{x}$ that minimizes $x^T M x$ subject to $Cx = b$.

2 pts Show how to compute an M-orthonormal basis V for the null space of C (this can be a MATLAB or Julia fragment).

2 pts Argue that $x = \bar{x} + Vy$ for some y, and show that y can be computed via an eigenvalue problem (you may refer to HW solutions from class).