# CS 6210 Take-Home Midterm:Comments

Average was 23.5 out of 30, roughly (P1,P2,P3) = (7.36,7.46,8.66). If you would like a letter grade interpretation: A = [25,30], B = [18,20], C = [12,15]. Solution/test scripts on website. The fastest solution framework for P1 is outlined below. Only about 2 students were on the right track for that. But the problem was basically graded against the "obvious" divide and conquer strategy.

## 1 Fast Matrix Multiply

Some facts. First,

$$\texttt{tril}(ST^T, -1) \;=\; \texttt{tril}(S(:,1)T(:,1)^T, -1) \;+\; \texttt{tril}(S(:,2)T(:,2)^T, -1)$$

$$\texttt{triu}(TS^T, 1) \;=\; \texttt{triu}(T(:,1)S(:,1)^T, 1) \;+\; \texttt{triu}(T(:,2)S(:,2)^T, 1)$$

Second, if $s, t \in \mathbb{R}^5$ then

$$\begin{aligned}
\texttt{tril}(st^T, -1)x &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ s_2t_1 & 0 & 0 & 0 & 0 \\ s_3t_1 & s_3t_2 & 0 & 0 & 0 \\ s_4t_1 & s_4t_2 & s_4t_3 & 0 & 0 \\ s_5t_1 & s_5t_2 & s_5t_3 & s_5t_4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \\
&= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ s_2 & 0 & 0 & 0 & 0 \\ s_3 & s_3 & 0 & 0 & 0 \\ s_4 & s_4 & s_4 & 0 & 0 \\ s_5 & s_5 & s_5 & s_5 & 0 \end{bmatrix} \begin{bmatrix} t_1x_1 \\ t_2x_2 \\ t_3x_3 \\ t_4x_4 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ s_1(t_1x_1) \\ s_2(t_1x_1 + t_2x_2) \\ s_3(t_1x_1 + t_2x_2 + t_3x_3) \\ s_4(t_1x_1 + t_2x_2 + t_3x_3 + t_4x_4) \end{bmatrix} \\
&= \begin{bmatrix} 0 \\ s(1{:}n-1).*\texttt{cumsum}(t(1{:}n-1).*x(1{:}n-1)) \end{bmatrix}
\end{aligned}$$

This is $O(n)$. Similarly

$$\begin{aligned}
\texttt{triu}(ts^T, 1)x &= \begin{bmatrix} 0 & t_1s_2 & t_1s_3 & t_1s_4 & t_1s_5 \\ 0 & 0 & t_2s_3 & t_2s_4 & t_2s_5 \\ 0 & 0 & 0 & t_3s_4 & t_3s_5 \\ 0 & 0 & 0 & 0 & t_4s_5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} t_1(s_2x_2 + s_3x_3 + s_4x_4 + s_5x_5) \\ t_2(s_3x_3 + s_4x_4 + s_5x_5) \\ t_3(s_4x_4 + s_5x_5) \\ t_4(s_5x_5) \\ 0 \end{bmatrix} \\
&= \begin{bmatrix} t(1{:}n-1).*\texttt{flip}(\texttt{cumsum}(\texttt{flip}(s(2{:}n).*x(2{:}n)))) \\ 0 \end{bmatrix}
\end{aligned}$$

This is $O(n)$ too. So overall we have

```
   function y = FastProdCVL0(d,S,T,x)
% d and x are column n vectors, S and T are n-by-2, n is a positive power of two.
% y = A*x where A = diag(d) + tril(S*T',-1) + triu(T*S',1)
n = length(d);
y = d.*x + ...
   [0;S(2:n,1).*cumsum(T(1:n-1,1).*x(1:n-1))] + ...
   [0;S(2:n,2).*cumsum(T(1:n-1,2).*x(1:n-1))] + ...
   [T(1:n-1,1).*flip(cumsum(flip(S(2:n,1).*x(2:n))));0] + ...
   [T(1:n-1,2).*flip(cumsum(flip(S(2:n,2).*x(2:n))));0];
```