CS 6210 Take-Home Midterm Due: 10/30/15 (Fri) at 6pm

You are not allowed to communicate with anybody about this exam.

1 Fast Matrix Multiply

Develop the most efficient $O(n)$ implementation that you can of the following function:

```
  function y = FastProd(d,S,T,x)
% d and x are column n vectors, S and T are n-by-2, n is a positive power of two.
% y = A*x where A = tril(S*T',-1) +  diag(d) +  triu(T*S',1)
```

The matrix A has the property that any submatrix of the form $A(i_1{:}i_2, j_1{:}j_2)$ that does *not* contain a diagonal entry has rank 2. Efficiency will be measured with `tic` and `toc`. You may assume that $n \leq 2^{20}$. Submit `FastProd` to CMS.

2 Max Diagonal Entry

Develop the most efficient implementation that you can of the following function:

```
  function y = MaxDiag(d,C)
% d is column n vectors with positive entries and C is n-by-r with r<<n.
% mu is the largest entry along the diagonal of inv(diag(d) + C*C').
```

The test script will check things out for $100 \leq n \leq 10000$ and $1 \leq r \leq 10$. Submit `MaxDiag` to CMS.

3 Rotations

This problem is about rotating one vector into another using a sequence of Givens rotations. (a) Implement the following function so that it performs as specified:

```
  function [c,s] = TwoVecRotate(x,alfa)
% x is a nonzero column 2-vector and 0<=alfa<=norm(x,2)
% c and s satisfy c^2 + s^2 = 1 and have the property that if [c s,-s,c]'*x = y,
% then y(1) = alfa.
```

(b) Suppose $c^2 + s^2 = 1$ and $1 \leq i < j \leq n$. Let $G(c, s, i, j) \in \mathbb{R}^{n \times n}$ denote the n-by-n Givens rotation in planes i and j, e.g.,

$$G(c,s,2,4) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & c & 0 & s & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -s & 0 & c & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

By making effective use of `TwoVecRotate(x,alfa)` implement the following function so that it performs as specified:

```
  function [c,s,idx,jdx] = Uniform(v)
% v is a column n-vector with unit 2-norm.
% c and s are column (n-1)-vectors with c.^2 + s.^2 = ones(n-1,1) and
% idx and jdx are column (n-1)-vectors of integers with the property that
% 1<= idx(k)<jdx(k)<=n, k=1:n-1 and
%
%    G(c(n-1),s(n-1),idx(n-1),jdx(n-1))'*...*G(c(1),s(1),1,idx(1),jdx(1))'*v = ones(n,1)/sqrt(n)
```

Submit `TwoVecRotate` and `Uniform` to CMS.