CS 6210: Assignment 7

Due: Wednesday, November 17, 2010 (In Class or in Upson 5153 by 4pm)

Scoring for each problem is on a 0-to-5 scale (5 = complete success, 4 = overlooked a small detail, 3 = good start, 2 = right idea, 1 = germ of the right idea, 0 = missed the point of the problem.) One point will be deducted for insufficiently commented code. Unless otherwise stated, you are expected to utilize fully MATLAB'S vectorizing capability subject to the constraint of being flop-efficient. Test drivers and related material are posted at `http://www.cs.cornell.edu/courses/cs6210/2010fa/`. For each problem submit output and a listing of all scripts/functions that *you* had to write in order to produce the output. You are allowed to discuss *background* issues with other students, but the codes you submit must be your own.

P1. (A Specified Hessenberg Reduction)

Complete the following function so that it performs as specified.

```
  function [Q,H] = SpecialHess(A,v)
 % v is a column n-vector with unit 2norm
 % Q is orthogonal with the property that Q(:,1) = v and
 % Q'*A*Q = H is upper Hessenberg.
```

You are allowed to make use of the built-in MATLAB function `hess`. Test your implementation with `P1`.

P2. (Matrix Exponential)

Develop a closed form expression for

$$F = \exp\left(\begin{bmatrix} \lambda & \beta \\ \alpha & \lambda \end{bmatrix}\right) \qquad \alpha\beta < 0.$$

We say that a quasi-triangular matrix is normalized if all the 2-by-2 bumps have constant diagonals. Implement the following function

```
  function F = MyExpM(T)
 % T is a normalized upper quasitriangular matrix with distinct eigenvalues.
 % F = exp(T)
```

Make effective use of the ideas in §11.1.4. You may use \ to solve the Sylvester systems that arise in (11.1.5). Test your implementation with the script `P2`.

P3. (Block Swap)

Subject to the coupling of complex conjugate eigenvalue pairs, the eigenvalues can be arbitrarily ordered in the real schur form. For example, if

$$T_0 = \begin{bmatrix} \lambda & a & b \\ 0 & c & d \\ 0 & e & f \end{bmatrix}$$

then it is possible to find an orthogonal Q such that

$$Q^T T_0 Q = T = \begin{bmatrix} \tilde{c} & \tilde{d} & \tilde{a} \\ \tilde{e} & \tilde{f} & \tilde{b} \\ 0 & 0 & \lambda \end{bmatrix}$$

with $\lambda(T(1{:}2, 1{:}2)) = \lambda(T_0(2{:}3, 2{:}3))$. Implement the following function

```
  function [Q,T] = Swap3(T0)
% T0 is a 3-by-3 matrix with the property that T0(2:3,1) = 0 and T0(2:3,2:3)
% has complex eigenvalues.
% Q is orthogonal with the property that Q'*T0*Q = T and T(3,1:2) = 0.
```

Test your implementation with the script P3.

C7. (A Generalized Eigenvalue Problem)

Complete the following function so that it performs as specified.

```
  function d = GenEig(A,B)
% A and B are n-by-n nonsingular matrices.
% d is a column n-vector consisting of the eigenvalues of the matrix
%          M = (A*inv(B) + B*inv(A))/2
```

To receive full credit, your method must (a) not form the matrix AB^{-1}, (b) not form the matrix BA^{-1}, and (c) apply only orthogonal transformations to A and B. Test your implementation with the script C7.