CS 6210: Assignment 4

Due: Friday, October 8, 2010 (In Class or in Upson 5153 by 4pm)

Scoring for each problem is on a 0-to-5 scale (5 = complete success, 4 = overlooked a small detail, 3 = good start, 2 = right idea, 1 = germ of the right idea, 0 = missed the point of the problem.) One point will be deducted for insufficiently commented code. Unless otherwise stated, you are expected to utilize fully MATLAB'S vectorizing capability subject to the constraint of being flop-efficient. Test drivers and related material are posted at `http://www.cs.cornell.edu/courses/cs6210/2010fa/`. For each problem submit output and a listing of all scripts/functions that *you* had to write in order to produce the output. You are allowed to discuss *background* issues with other students, but the codes you submit must be your own.

P1. (Nearest Vector in the Orthogonal Complement)

If S is a subspace in R^m, then its orthogonal complement $S^\perp$ is defined by

$$S^\perp = \{ x \mid x^T v = 0 \text{ for every } v \in S\}.$$

If $A \in \mathbb{R}^{m\times n}$, then the range of A is defined by

$$\text{Ran}(A) = \{Ax \mid x \in \mathbb{R}^n \}.$$

Making efficient use of the MATLAB function `QR`, complete the following so that it performs as specified:

```
  function v = Nearest1(A,b)
% A is m-by-n and assume rank(A) = n < m.
% v is the vector in the orthogonal complement of Ran(A) that is nearest
% to b in the 2-norm
```

Note: If $A \in \mathbb{R}^{m\times n}$ then `[Q,R] = qr(A)` requires $O(m^2n)$ flops while `[Q,R] = qr(A,0)` requires $O(mn^2)$. So think about how you use `QR`. Test your implementation on the script `P1`.

2. (A Trigonometric Fitting Problem)

Consider the problem of fitting the data $(\tau_1, f_1), \ldots (\tau_m, f_m)$ with a function of the form

$$F(t) = \alpha + \sum_{k=1}^{n} \beta_k \cos\left(\frac{2\pi k}{P} t\right) + \gamma_k \sin\left(\frac{2\pi k}{P} t\right)$$

Here, P is given. If we use least squares, then the goal is to choose the parameters $\alpha, \beta_1, \ldots, \beta_n, \gamma_1, \ldots, \gamma_n$ so that

$$\psi(\alpha, \beta, \gamma) = \sum_{i=1}^{m} (F(\tau_i) - f_i)^2$$

is minimized. Complete the following function so that it performs as specified:

```
  function [alfa,beta,gamma,Predict] = TrigLS(tau,f,P,n)
% tau and f are column m-vectors, P is a positive real, and
% n is a positive integer (2n+1<=m).
% Determines alfa (scalar), beta (column n-vector), and gamma (column n-vector)
% so that if
%
%       F(t) = alfa +  sum_{k=1:n} of (beta(k)cos(2pi*k*t/P) + gamma(k)*sin(2*pi*k*t/P) )
% then
%       sum_{i=1:m} of ( F(tau(i)) - f(i) )^2
%
% is minimized. In other words, the data (tau(1),f(1)),...,(tau(m),f(m)) is being
% fit in the least squares sense with a sum of sines and cosines.
% Predict is the column m-vector F(tau), i.e., Predict(i) = F(tau(i)), i=1:m.
```

Test your implementation on the script P2. You may use \ to solve the least square problem.

3. (A Matrix LS Problem)

Complete the following function so that it performs as specified:

```
   function X = MatLS(A1,A2,B)
 % A1 and A2 are each m-by-n with m > n = rank(A1) = rank(A2)
 % B is m-by-m
 % X minimizes norm(A1*X*A2' - B,'fro')
```

Make effective use of the MATLAB function svd. Submit listing and output when the test script P3 is applied.

Challenge Problem (Gauss-Seidel for Normal Equations)

Recall that if $A \in \mathbb{R}^{m\times n}$ has full column rank, then the solution to $A^T Ax = A^T b$ minimizes $\| Ax - b \|_2$. Complete the following function so that it performs as specified:

```
   function x = LSviaGS(A,b,itMax)
  % A is an m-by-n matrix and rank(A) = n.
  % Assume that A is represented in sparse format.
  % b is a column m-vector
  % x is obtained by applying itMax steps of Gauss-Seidel  to the linear system
  % A'*A*x = A'*b with starting vector x = b.
```

Submit listing and output when the test script C4 is applied.