Hella Crunk Opfibrations II

Formal Definition

$$\begin{tikzcd} D \arrow[r, "\bar{f}"] \arrow[rr, bend right, "h"'] & D' \arrow[r, dashed, "\bar{g}"] & D'' \end{tikzcd}$$

$$S \xrightarrow{f} S' \xrightarrow{g} S''$$

$\bar{f}$ is opcartesian if given an $g : S' \to S''$ and $h : D \to D'$ with $Uh = f;g$ then there exists a unique $\bar{g} : D' \to D''$ over g

Example: Relations ($Rel(2)$)

Objects: $\langle X, R \rangle$ a set X and subset $R \subseteq X \times X$

Morphisms: $\langle f, p \rangle : \langle X, R \rangle \to \langle X', R' \rangle$ a function $f : X \to X'$ a proof p that $\forall x, y \in X.\ xRy \to f(x) R' f(y)$

relation-preserving functions

Is $Rel(2)$ an opfibration?

$$\begin{tikzcd} \langle X, R \rangle \arrow[r, "\langle f, p \rangle"] \arrow[dr, "\langle f;g, p'' \rangle"'] & \langle Y, yR'y' = \exists x, x' \in X.\ f(x) = y \wedge f(x') = y' \wedge xRx' \rangle \arrow[d, dashed, "\langle g, p' \rangle"] \\ & \langle Z, R'' \rangle \end{tikzcd}$$

$$X \xrightarrow{f} Y \xrightarrow{g} Z$$

Yes!

Is Irrefl an opfibration?

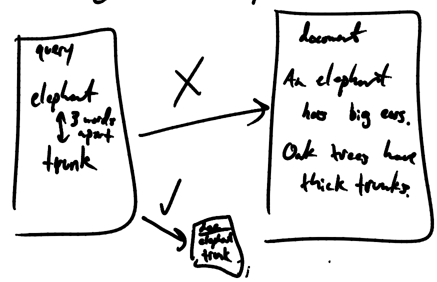
irreflexive relations

No, in general, but has opcartesian liftings of injections

$$\{a, b\} \xrightarrow{a, b \mapsto c} \{c\}$$

Another example: metric spaces (kinda of)

contractive mappings



Does Met_c form an opfibration?

$$\langle X, d\rangle \xrightarrow{\langle f, 1\rangle} \langle Y, d'(y,y') = \min_{x,x' \in X \wedge f(x)=y \wedge f(x')=y'} d(x,x')\rangle$$

$$X \xrightarrow{f} Y$$