



! - Modality

!A

more

as many copies of A
as the consumer wants
(each produced separately)

! Rules (Left)

$$\frac{P, A + \Delta}{P, !A + \Delta} \quad (\text{clone one copy})$$

$$\frac{P + \Delta}{P, !A + \Delta} \quad (\text{clone zero copies})$$

$$\frac{!A, !A + \Delta}{!A, !A + \Delta} \quad (\text{double your copies})$$

? - modality

?A

more

as many As as
the producer wants
all consumed separately

? Right Rules

$$\frac{P + \Delta}{P + ?A, \Delta} \quad (\text{clone zero copies})$$

$$\frac{P + A, \Delta}{P + ?A, \Delta} \quad (\text{clone one copy})$$

$$\frac{P + ?A, ?A, \Delta}{P + ?A, \Delta} \quad (\text{clone double})$$

Promotion Rules

$$\frac{!P + ?\Delta, A}{!P + ?\Delta, !A} \qquad \frac{!P, A + ?\Delta}{!P, ?A + ?\Delta}$$

! as Data

$$\begin{array}{ccc}
 \frac{\overline{F1}}{+101} \text{ left} & \frac{\overline{F1}}{+101} \text{ right} & \\
 \frac{\overline{F!(101)}}{1+(101)} & \frac{\overline{F!(101)}}{1+(101)} & \\
 \frac{1+(101)}{101+(101)} & \frac{1+(101)}{1+(101)} & \frac{101+101}{!(101)+101} \\
 \leftarrow \text{expand} & \rightarrow \text{justify} &
 \end{array}$$

Non-Linear Intuitionistic Type Theory

$$\begin{array}{l}
 \Gamma \vdash A \xrightarrow{\text{trivial}} !\Gamma \vdash A \\
 \Gamma \vdash \Delta \xrightarrow{\quad\quad} !\Gamma \vdash \Delta \\
 \frac{\Gamma \vdash \Delta, A \quad A, \Gamma' \vdash \Delta'}{\Gamma, \Gamma' \vdash \Delta, \Delta'} \text{ oppoing } \Delta = \emptyset \xrightarrow{\quad\quad} \frac{!\Gamma \vdash \Delta, A \quad !\Gamma' \vdash \Delta'}{!\Gamma, !\Gamma' \vdash \Delta, \Delta'}
 \end{array}$$

Non-Linear Classical Logic

$$\begin{array}{l}
 \Gamma \vdash A \xrightarrow{\text{trivial}} !\Gamma \vdash ?A \\
 \vdots \\
 \frac{\Gamma \vdash A \quad A, \Gamma' \vdash A}{\Gamma, \Gamma' \vdash A \circ A} \xrightarrow{\quad\quad} \frac{!\Gamma \vdash ?A \quad !\Gamma' \vdash ?A}{!\Gamma, !\Gamma' \vdash ?(A \circ A)}
 \end{array}$$