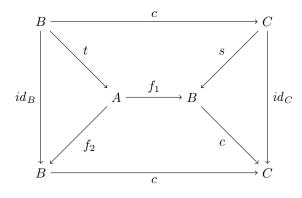
Assignment 15

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Definition (Split Fork). Given morphisms $A \xrightarrow[f_2]{i} B \xrightarrow[f_2]{i} C$ of a category, they form a split fork if there exist morphisms $s: C \to B$ and $t: B \to A$ such that the following diagram commutes:



Exercise 1. Show that, for every split fork $A \xrightarrow{f_1}{f_2} B \xrightarrow{c} C$, the morphism c is a coequalizer of f_1 and f_2 .

Definition (Absolute (Co)Limit). A (co)limit in a given category C is called *absolute* if every functor from C preserves it.

Exercise 2. A coequalizer $c: B \to C$ of a pair of morphisms $A \xrightarrow{f_1}{f_2} B$ is called split if $A \xrightarrow{f_1}{f_2} B \xrightarrow{c} C$ is a split fork. Prove that every split coequalizer is an absolute coequalizer. That is, show that every split coequalizer $A \xrightarrow{f_1}{f_2} B \xrightarrow{c} C$ in any category **C** has the property that, for any category **D** and functor $F: \mathbf{C} \to \mathbf{D}$, the morphism $Fc: FB \to FC$ is a coequalizer of the pair of morphisms $FA \xrightarrow{Ff_1}{Ff_2} FB$. You may use the fact that you proved above.

Exercise 3. Prove that the multicategory **Prost** is left closed. However, for the existence and uniqueness proofs of λf , for the sake of readability only show this for the case where f is binary.