Security

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Exercise 1. Prove that the 2-category **MULTICAT** is powered. I am content if you can construct the power multicategory $\mathbf{E} \pitchfork \mathbf{M}$ and the necessary functor $\mathbf{E} \to \mathbf{M}_{\mathbf{MULTICAT}}(\mathbf{E} \pitchfork \mathbf{M}, \mathbf{M})$ and show that any multicategory $\mathbf{D}$ with a functor $\mathbf{E} \to \mathbf{M}_{\mathbf{MULTICAT}}(\mathbf{D}, \mathbf{M})$ has a 1-cell from $\mathbf{D}$ to $\mathbf{E} \pitchfork \mathbf{M}$. The remaining requirements need to hold but do not need to be proved.

Proof. Let $\mathbf{C_M}$ be the category whose objects are the same as $\mathbf{M}$ and whose morphisms are the unary morphisms of $\mathbf{M}$ with the obvious identity and composition. Suppose $F_1, \ldots, F_n$ and G are functors from $\mathbf{E}$ to $\mathbf{C_M}$, then let a multitransformation α from $\vec{F}$ to G map an object $\mathcal{E}$ of $\mathbf{E}$ to morphism of $\mathbf{M}$ from $[F_1(\mathcal{E}), \ldots, F_n(\mathcal{E})]$ to $G(\mathcal{E})$, and let α be natural if for every morphism $e : \mathcal{E} \to \mathcal{E}'$ of $\mathbf{E}$ the composition $\alpha_{\mathcal{E}} \,; G(e)$ equals the composition $[F_1(e), \ldots, F_n(e)] \,; \alpha_{\mathcal{E}'}$.

Define $\mathbf{E} \pitchfork \mathbf{M}$ to be the category whose objects are functors from $\mathbf{E}$ to $\mathbf{C_M}$ and whose morphisms are natural multitransformations with composition and identity each defined pointwise (which obviously always results in a natural multitransformation). Given an object $\mathcal{E}$ of $\mathbf{E}$, let $\pi_{\mathcal{E}} : \mathbf{E} \pitchfork \mathbf{M} \to \mathbf{M}$ map the functor $F : \mathbf{E} \to \mathbf{C_M}$ to the object $F(\mathcal{E})$ and the multitransformation α to the morphism $\alpha_{\mathcal{E}}$, which defines a functor of multicategories because composition and identity in $\mathbf{E} \pitchfork \mathbf{M}$ are defined pointwise. Given a morphism $e : \mathcal{E} \to \mathcal{E}'$ of $\mathbf{E}$, let $\pi_e : \pi_{\mathcal{E}} \Rightarrow \pi_{\mathcal{E}'}$ be the transformation mapping a functor $F : \mathbf{E} \to \mathbf{C_M}$ to the morphism $F(e) : [F(\mathcal{E})] \to F(\mathcal{E}')$, which is natural due to the naturality requirement on the morphisms of $\mathbf{E} \pitchfork \mathbf{M}$. $\pi : \mathbf{E} \to \mathbf{M}_{\mathbf{MULTICAT}}(\mathbf{E} \pitchfork \mathbf{M}, \mathbf{M})$ is functorial because each object of $\mathbf{E} \pitchfork \mathbf{M}$ is functorial. $\square$