Internalization

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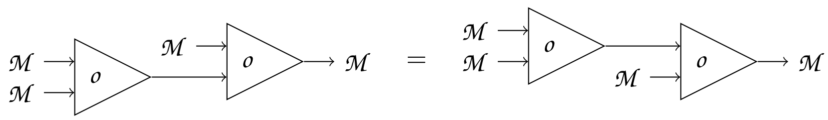
September 9, 2014

Definition (Internal Biased Monoid of a Multicategory $\mathbf{M}$)**.** A tuple $\langle \mathcal{M}, o, \mathfrak{a}, e, \mathfrak{i} \rangle$ where the components have the following types:

Underlying Object $\mathcal{M}$**:** $O_{\mathbf{M}}$

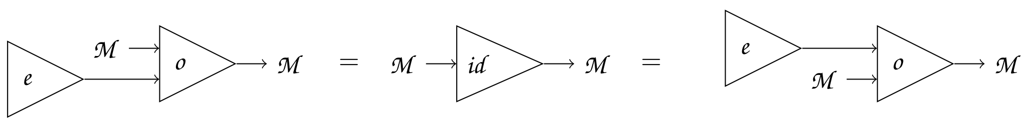
Operator o**:** $[\mathcal{M}, \mathcal{M}] \to \mathcal{M}$

Associativity $\mathfrak{a}$**:**



Identity Morphism e**:** $[\,] \to \mathcal{M}$

Identity $\mathfrak{i}$



Example. The following are equivalent to internal monoids of respective multicategories:

Set: A monoid

Prost: A monoid with a congruent preorder, meaning a preorder $\leq$ on the underlying set M such that:

$$\forall m_1, m_1', m_2, m_2' : M.\ m_1 \leq m_1' \wedge m_2 \leq m_2' \implies m_1 * m_2 \leq m_1' * m_2'$$

$\mathcal{M}$ **where** $\mathcal{M}$ **is a monoid:** The identity element of $\mathcal{M}$

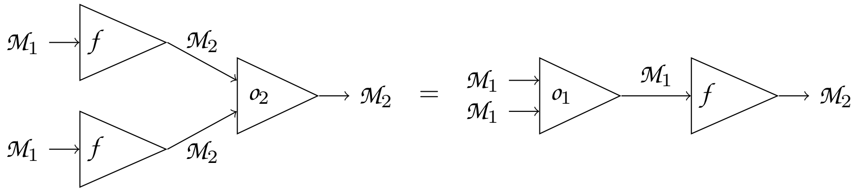
BinRel: A set and a preorder on that set

SplitGraph: A small category

Definition (Internal (Biased) Monoid Homomorphism from $\langle \mathcal{M}_1, o_1, \_, e_1, \_ \rangle$ to $\langle \mathcal{M}_2, o_2, \_, e_2 \rangle$)**.** A tuple $\langle f, \mathfrak{d}, \mathfrak{i} \rangle$ where the components have the following types:

Underlying Morphism f**:** $[\mathcal{M}_1] \to \mathcal{M}_2$

Distributivity $\mathfrak{d}$**:**



Identity $\mathfrak{i}$**:**