Transformations

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Remark. Proofs by contraditiction or law of excluded middle are not permitted.

Exercise 1. Prove that for any category $\mathbf{C}$ and any object $\mathcal{C} : \mathbf{C}$, the category $\mathbf{Sub}(\mathcal{C})$ is thin, meaning there is at most one morphism between any two objects.

Exercise 2. Prove that **Prost** is a reflective subcategory of $\mathbf{Rel}(2)$ (the category whose objects are sets with a binary relation and whose morphisms are relation-preserving functions).

Remark. To get an early start on Exercise 3 below, look at Exercise 6 in the lecture notes for Nulls.

Exercise 3. Suppose a subcategory $\mathbf{S} \overset{I}{\hookrightarrow} \mathbf{C}$ has a mapping from each object $\mathcal{C} : \mathbf{C}$ to a reflection arrow $\mathcal{C} \xrightarrow{r_{\mathcal{C}}} I(R(\mathcal{C}))$. Prove that there is a unique way to extend the function R to a functor from $\mathbf{C}$ to $\mathbf{S}$ such that the reflection arrows form a natural transformation $r : \mathbf{C} \Rightarrow R\,;I$.

Exercise 4. Prove that the category **Cat** can be enriched in the multicategory **CAT**.