

Proofs

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Exercise 1. Let $trans : \langle \{a, b, c\}, \{\langle a, b \rangle, \langle b, c \rangle\} \rangle \rightarrow \langle \{a, b, c\}, \{\langle a, b \rangle, \langle b, c \rangle, \langle a, c \rangle\} \rangle$ be a morphism of $\mathbf{Rel}(2)$ whose underlying function is the identity. Call its domain \mathcal{P} and its codomain \mathcal{R} . Describe, in standard set-theoretic terms, what the pushout of the following is, for any object \mathcal{X} and morphism $f : \mathcal{P} \rightarrow \mathcal{X}$:

$$\begin{array}{ccc} \mathcal{P} & \xrightarrow{trans} & \mathcal{R} \\ \downarrow f & & \\ \mathcal{X} & & \end{array}$$

Prove that your description is actually a pushout.