Monads

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Exercise 1. Prove that for any 2-category $\mathbf{C}$ and any adjunction $f \dashv g$ in $\mathbf{C}$, one can build a monad in $\mathbf{C}$ whose underlying morphism is $f \mathbin{;} g$.

Exercise 2. Prove that, in the 2-category $\mathbf{CAT}$, for every monad $\mathcal{M}$ with underlying functor M on a category $\mathbf{C}$ there is some adjunction $F \dashv U$ such that M equals $F \mathbin{;} U$. Hint: use the underlying functor $U : \mathbf{Alg}(\mathcal{M}) \to \mathbf{C}$ as the right adjoint.

Remark. The above theorem holds for monads in $\mathbf{CAT}$ but not necessarily for monads in other 2-categories.