Limits

Ross Tate

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Exercise 1. Prove that for any object $\mathscr{A}$ of any category $\mathbf{C}$, the object $\mathscr{A} \,\&\, \top$ (if it exists) is isomoprhic to $\mathscr{A}$.

Exercise 2. Prove that, in any 2-category, if morphisms $\mathcal{C}_1 \xrightarrow{f_1} \mathcal{C}_2$ and $\mathcal{C}_2 \xrightarrow{f_2} \mathcal{C}_3$ are both left adjoints, then their composition $f_1 \,;\, f_2$ is also a left adjoint.

Exercise 3. The monoid $\mathcal{A} \,\&\, \mathcal{B}$ is commutative if both $\mathcal{A}$ and $\mathcal{B}$ are commutative, and in that case is (with the appropriate projection homomorphisms) also the product of $\mathcal{A}$ and $\mathcal{B}$ in **CommMon**. Prove that there are morphisms $\kappa_{\mathcal{A}}$ and $\kappa_{\mathcal{B}}$ demonstrating that $\mathcal{A} \,\&\, \mathcal{B}$ is also the coproduct of $\mathcal{A}$ and $\mathcal{B}$ in **CommMon**. That is, prove that **CommMon** has *biproducts*, meaning it has products and coproducts and they coincide on objects.