

Existentials

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Exercise 1. Prove that the existential subtyping relation for a category **Bnd** and functor $\mathbf{Bnd} \rightarrow \mathbf{Prost}$ is always a preorder. Be detailed in your proof.

Exercise 2. Prove that **Prost** has an $(\mathcal{E}, \mathcal{M})$ factorization structure, where \mathcal{E} is the set of all epimorphisms and \mathcal{M} is the set of all *initial* mono-sources. A **Prost**-source $(C \xrightarrow{f_i} C_i)_{i:I}$ is initial when for all pairs $c_1, c_2 : C$, if $\forall i : I. f_i(c_1) \leq f_i(c_2)$ holds then $c_1 \leq c_2$ holds as well (note that the reverse implication always holds because each function must be relation-preserving). You may assume that **Set** has a epi-mono factorization structure, that a morphism in **Prost** is an epimorphism iff its underlying function is an epimorphism, and that a source in **Prost** is a mono-source iff its underlying source is a mono-source.