# Categories

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**Exercise 1.** Give, for any category $\mathbf{C}$ and any object $\mathcal{C} : \mathbf{C}$, a monoidal structure on the set $\mathcal{C} \to \mathcal{C}$.

**Exercise 2.** Prove that for any monoid $\mathcal{M}$ there is a category with one object $\star$ such that $\star \to \star$ equals $M$.

**Exercise 3.** Show that the above extends to a functor from **Mon** to **Cat**.

**Exercise 4.** Show that there is a functor $F : \mathbf{Set} \to \mathbf{Mon}$ and a functor $U : \mathbf{Mon} \to \mathbf{Set}$ such that $F\,;U$ equals $\mathbb{L}$. Hint: $U$ maps a monoid to its underlying set.

**Exercise 5.** Prove that any category that has exactly one functor to it from each other category must be isomorphic to the category $\mathbf{1}$.

**Exercise 6.** Prove that any category that has exactly one functor from it to each other category must be isomorphic to the category $\mathbf{0}$.

**Exercise 7.** Given categores $\mathbf{A}$ and $\mathbf{B}$, define a category $\mathbf{A} \times \mathbf{B}$ with "projection" functors $\pi_A$ and $\pi_B$ from it to $\mathbf{A}$ and $\mathbf{B}$ respectively.