How Reasonable are the Axioms?

All the axioms that Savage and von Neumann-Morgenstern use seem so reasonable.

- Savage views his axioms as characterizing rationality

Is that reasonable?

To make matters worse, people make lots of systematic probability errors.

Allais Paradox

The set of prizes is $X = \{\$0, \$1,000,000, \$5,000,000\}$.

- Which probability do you prefer:
  $p_1 = (0.00, 1.00, 0.00)$ or $p_2 = (0.01, 0.89, 0.10)$?
- Which probability do you prefer:
  $p_3 = (0.90, 0.00, 0.10)$ or $p_4 = (0.89, 0.11, 0.00)$?

Many subjects report: $p_1 \succ p_2$ and $p_3 \succ p_4$

Inconsistent with EUT

Suppose (u_0, u_1, u_5) represents $\succ$.

Then $p_1 \succ p_2$ implies

$$u_1 > .01u_0 + .89u_1 + .1u_5$$

$$.11u_1 - .01u_0 > .1u_5$$

$$.11u_1 + .89u_0 > .1u_5 + .9u_0.$$

So $p_4 \succ p_3$.

Which axiom is violated?

Independence: $a \succ b$ iff $\alpha a + (1-\alpha)c \succ \alpha b + (1-\alpha)c$.

- homework – explain exactly how.

Ellsberg Paradox

There is one urn with with 300 balls: 100 of these balls are red (R) and the rest are either blue (B) or yellow(Y). Consider the following two choice situations:

I: a. Win \$100 if a ball drawn from the urn is R and nothing otherwise.

a'. Win \$100 if a ball drawn from the urn is B and nothing otherwise.

II: b. Win \$100 if a ball drawn from the urn is R or Y and nothing otherwise.

b'. Win \$100 if a ball drawn from the urn is B or Y and nothing otherwise.

Inconsistent with SEU

Suppose a decision maker's preferences are such that $a \succ a'$ and $b' \succ b$.

If there are subjective probabilities then the first choice implies that the probability of a red ball is greater than the probability of a blue ball and the second choice implies the reverse.

Which of Savage's axioms is violated?

- Independence: Remember that an act is a function from states to outcomes. Let $T \subseteq S$ be a subset of states. Then

$$f_T g \succeq f'_T g \text{ iff } f_T h \succeq f'_T h.$$

Homework: prove that the standard choices in the Ellsberg paradox violate this.

Maxmin Expected Utility Rule

Suppose that the decision maker's uncertainty can be represented by a set $\mathcal{P}$ of probabilities . Let

$$\underline{E}_{\mathcal{P}}(u_a) = \inf_{\Pr \in \mathcal{P}} \{E_{\Pr}(u_a) : \Pr \in \mathcal{P}\}$$

Recall the maximin expected utility rule: (covered earlier in the course):

- $a >^1_{\mathcal{P}} a'$ iff $\underline{E}_{\mathcal{P}}(u_a) > \underline{E}_{\mathcal{P}}(u_{a'})$

This is like maximin:

- Optimizing the worst-case expectation

This could explain the Ellsberg Paradox:

- Let $\mathcal{P} = \{(1/3, p_B, p_Y) : 0 \leq p_B \leq 2/3\}$

Gilboa and Schmeidler axiomatized the maxmin expected utility rule

- It does *not* satisfy independence
- Gilboa and Schmeidler replaced independence by a weaker axiom.

Framing Effects—Kahneman and Tversky

A disease is expected to kill 600 people. Two alternative programs have been proposed:

- Program A: 200 people will be saved
- Program B: probability 1/3: 600 people will be saved probability 2/3: no one will be saved

Which program would you favor?

Framing Effects—Kahneman and Tversky

A disease is expected to kill 600 people. Two alternative programs have been proposed:

- Program C: 400 people will die
- Program D: probability 1/3: no one will die
  probability 2/3: 600 will die

Which program would you favor?

Framing Effects—Kahneman and Tversky

Kahneman and Tversky found:

- 72% chose A over B.
- 22% chose C over D.

But if 200 people will be saved out of 600 is the same to the decision-maker as 400 people will die out of 600, and so on, then A and C are identical and so are B and D.

Conjunction Fallacy or Failure of Extensionality

Tom is a rancher from Montana.

Which bet would you prefer?

- Win $10 if Tom drives either a Ford or a Chevy, otherwise win nothing
- Win $10 if Tom drives either a Chevy truck or Ford truck, otherwise win nothing

Kahneman and Tversky experiment:

Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations. Which is more probable?

- Linda is a bank teller.
- Linda is a bank teller and is active in the feminist movement.

85% of subjects chose the second option.

Another systemic error: ignoring priors

A cab was involved in a hit and run accident last night. Two cab companies, Green and Blue, operate in the city.

You know:

- A witness identified the cab as Blue.
- Witnesses are pretty reliable: Tests have shown that in similar cirumstances witnesses correctly identify each of the two cabs 80% of the time and misidentify them 20% of the time.
- 85% of the cabs in the city are Green the rest are Blue.

What is the probability that the cab involved in the accident was Blue?

The correct answer requires Bayes rule:

$$\begin{aligned} Pr(B|idB) &= \frac{Pr(idB|B)Pr(B)}{Pr(idB)} \\ &= \frac{(.8)(.15)}{(.8)(.15)+(.2)(.85)} \\ &= .41 \end{aligned}$$