# Decision Theory Prelim
## October 16,2008

There are three questions. Please answer each question in a separate blue book. The test is out of 100; you have 75 minutes. Good luck!

I (30 points) Provide a brief answer for each of the following.

(a) Let $X = \{1, 2, \cdots, N\}$, $N \geq 3$, and suppose that $x \succ y$ if and only if $x > y + 1$. As usual, define $\succeq$ by $x \succeq y$ if $not[y \succ x]$. Is this binary relation complete?

(b) Let $X = \{a, b, c\}$ and suppose we have a choice function such that $c(\{a, c\}) = \{c\}, c(\{a, b\}) = \{a\}, c(\{b, c\}) = \{c\}, c(\{a, b, c\}) = \{a, c\}$. Is there a preference relation $\succ$ such that $C(\cdot) = C(\cdot, \succ)$?

(c) Suppose someone is deciding whether to quit smoking, having heard that smoking might shorten his life span. Of course, he'd prefer to live to a ripe old age, but he enjoys smoking, and if he does make it to old age, he'd prefer to have smoked to not having smoked; likewise if he's going to die early in any case. Thus, he constructs the following decision table, where Q represents quit, C represents continue smoking, L represents live to an old age, and D represents die early.

|   | L | D |
|---|---|---|
| Q | 95 | -5 |
| C | 100 | 0 |

He notices that continuing to smoke dominates quitting (no matter whether he lives to a ripe old age or dies early he is better off smoking).

Under what circumstances is this an appropriate representation of the problem; under what circumstances is it not?

II (30 Points) Consider the following decision matrix:

|       | $s_1$ | $s_2$ | $s_3$ | $s_4$ |
|-------|-------|-------|-------|-------|
| $a_1$ | 6     | 8     | 9     | 4     |
| $a_2$ | 2     | 9     | 9     | 11    |
| $a_3$ | 4     | 4     | 4     | 4     |
| $a_4$ | 0     | 5     | 15    | 6     |

(a) Order the acts according to each of the following decision rules:

   (i) maximin;
   (ii) optimism-pessimism, with index $\alpha = 1/3$;
   (iii) minimax regret;
   (iv) principle of insufficient reason.

(b) Suppose that we multiplied all the utilities by 2 and added 5 (so that, for example, the first row in the matrix would become 17, 21, 23, 13). Which, if any, of the orders in part (a) would change.

(c) [GRAD: for those taking CS5846 or ECON6760 only:] Suppose that we replaced each utility $n$ by $10^n$ (so that, for example, the first row in the matrix would become $10^6$, $10^8$, $10^9$, $10^4$). Now which of the orders in part (a) would change?

III (40 points)

Let $X = \{(i,j) : i \text{ and } j \text{ are non-negative integers.}\}$. Suppose that $x \succ y$ if and only if $x_1 + x_2 > y_1 + y_2 + 2$.

(a) Show that $\succ$ is transitive.

(b) Find a weak representation for $\succ$, i.e. a function $u : X \rightarrow R$ such that if $x \succ y$ then $u(x) > u(y)$.

(c) Show that there does not exist a utility representation for $\succ$.

(d) Let $A = \{x \in X : x_1 + 2x_2 \leq 10\}$. What is $c(A, \succ)$? [$c(A, \succ)$ is the set of undominated elements of $A$ according to $\succ$.]