

CS5670: Computer Vision

Lecture 2: Edge detection

SHADOW

From [Sandlot Science](#)

The logo for ZINNI, featuring the word "ZINNI" in a stylized, white, sans-serif font with a registered trademark symbol (®) to the right. The letters are set against a solid teal background. The 'Z' is composed of three diagonal strokes, and the 'I' is a simple vertical bar.

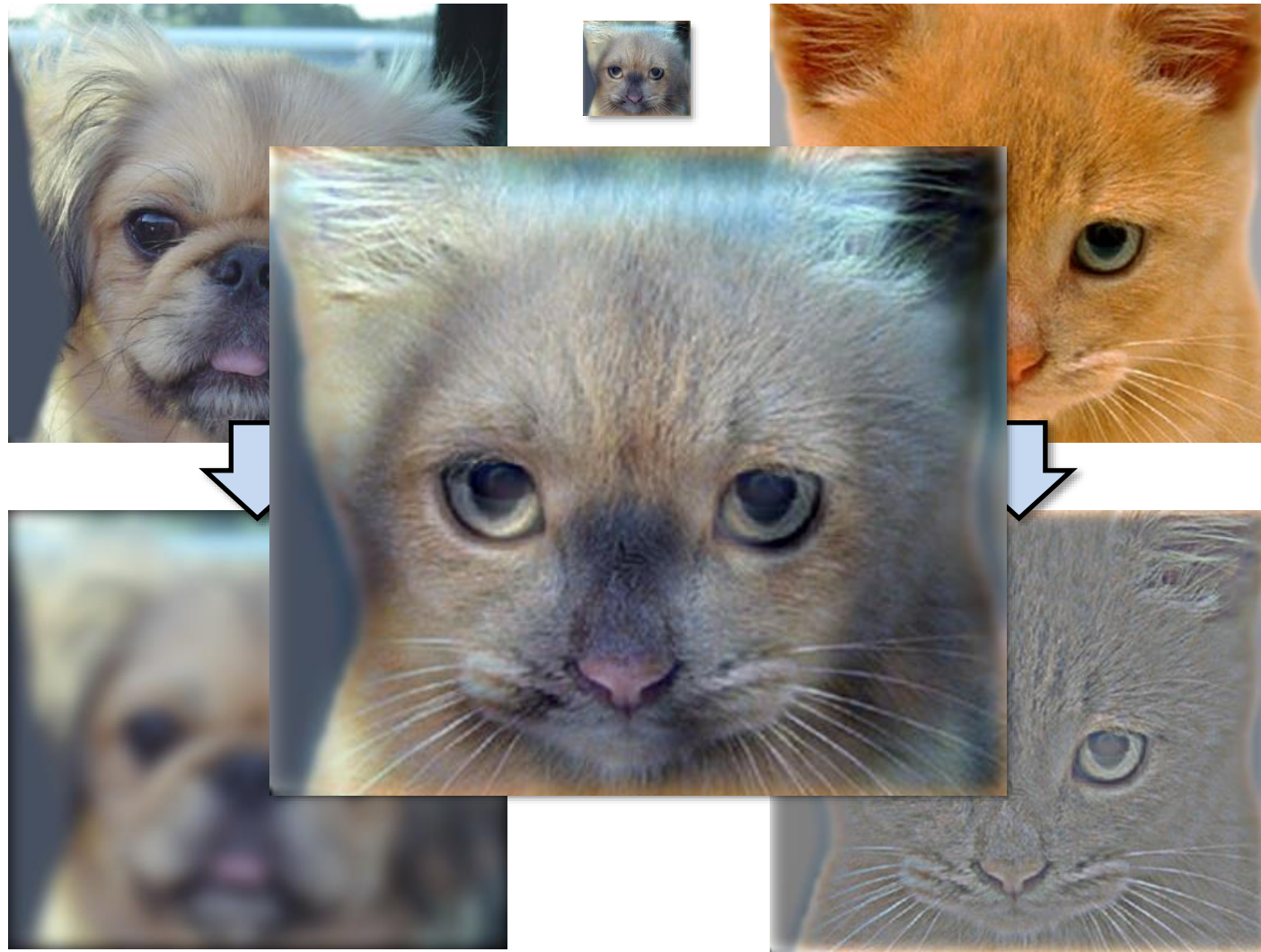
Announcements

- Project 1 (Hybrid Images) is now on the course webpage (see *Projects* link)
 - Due Thursday, Feb 25, by 11:59pm on CMS
 - Artifact due Monday, March 1, by 11:59pm
 - Project to be done individually
 - Voting system for favorite artifacts (with small amount of extra credit)
 - Skeleton code available on Github Classroom – instructions for setting up Python environment on the project webpage

Announcements

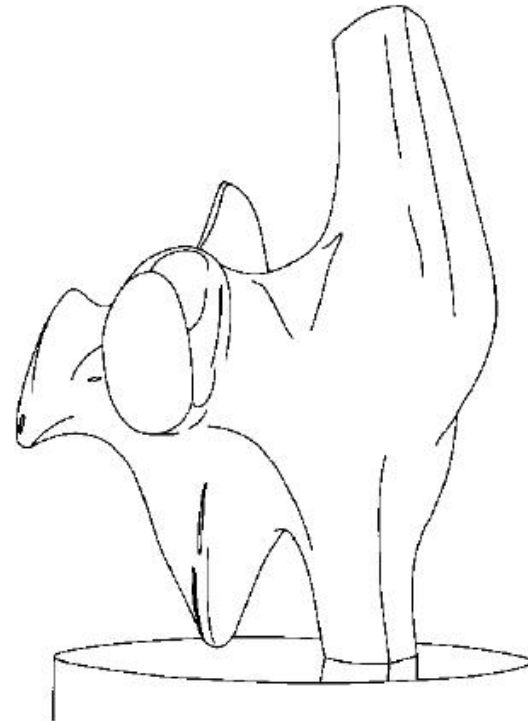
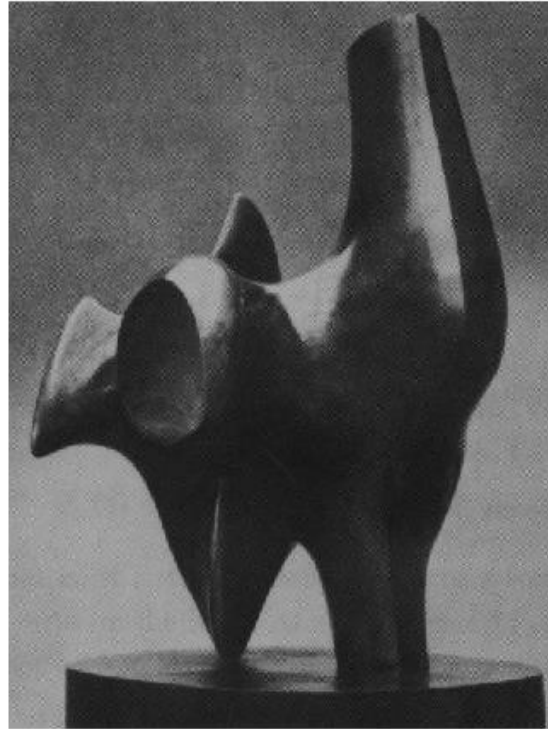
- Quizzes to be given through Canvas or Gradescope
- Still working out the details

Project 1: Hybrid Images



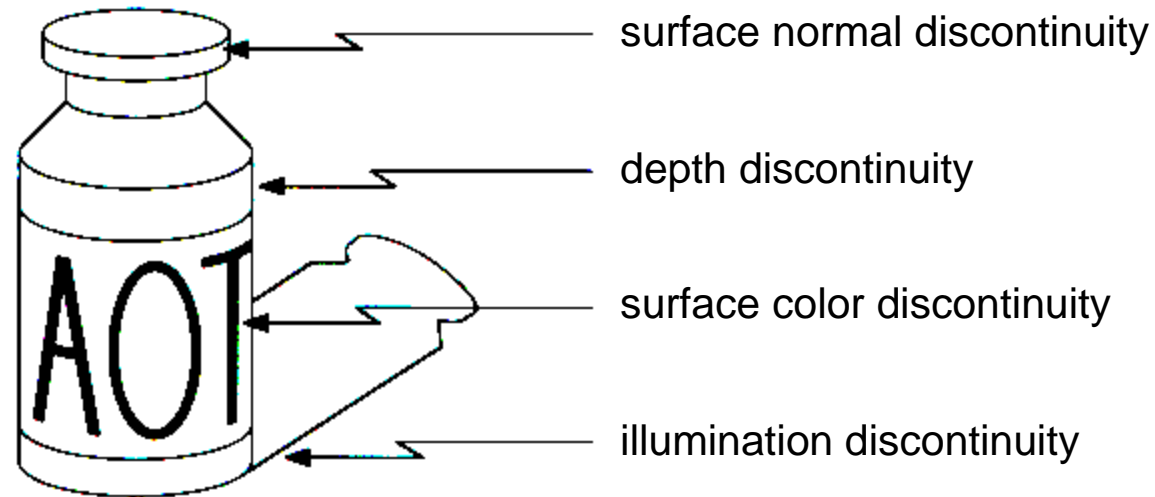
Project 1 Demo

Edge detection



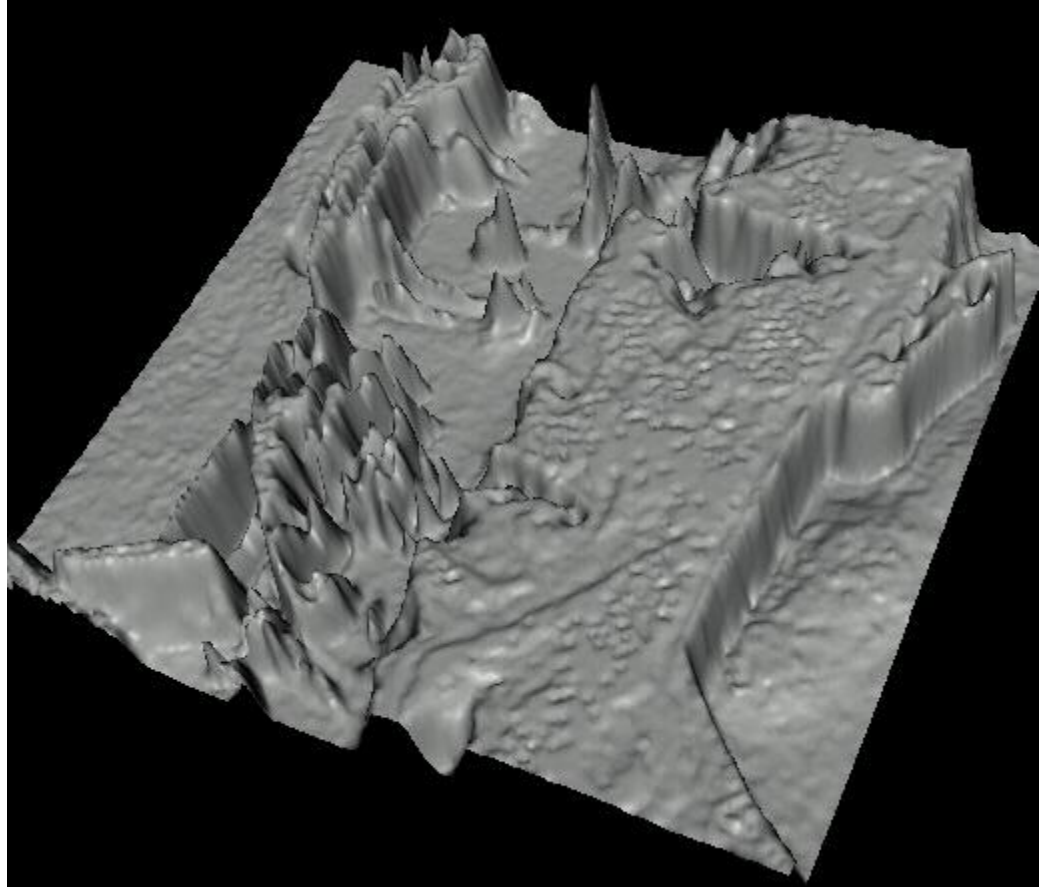
- Convert a 2D image into a set of curves
 - Extracts salient features of the scene
 - More compact than pixels

Origin of edges



- Edges are caused by a variety of factors

Images as functions...



- Edges look like steep cliffs

Characterizing edges

- An edge is a place of *rapid change* in the image intensity function

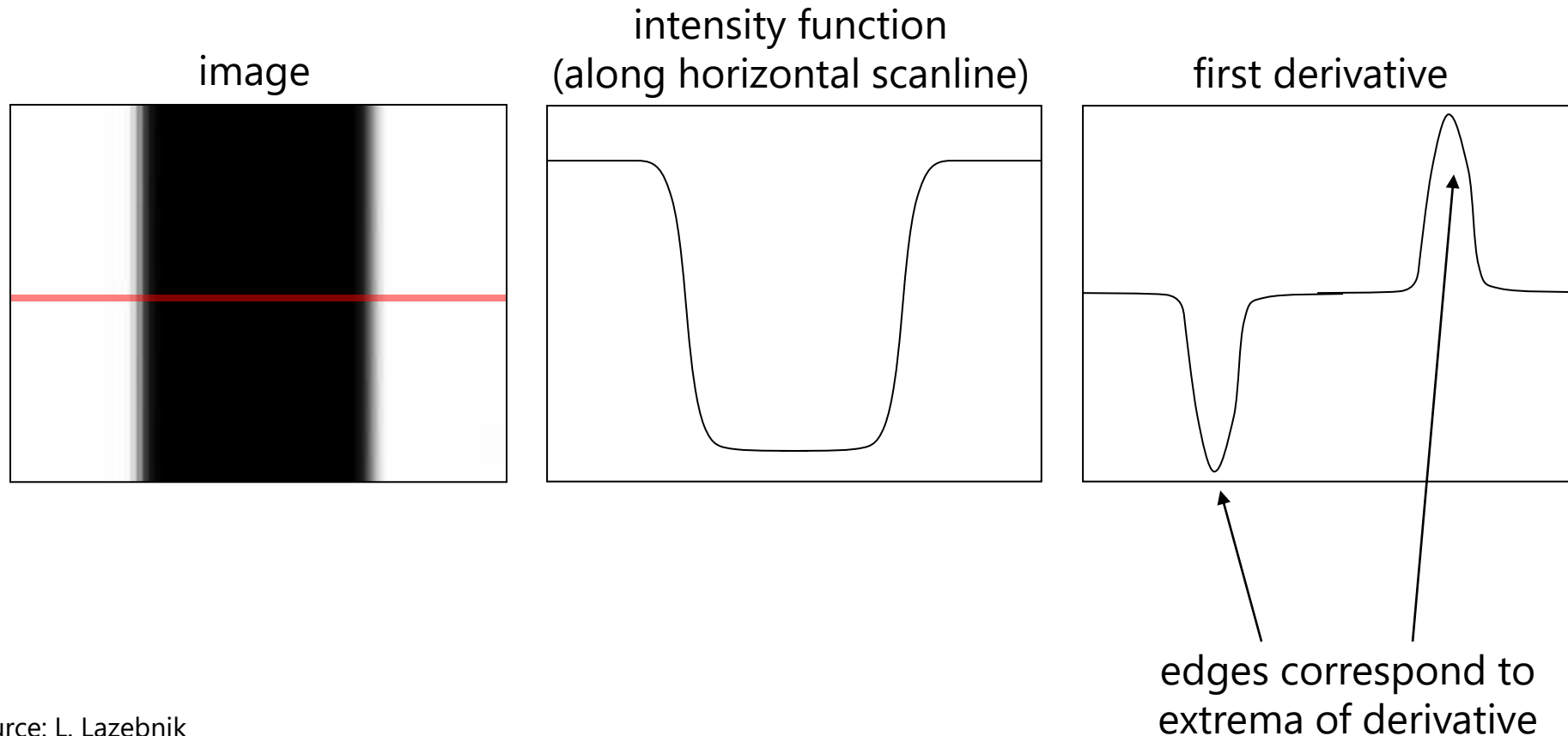


Image derivatives

- How can we differentiate a *digital* image $F[x,y]$?
 - Option 1: reconstruct a continuous image, f , then compute the derivative
 - Option 2: take discrete derivative (finite difference)

$$\frac{\partial f}{\partial x}[x, y] \approx F[x + 1, y] - F[x, y]$$

How would you implement this as a linear filter?

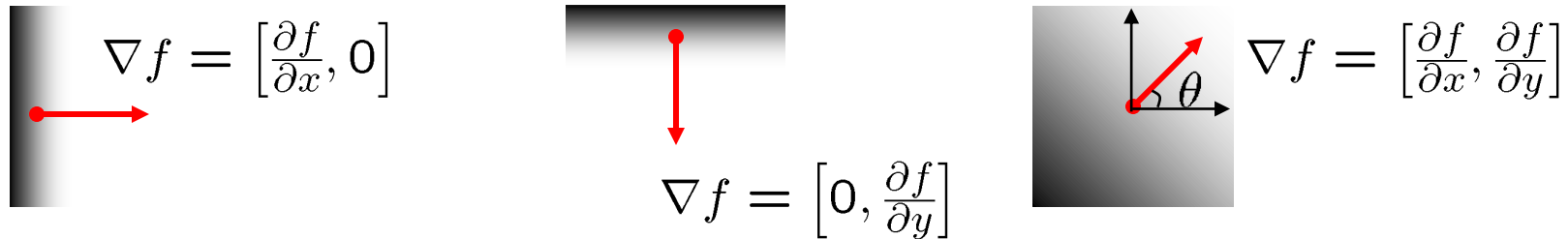
$$\frac{\partial f}{\partial x} \cdot \begin{array}{|c|c|c|} \hline & & \\ \hline & & \\ \hline & & \\ \hline \end{array} \\ H_x$$

$$\frac{\partial f}{\partial y} \cdot \begin{array}{|c|c|c|} \hline & & \\ \hline & & \\ \hline & & \\ \hline \end{array} \\ H_y$$

Image gradient

- The *gradient* of an image: $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$

The gradient points in the direction of most rapid increase in intensity



The *edge strength* is given by the gradient magnitude:

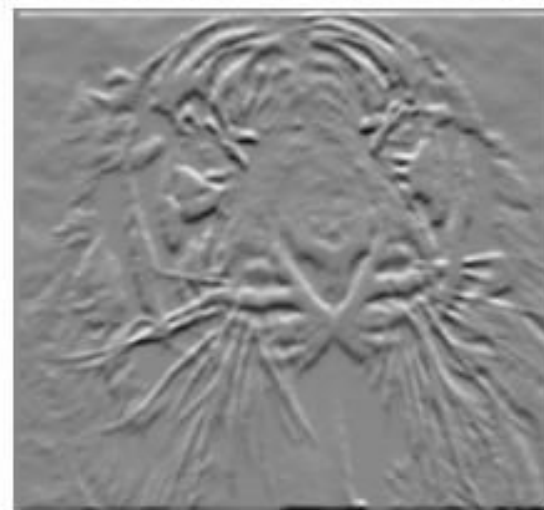
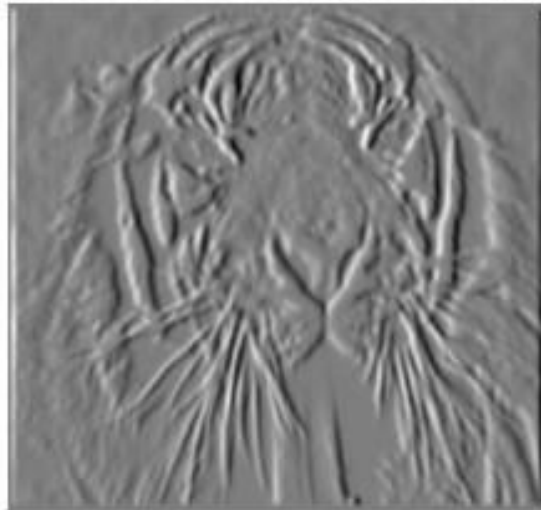
$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

The gradient direction is given by:

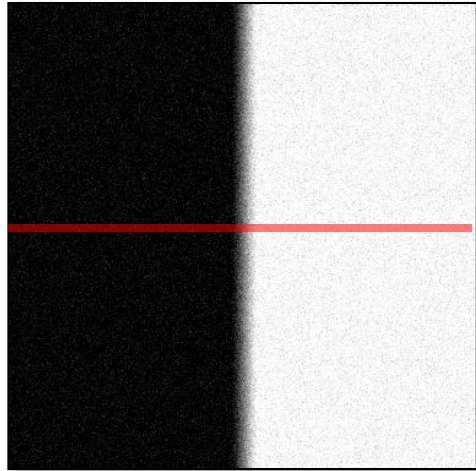
$$\theta = \tan^{-1} \left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x} \right)$$

- how does this relate to the direction of the edge?

Image gradient

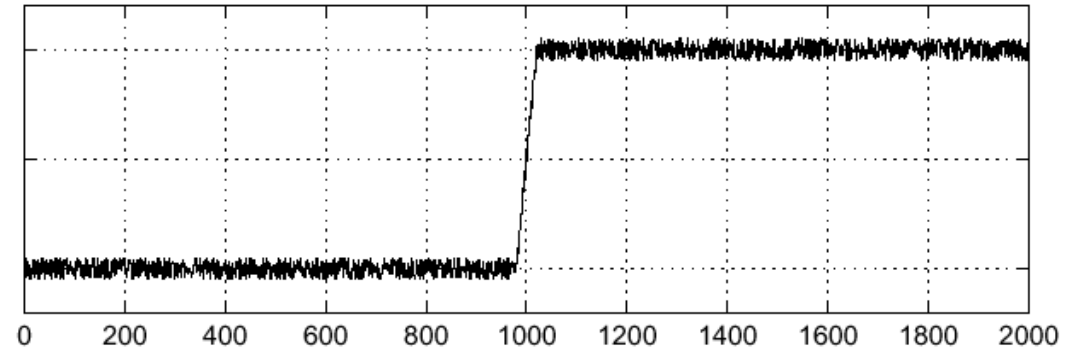


Effects of noise

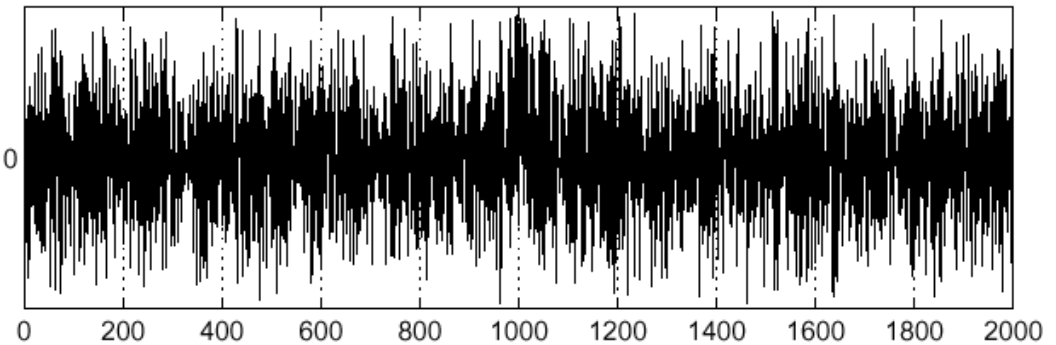


Noisy input image

$$f(x)$$

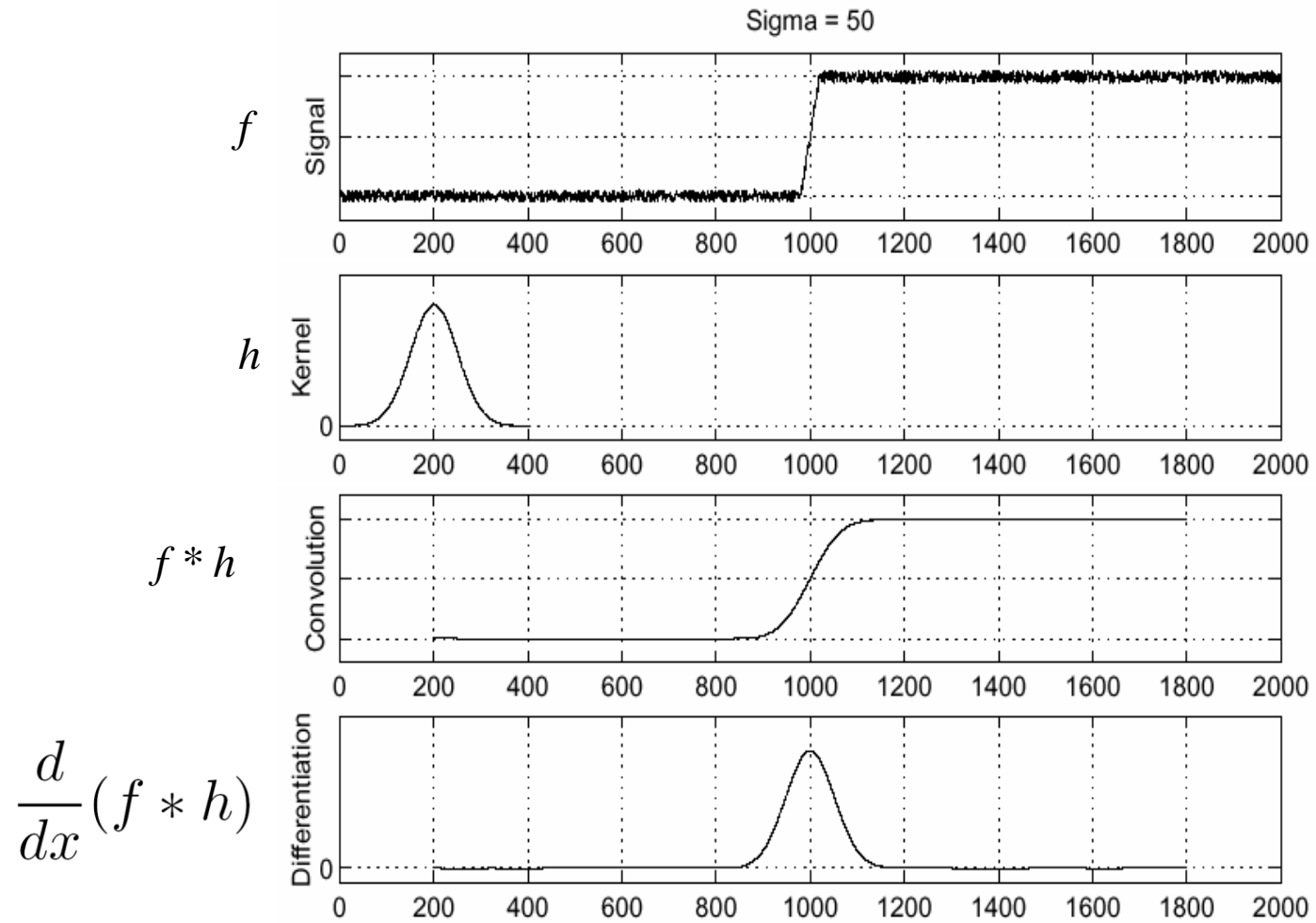


$$\frac{d}{dx} f(x)$$



Where is the edge?

Solution: smooth first

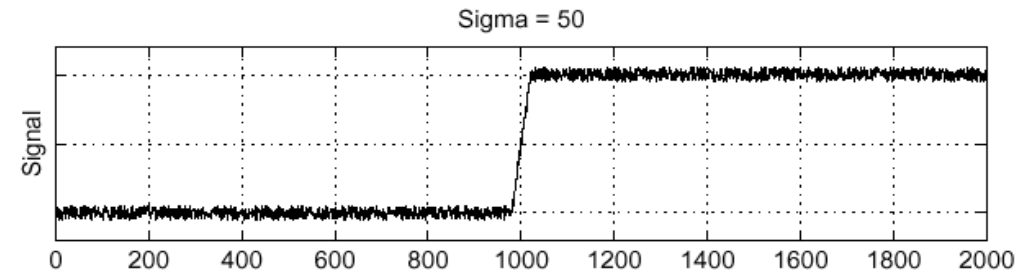


To find edges, look for peaks in $\frac{d}{dx}(f * h)$

Associative property of convolution

- Differentiation is convolution, and convolution is associative: $\frac{d}{dx}(f * h) = f * \frac{d}{dx}h$

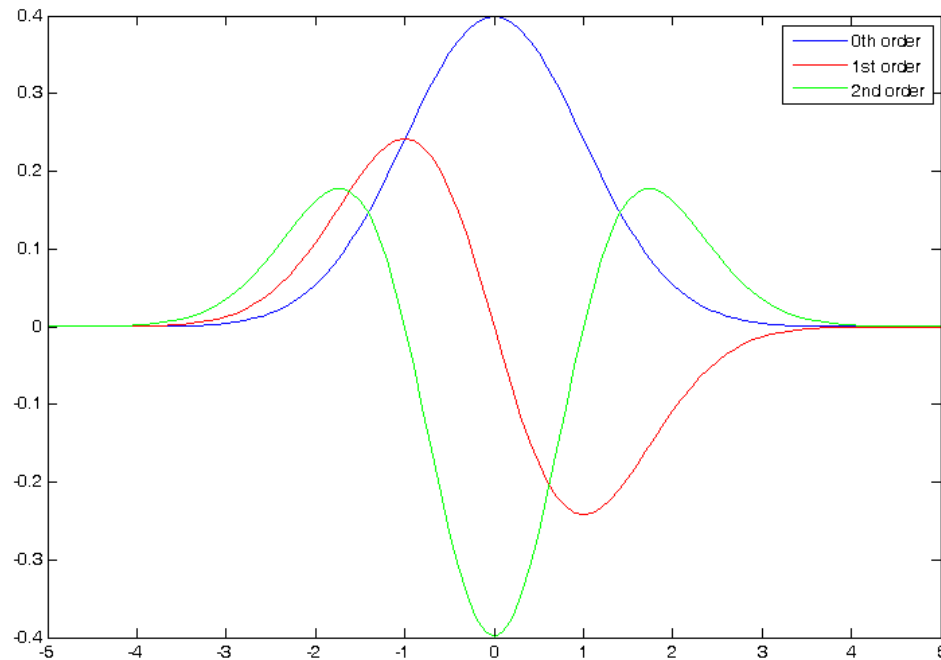
- This saves us one operation: f



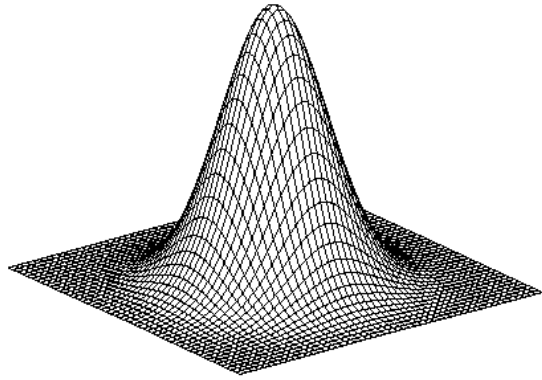
The 1D Gaussian and its derivatives

$$G_\sigma(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}$$

$$G'_\sigma(x) = \frac{d}{dx} G_\sigma(x) = -\frac{1}{\sigma} \left(\frac{x}{\sigma}\right) G_\sigma(x)$$

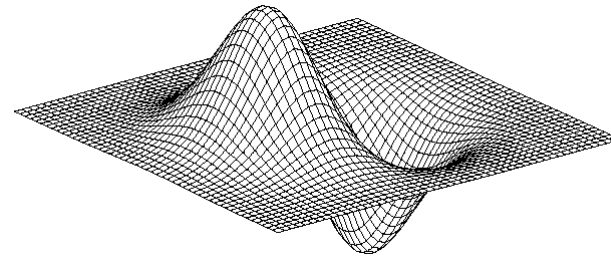


2D edge detection filters



Gaussian

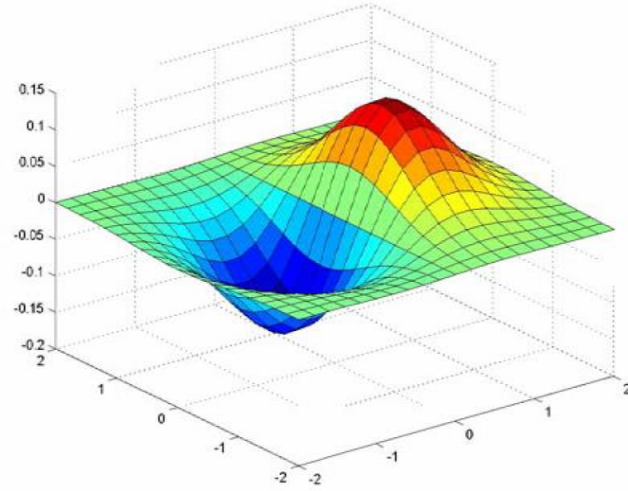
$$h_{\sigma}(u, v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$



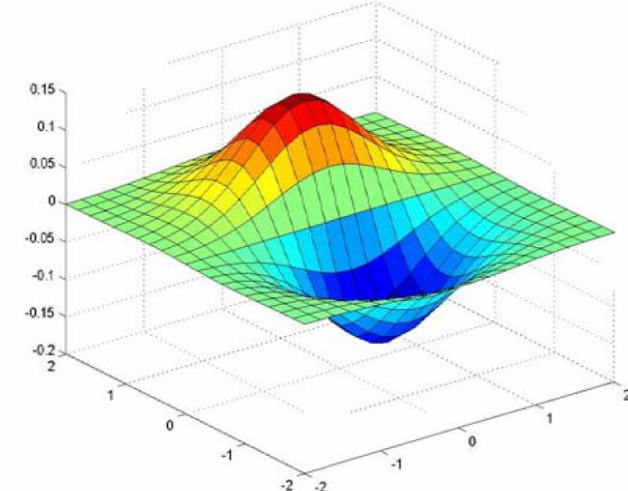
derivative of Gaussian (x)

$$\frac{\partial}{\partial x} h_{\sigma}(u, v)$$

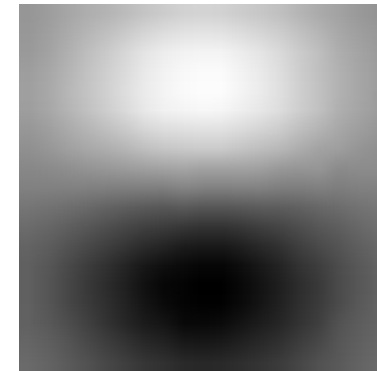
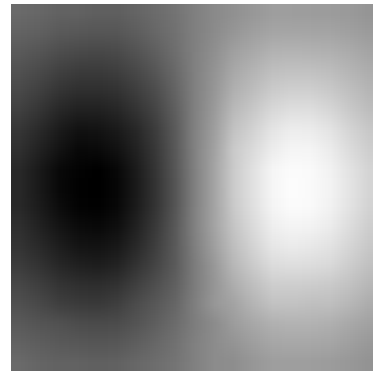
Derivative of Gaussian filter



x-direction



y-direction



The Sobel operator

- Common approximation of derivative of Gaussian

$$\frac{1}{8} \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

s_x

$$\frac{1}{8} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

s_y

- The standard definition of the Sobel operator omits the $1/8$ term
 - doesn't make a difference for edge detection
 - the $1/8$ term **is** needed to get the right gradient magnitude

Sobel operator: example

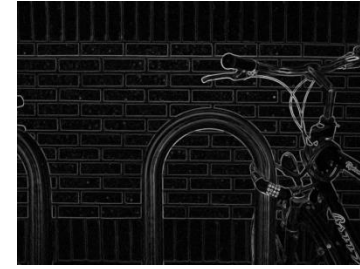
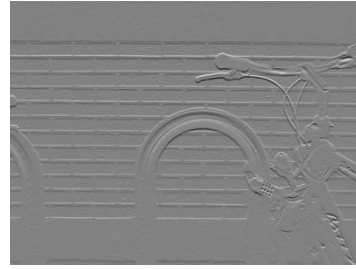
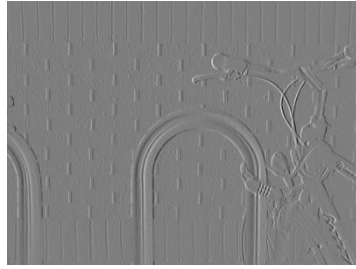
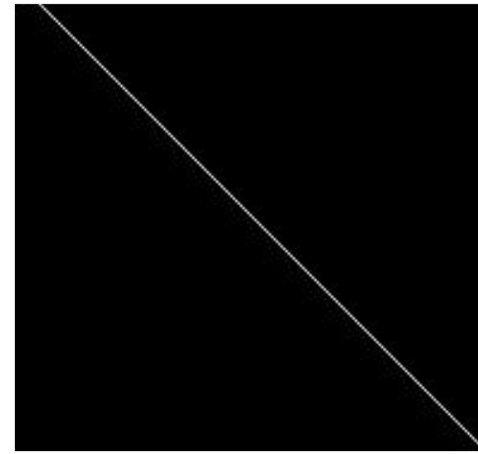




Image with Edge



Edge Location

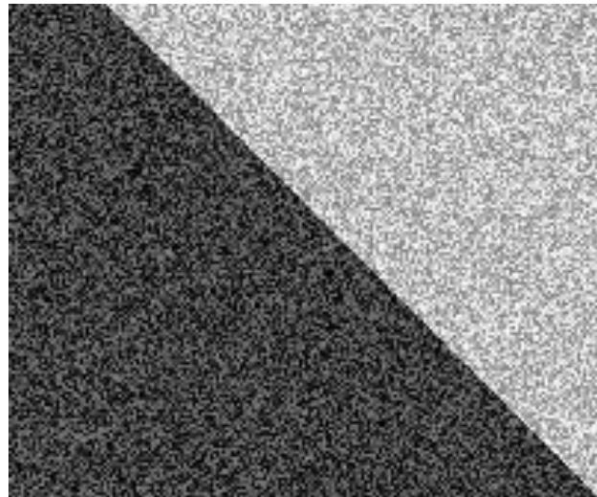
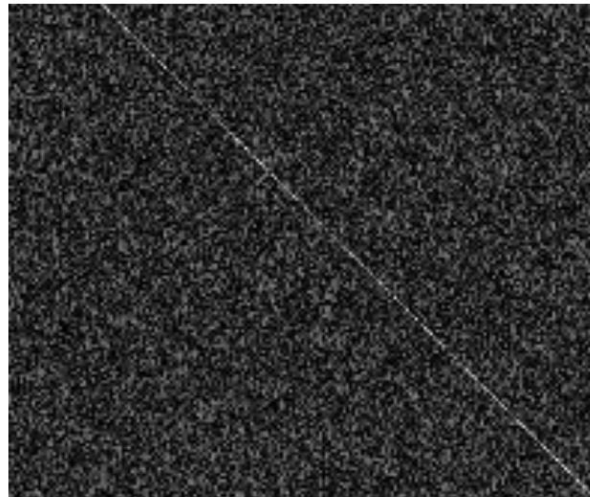
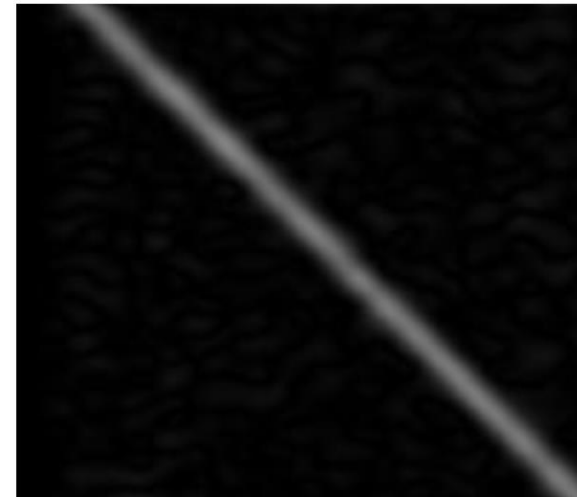


Image + Noise



Derivatives detect edge *and* noise



Smoothed derivative removes noise, but blurs edge

Example



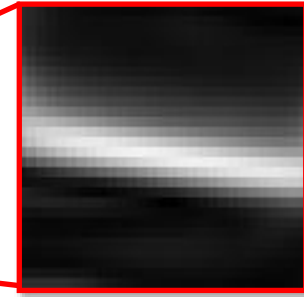
original image

Finding edges



smoothed gradient magnitude

Finding edges

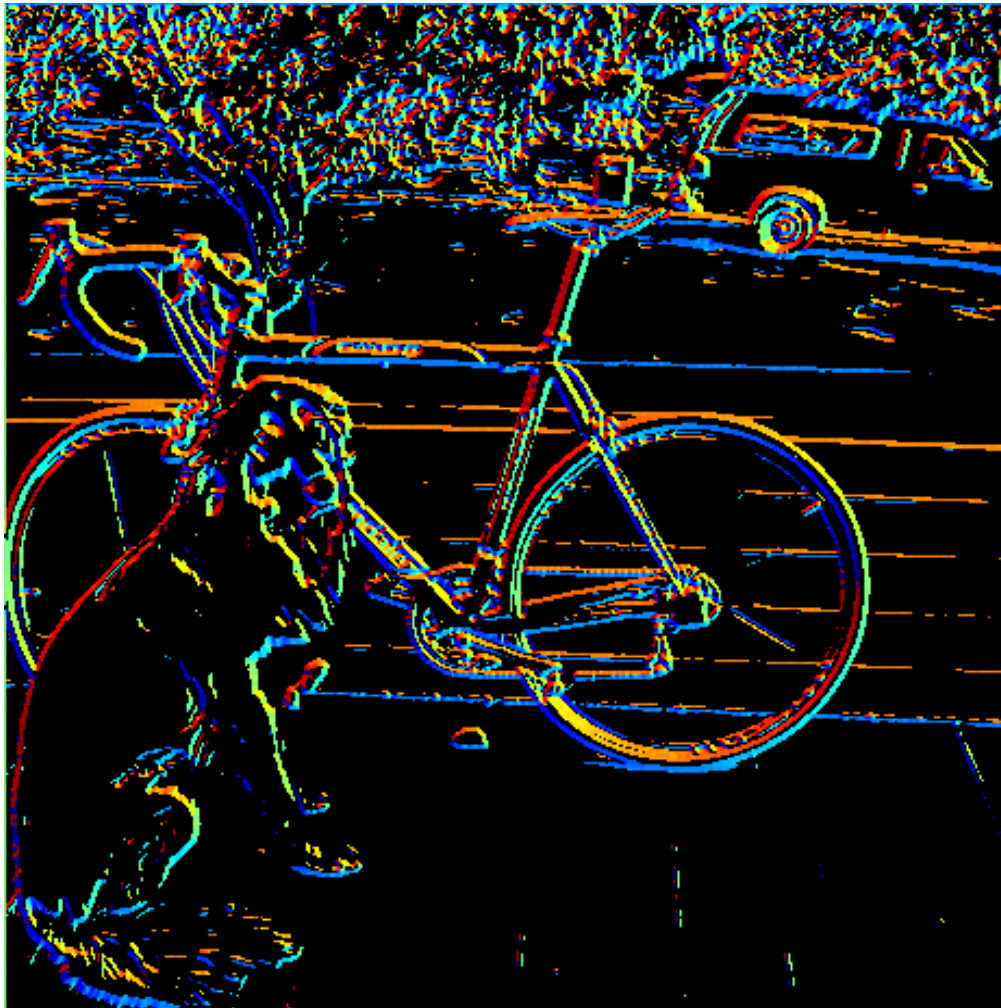


where is the edge?

thresholding

Get Orientation at Each Pixel

- Get orientation (below, threshold at minimum gradient magnitude)



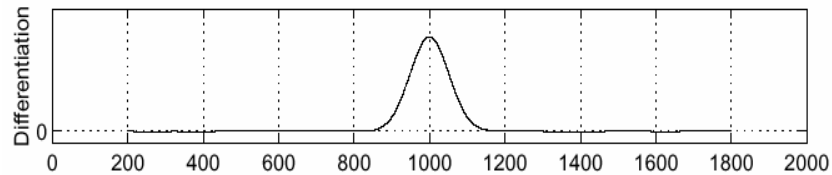
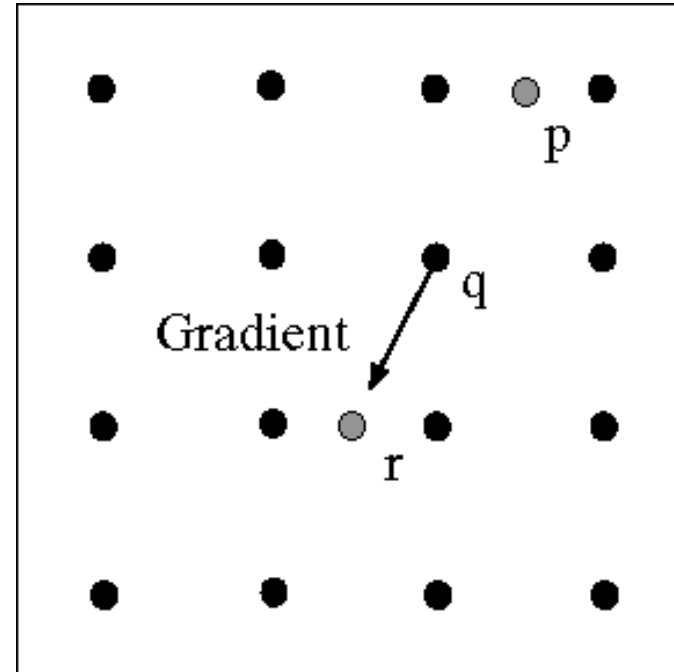
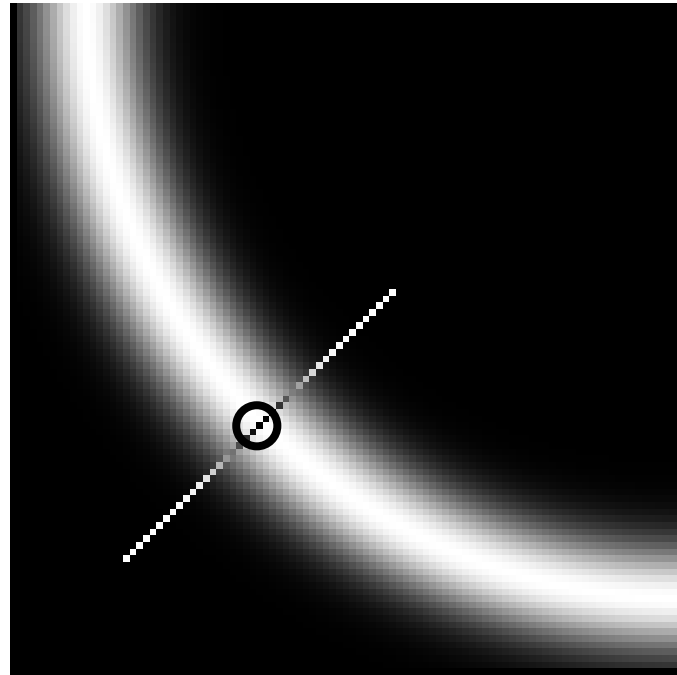
$$\text{theta} = \text{atan2}(g_y, g_x)$$

360

Gradient orientation angle

0

Non-maximum suppression



- Check if pixel is local maximum along gradient direction
 - requires *interpolating* pixels p and r

Before Non-max Suppression



After Non-max Suppression



Thresholding edges

- Still some noise
- Only want strong edges
- 2 thresholds, 3 cases
 - $R > T$: strong edge
 - $R < T$ but $R > t$: weak edge
 - $R < t$: no edge
- Why two thresholds?



Connecting edges

- Strong edges are edges!
- Weak edges are edges iff they connect to strong
- Look in some neighborhood (usually 8 closest)



Canny edge detector



MATLAB: `edge(image, 'canny')`



1. Filter image with derivative of Gaussian

2. Find magnitude and orientation of gradient



3. Non-maximum suppression

4. Linking and thresholding (hysteresis):

- Define two thresholds: low and high
- Use the high threshold to start edge curves and the low threshold to continue them



Canny edge detector

- Our first computer vision pipeline!
- Still a widely used edge detector in computer vision

J. Canny, [*A Computational Approach To Edge Detection*](#), IEEE Trans. Pattern Analysis and Machine Intelligence, 8:679-714, 1986.

- Depends on several parameters:
 - high threshold
 - low threshold
 - σ : width of the Gaussian blur

Canny edge detector



original



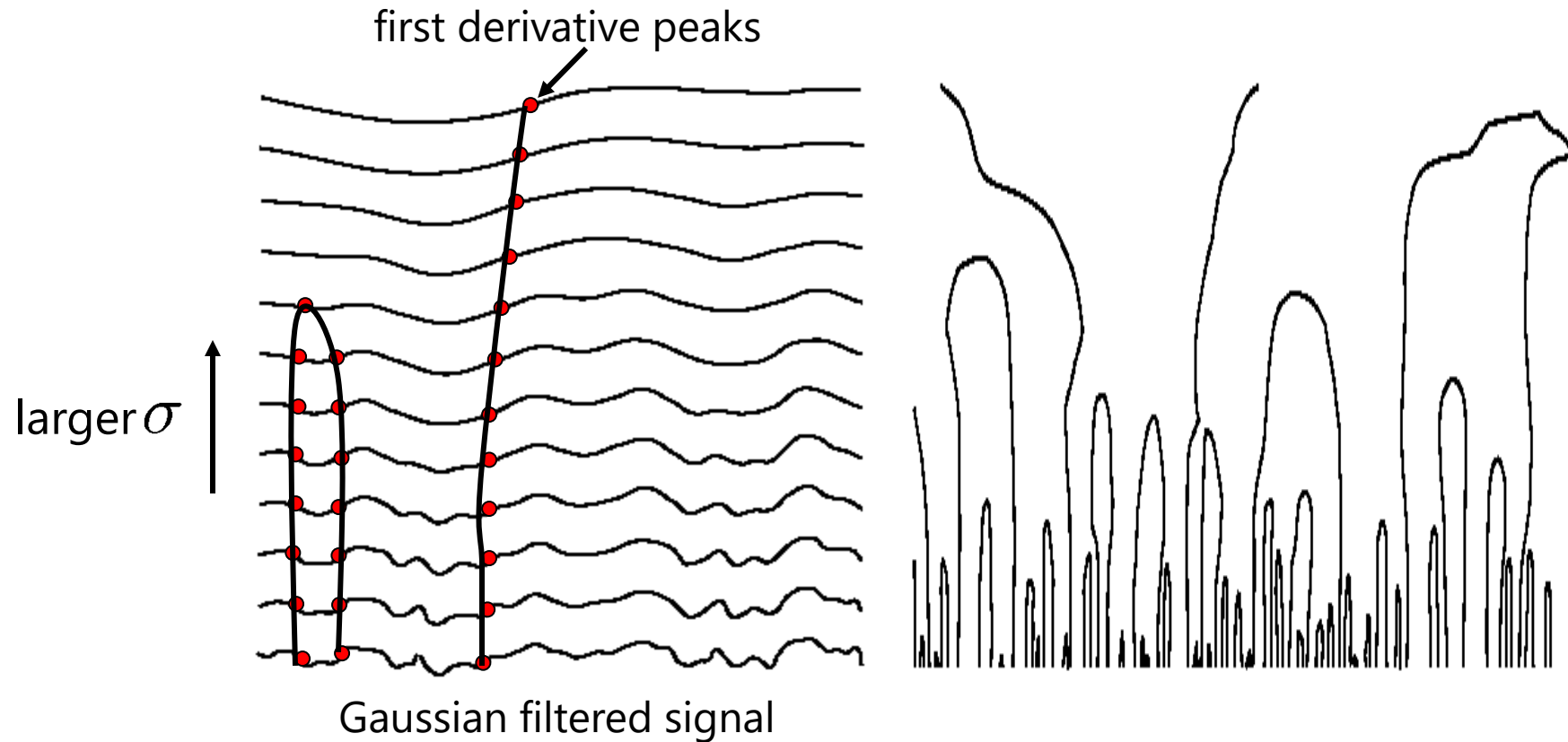
Canny with $\sigma = 1$



Canny with $\sigma = 2$

- The choice of σ depends on desired behavior
 - large σ detects “large-scale” edges
 - small σ detects fine edges

Scale space [Witkin 83]



- Properties of scale space (w/ Gaussian smoothing)
 - edge position may shift with increasing scale (σ)
 - two edges may merge with increasing scale
 - an edge may **not** split into two with increasing scale

Questions?