

CS5670: Computer Vision

Feature invariance



Reading

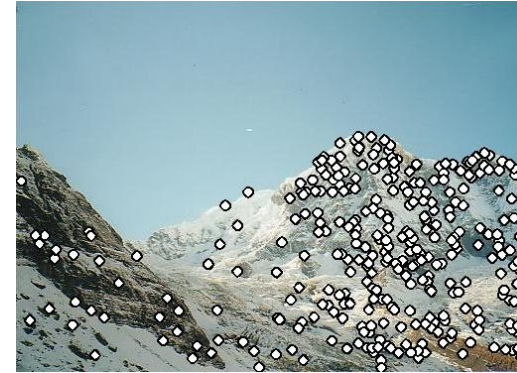
- Szeliski: 4.1

Announcements

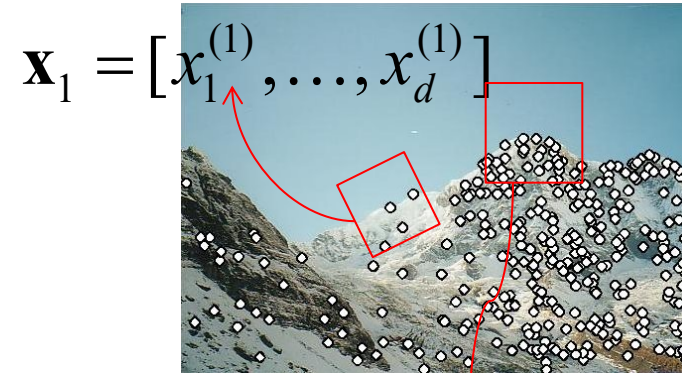
- Project 1 code due tonight at 11:59pm
- Project 1 artifact due Wednesday, 2/10, at 11:59pm
- Quiz 1 in class this Wednesday, 2/10 (first 10 minutes of class)
 - Closed book / closed note
- Project 2 (Feature Detection & Matching) will be out next week
 - To be done in groups of 2

Local features: main components

1) **Detection:** Identify the interest points



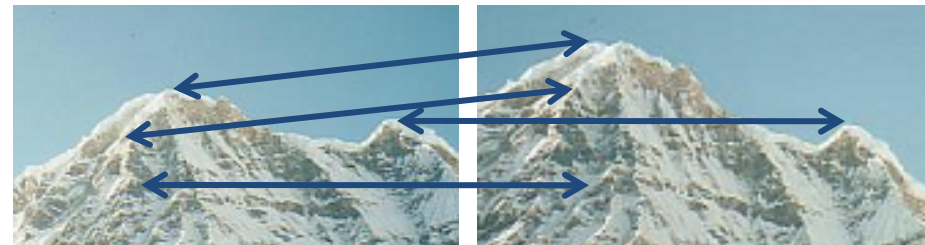
2) **Description:** Extract vector feature descriptor surrounding each interest point.



$$\mathbf{x}_1 = [x_1^{(1)}, \dots, x_d^{(1)}]$$

$$\mathbf{x}_2 = [x_1^{(2)}, \dots, x_d^{(2)}]$$

3) **Matching:** Determine correspondence between descriptors in two views



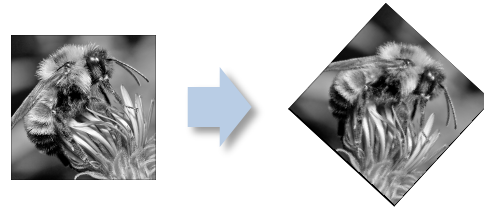
Harris features (in red)



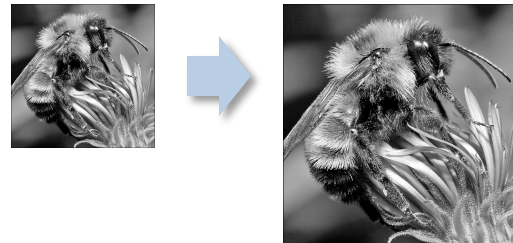
Image transformations

- Geometric

Rotation



Scale



- Photometric

Intensity change

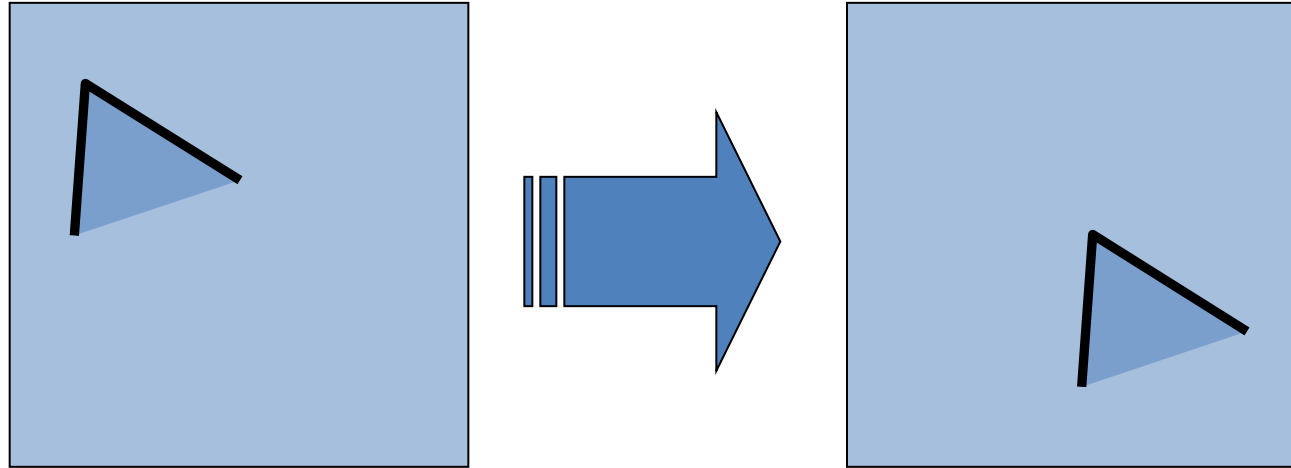


Invariance and equivariance

- We want corner locations to be *invariant* to photometric transformations and *equivariant* to geometric transformations
 - **Invariance:** image is transformed and corner locations do not change
 - **Equivariance:** if we have two transformed versions of the same image, features should be detected in corresponding locations
 - (Sometimes “invariant” and “equivariant” are both referred to as “invariant”)
 - (Sometimes “equivariant” is called “covariant”)



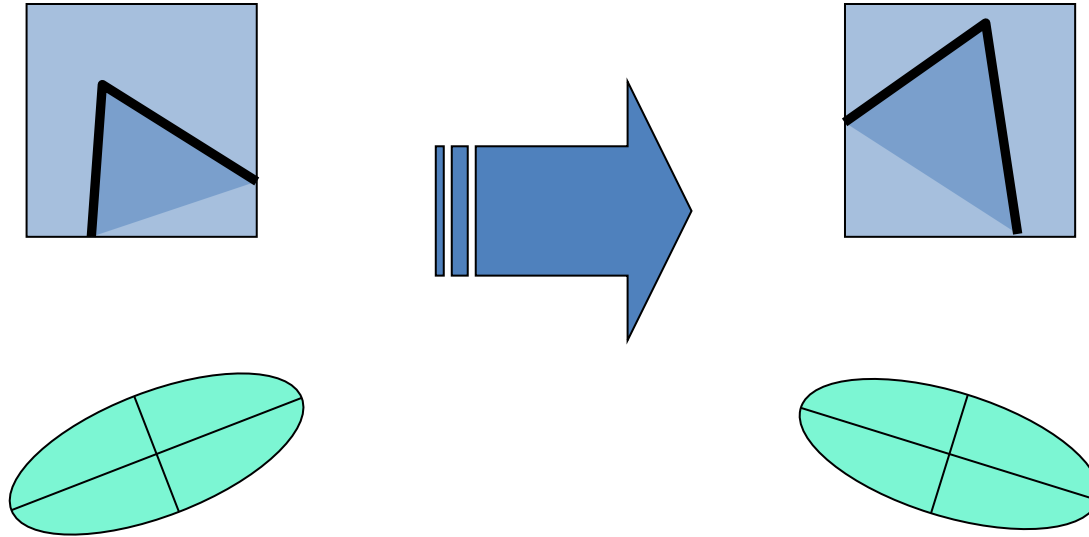
Harris detector invariance properties: image translation



- Derivatives and window function are equivariant

Corner location is equivariant w.r.t. translation

Harris detector invariance properties: image rotation



Second moment ellipse rotates but its shape (i.e. eigenvalues) remains the same

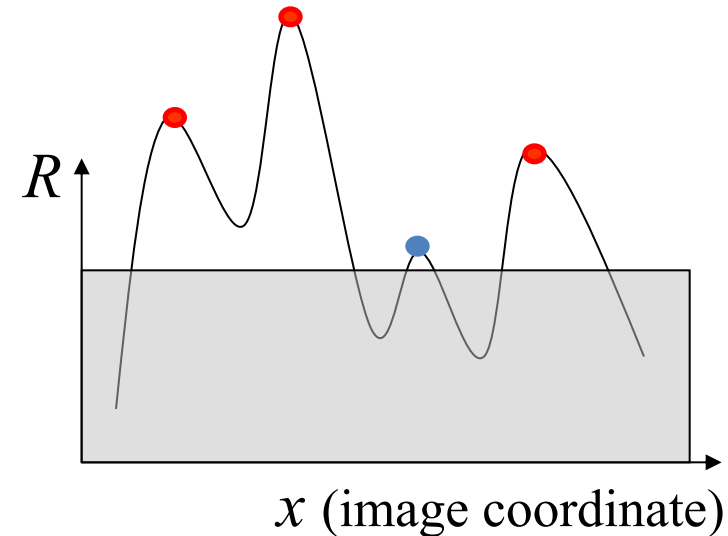
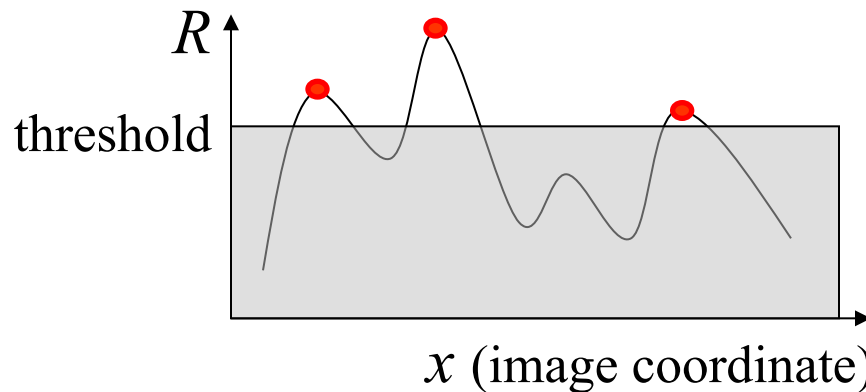
Corner location is equivariant w.r.t. image rotation

Harris detector invariance properties: Affine intensity change



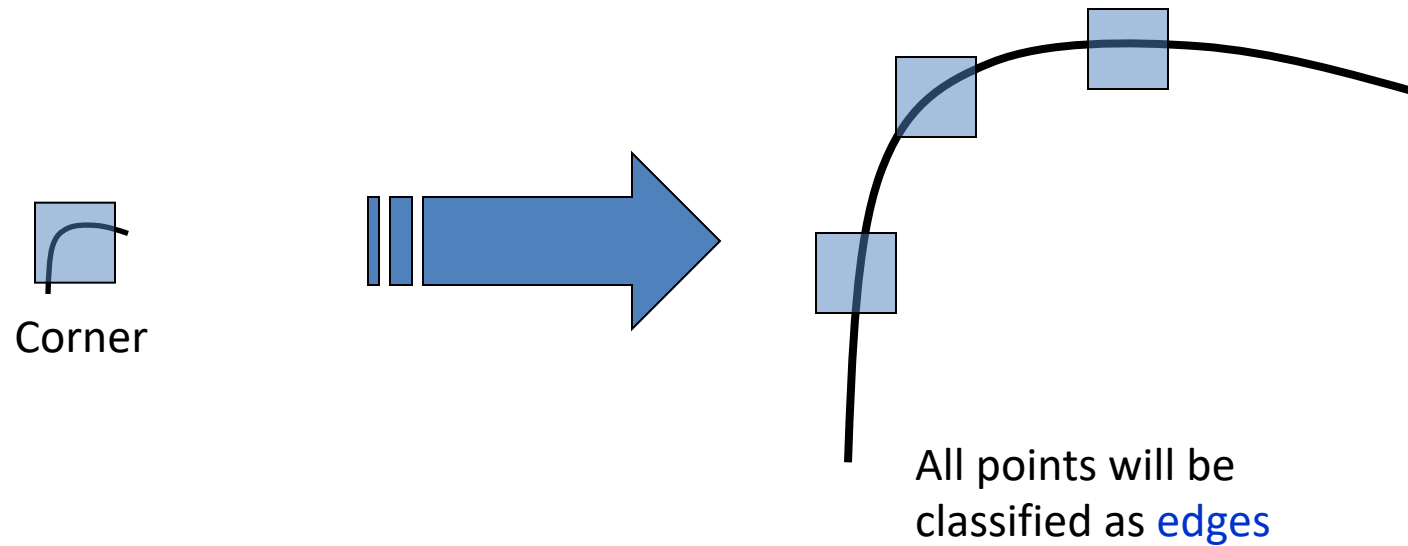
$$I \rightarrow aI + b$$

- Only derivatives are used \rightarrow invariance to intensity shift $I \rightarrow I + b$
- Intensity scaling: $I \rightarrow aI$



Partially invariant to affine intensity change

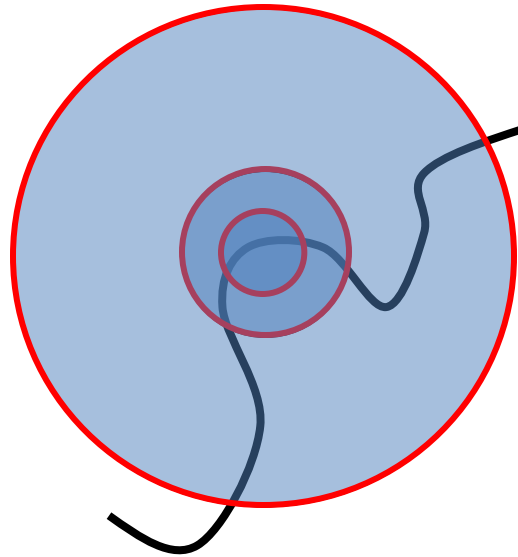
Harris detector invariance properties: scaling



Neither invariant nor equivariant to scaling

Scale invariant detection

Suppose you're looking for corners

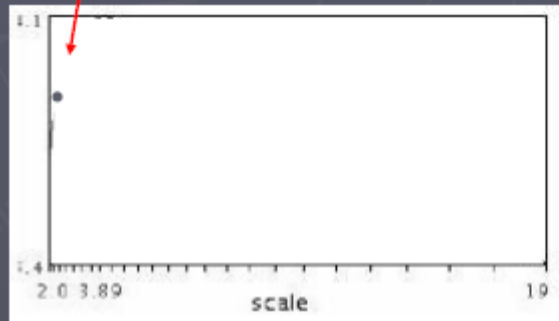


Key idea: find scale that gives local maximum of f

- in both position and scale
- One definition of f : the Harris operator

Automatic scale selection

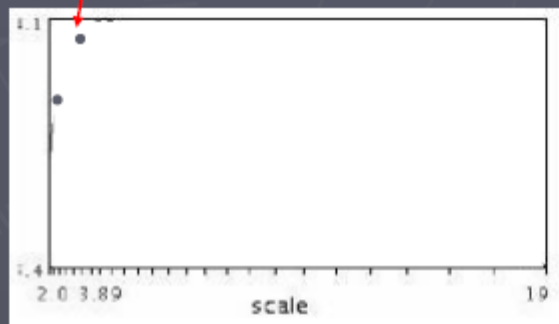
Lindeberg et al., 1996



$$f(I_{i_1...i_m}(x, \sigma))$$

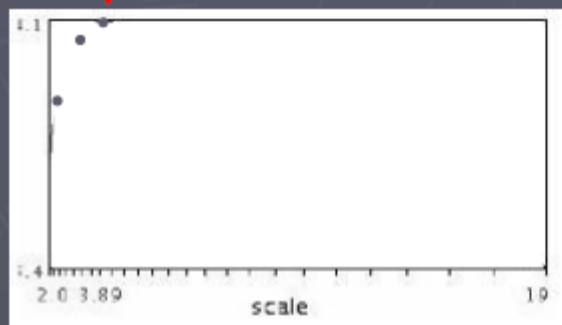
Slide from Tinne Tuytelaars

Automatic scale selection



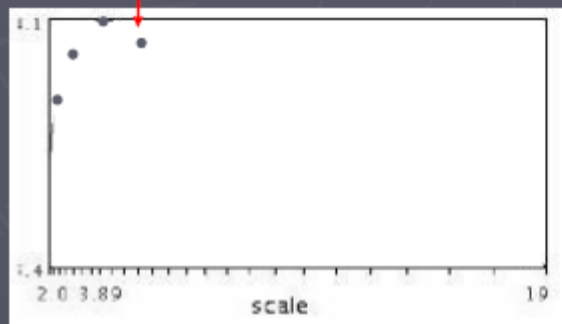
$$f(I_{i_1..i_m}(x, \sigma))$$

Automatic scale selection



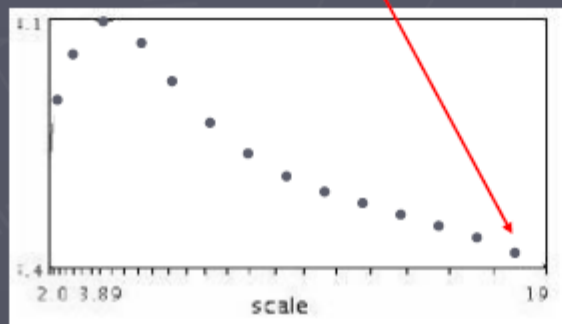
$$f(I_{i_1..i_m}(x, \sigma))$$

Automatic scale selection



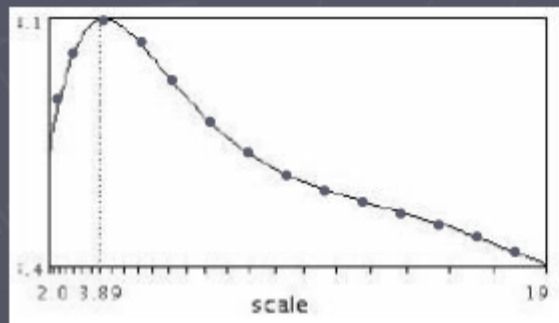
$$f(I_{i_1..i_m}(x, \sigma))$$

Automatic scale selection



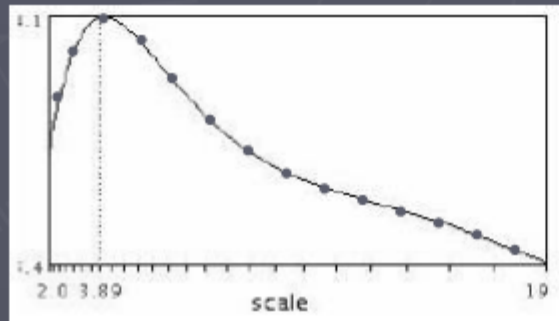
$$f(I_{i_1 \dots i_m}(x, \sigma))$$

Automatic scale selection

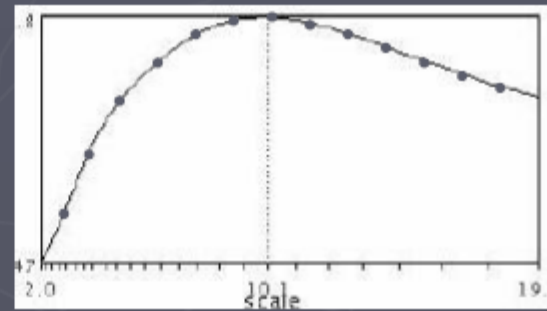


$$f(I_{i_1 \dots i_m}(x, \sigma))$$

Automatic scale selection



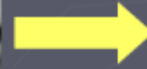
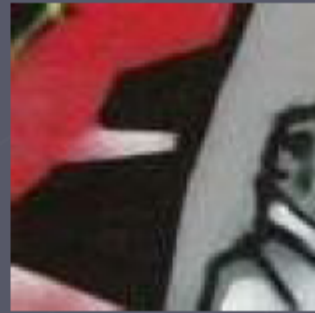
$$f(I_{i_1 \dots i_m}(x, \sigma))$$



$$f(I_{i_1 \dots i_m}(x', \sigma'))$$

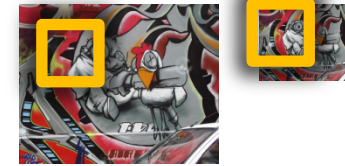
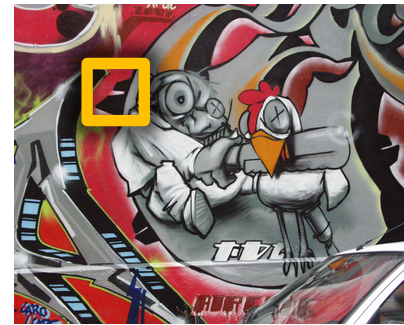
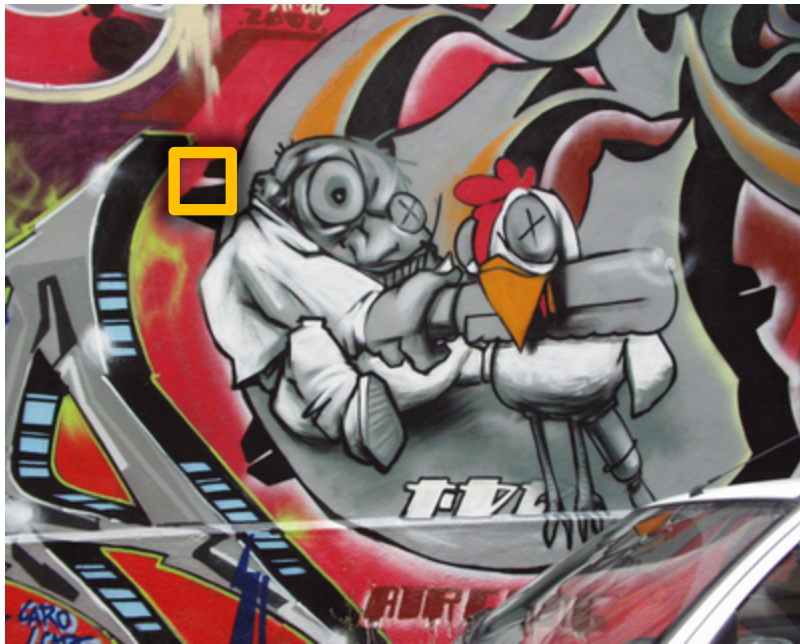
Automatic scale selection

Normalize: rescale to fixed size



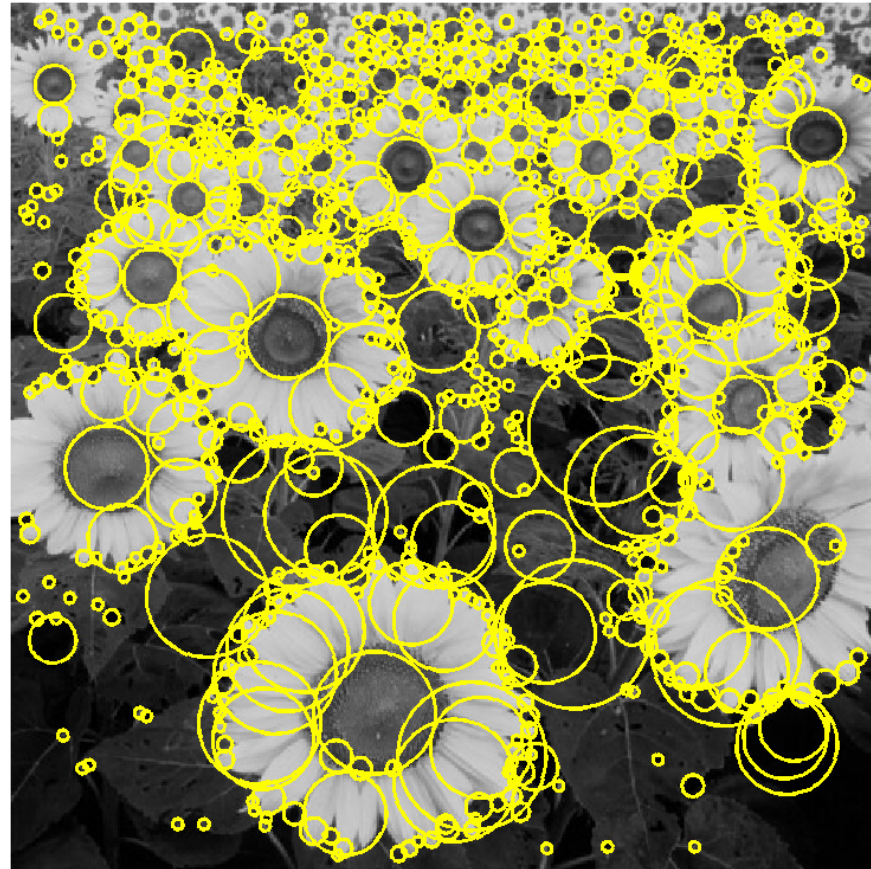
Implementation

- Instead of computing f for larger and larger windows, we can implement using a fixed window size with a Gaussian pyramid



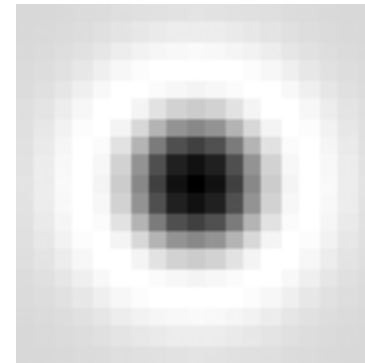
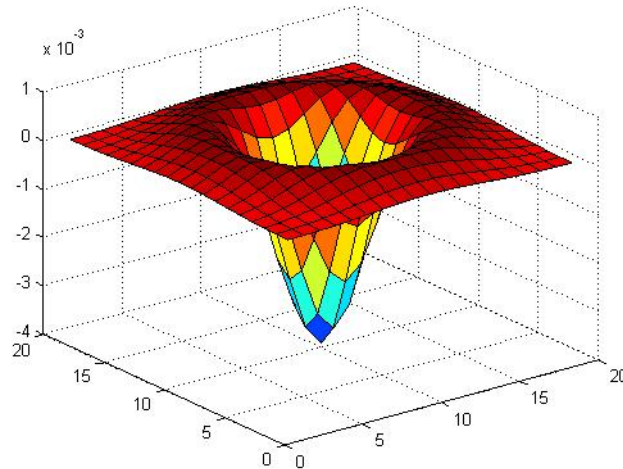
(sometimes need to create in-between levels, e.g. a $\frac{3}{4}$ -size image)

Feature extraction: Corners and blobs



Another common definition of f

- The *Laplacian of Gaussian (LoG)*

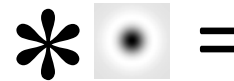


$$\nabla^2 g = \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2}$$

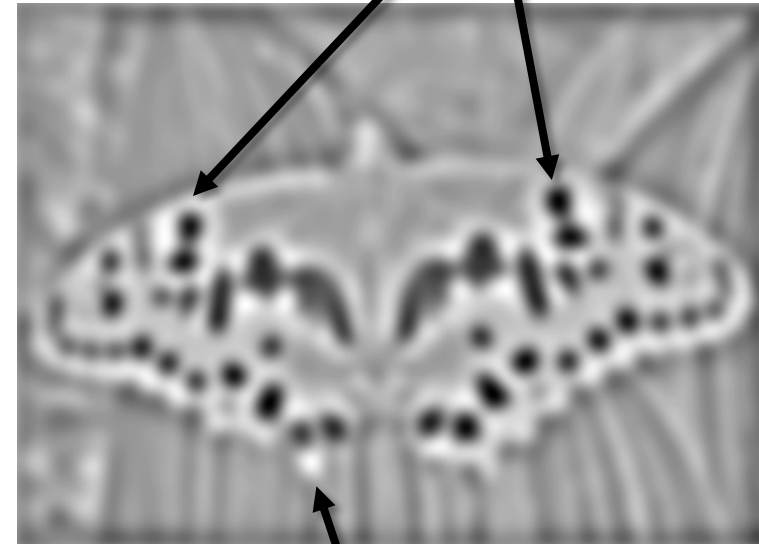
(very similar to a Difference of Gaussians (DoG) –
i.e. a Gaussian minus a slightly smaller Gaussian)

Laplacian of Gaussian

- “Blob” detector



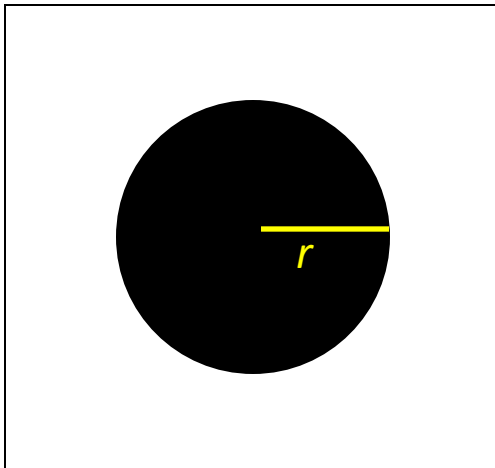
=



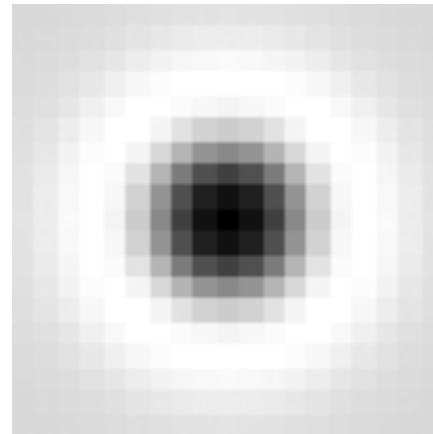
- Find maxima *and minima* of LoG operator in space and scale

Scale selection

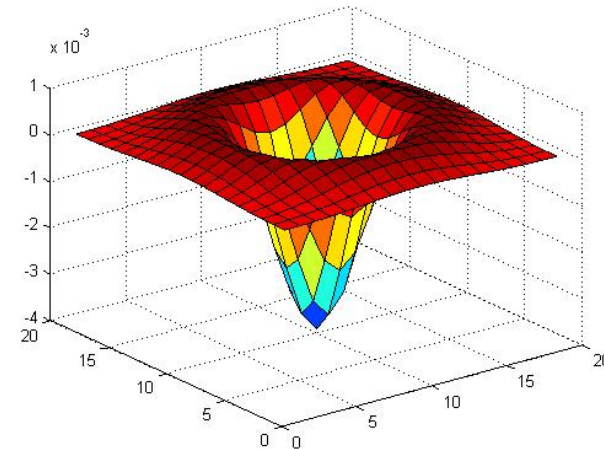
- At what scale does the Laplacian achieve a maximum response for a binary circle of radius r ?



image

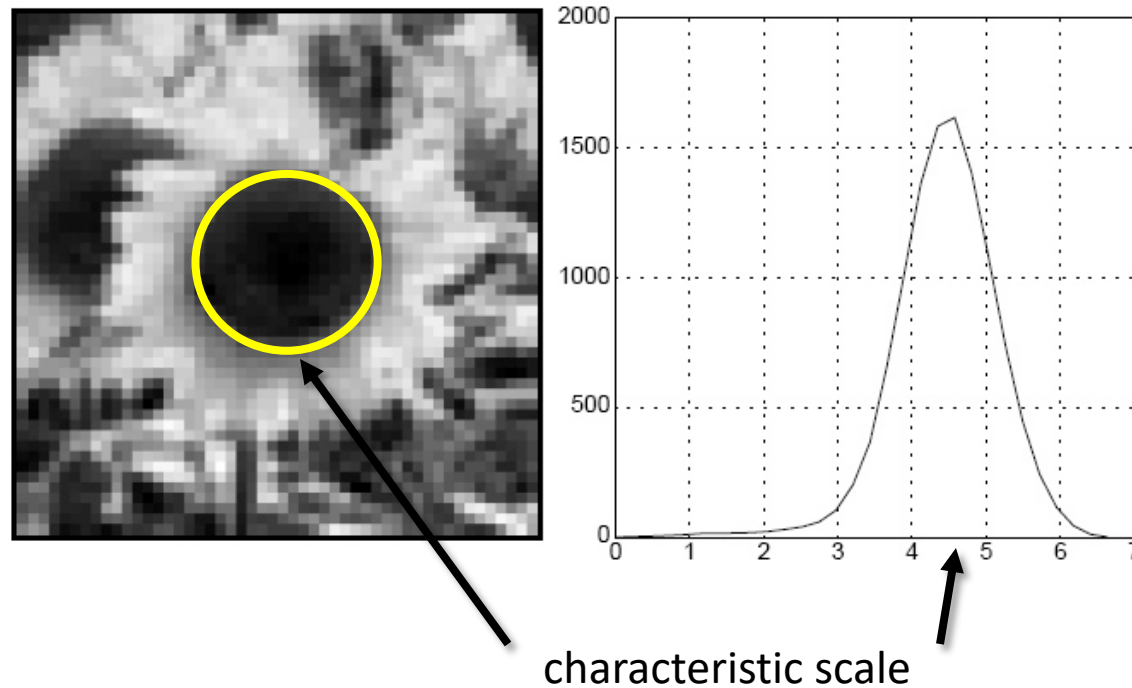


Laplacian



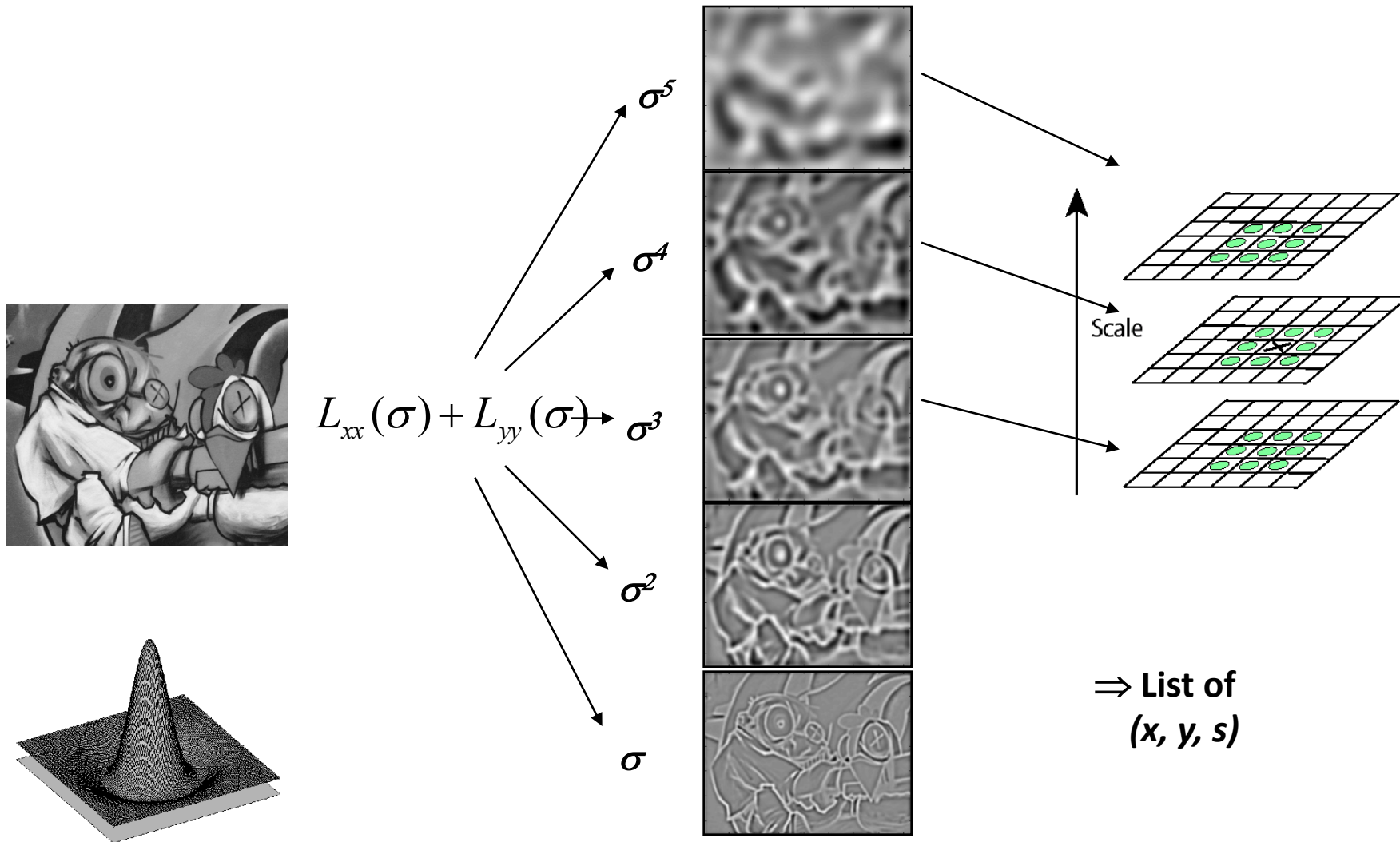
Characteristic scale

- We define the characteristic scale as the scale that produces peak of Laplacian response



T. Lindeberg (1998). ["Feature detection with automatic scale selection."](#)
International Journal of Computer Vision **30** (2): pp 77--116.

Find local maxima in 3D position-scale space



Scale-space blob detector: Example

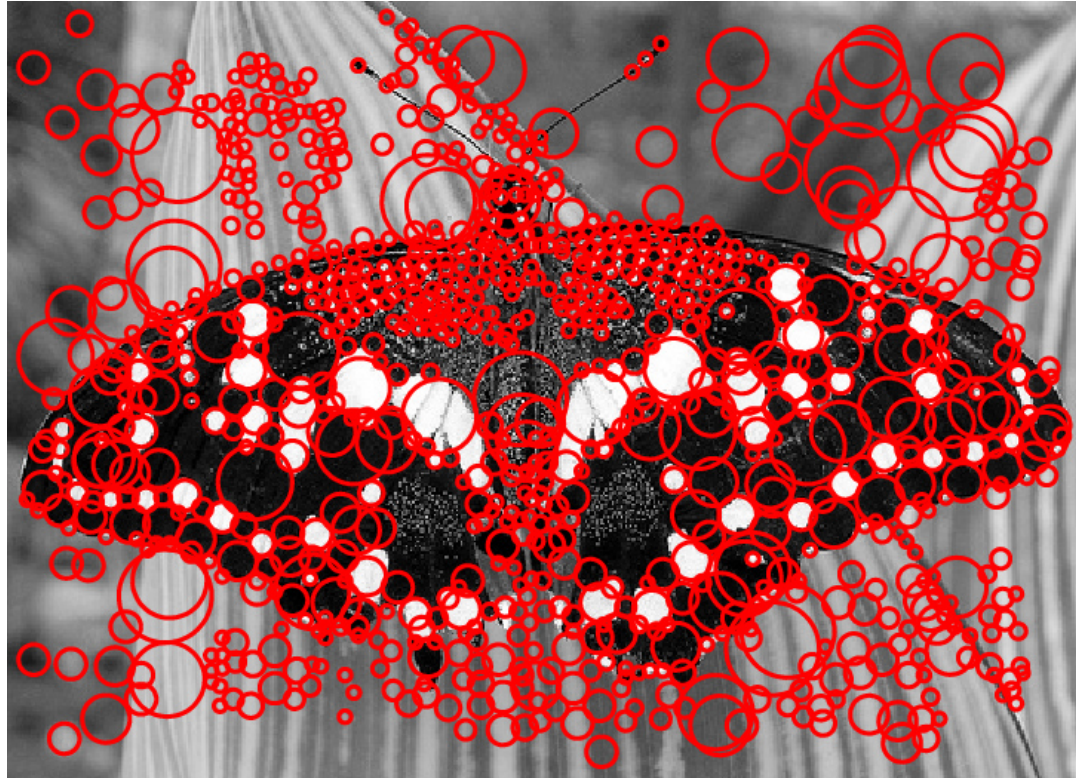


Scale-space blob detector: Example



sigma = 11.9912

Scale-space blob detector: Example



Scale Invariant Detection

- Functions for determining scale

$$f = \text{Kernel} * \text{Image}$$

Kernels:

$$\nabla^2 g = \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2}$$

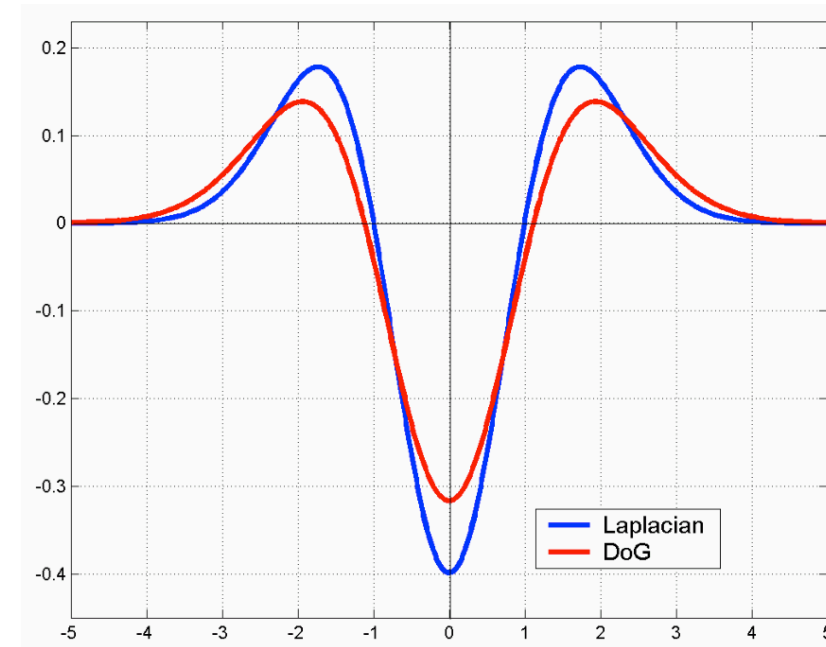
(Laplacian)

$$\text{DoG} = G(x, y, k\sigma) - G(x, y, \sigma)$$

(Difference of Gaussians)

where Gaussian

$$G(x, y, \sigma) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2 + y^2}{2\sigma^2}}$$



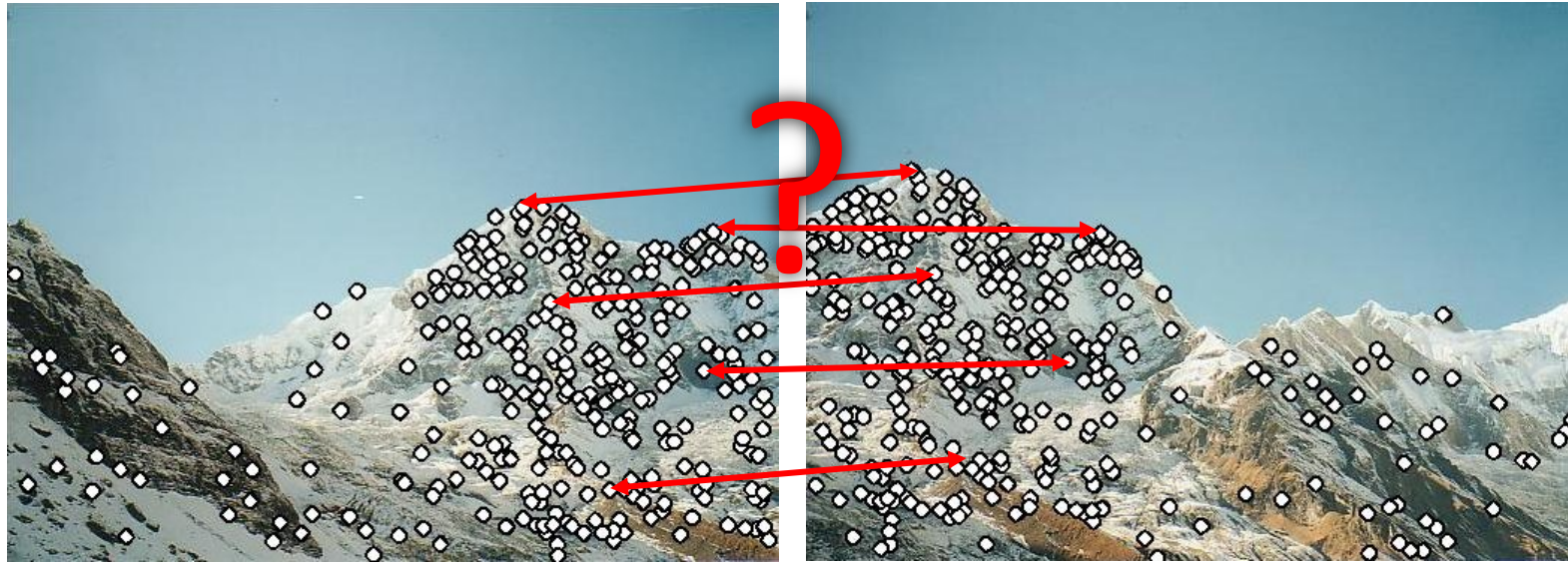
Note: The LoG and DoG operators are both rotation equivariant

Questions?

Feature descriptors

We know how to detect good points

Next question: **How to match them?**



Answer: Come up with a *descriptor* for each point,
find similar descriptors between the two images