CS5670: Computer Vision

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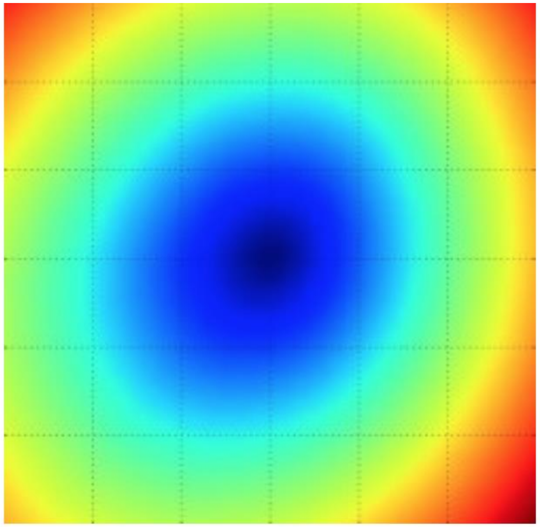
Lecture 23: Optimization and Neural Nets



Slides from Andrej Karpathy and Fei-Fei Li
http://vision.stanford.edu/teaching/cs231n/

Today

- Optimization
- Today and Thursday: Neural nets, CNNs
  - Mon: http://cs231n.github.io/classification/
  - Wed: http://cs231n.github.io/linear-classify/
  - Today:
    - http://cs231n.github.io/optimization-1/
    - http://cs231n.github.io/optimization-2/

Announcements

- Final project (P5) released, due Tuesday, 5/9, by 11:59pm, to be done in groups of two
- Final exam will be handed out in class Tuesday, due Friday, 5/12, by 5pm
- Project 3 voting results

Third Place

Boting Li and Ran Godrich

Second Place

Arpit Sabherwal and Jaldeep Acharya

First Place

Hong Gan and Renkai Xiang

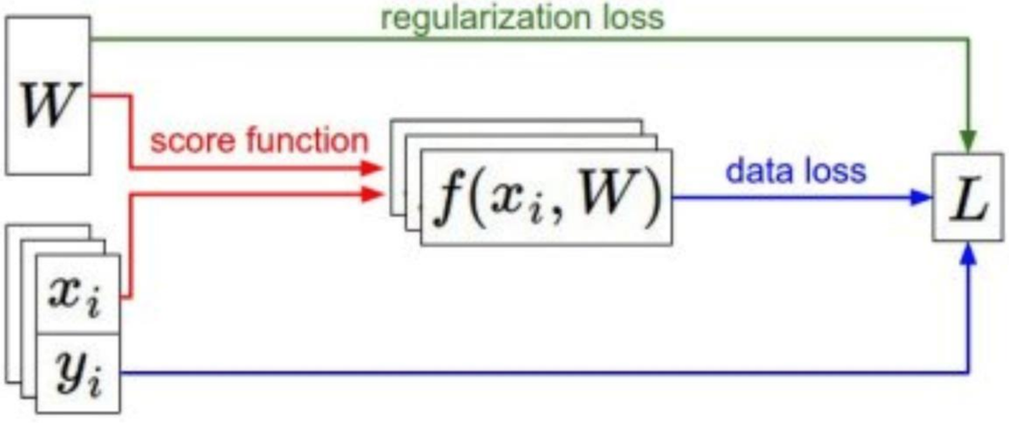
Summary

1. Score function

$$f(x_i, W, b) = Wx_i + b$$

2. Loss function

$$L = \frac{1}{N} \sum_i \sum_{j \neq y_i} \left[\max(0, f(x_i; W)_j - f(x_i; W)_{y_i} + \Delta) \right] + \lambda R(W)$$



Other loss functions

- Scores are not very intuitive

- Softmax classifier
  - Score function is same
  - Intuitive output: normalized class probabilities
  - Extension of logistic regression to multiple classes

Softmax classifier

$$f(x_i, W) = W x_i$$

score function is the same

$$\boxed{\frac{e^{f_{y_i}}}{\sum_j e^{f_j}}}$$

softmax function

$$[1, -2, 0] \to [e^1, e^{-2}, e^0] = [2.71, 0.14, 1] \to [0.7, 0.04, 0.26]$$

Interpretation: squashes values into range 0 to 1

$$P(y_i \mid x_i; W)$$

Cross-entropy loss

$$f(x_i, W) = Wx_i$$

score function is the same

$$L_i = -\log\left(\frac{e^{f_{y_i}}}{\sum_j e^{f_j}}\right)$$

$$L_i = -f_{y_i} + \log \sum_j e^{f_j}$$

$$\boxed{P(y_i \mid x_i; W)}$$

i.e. we're minimizing the negative log likelihood.

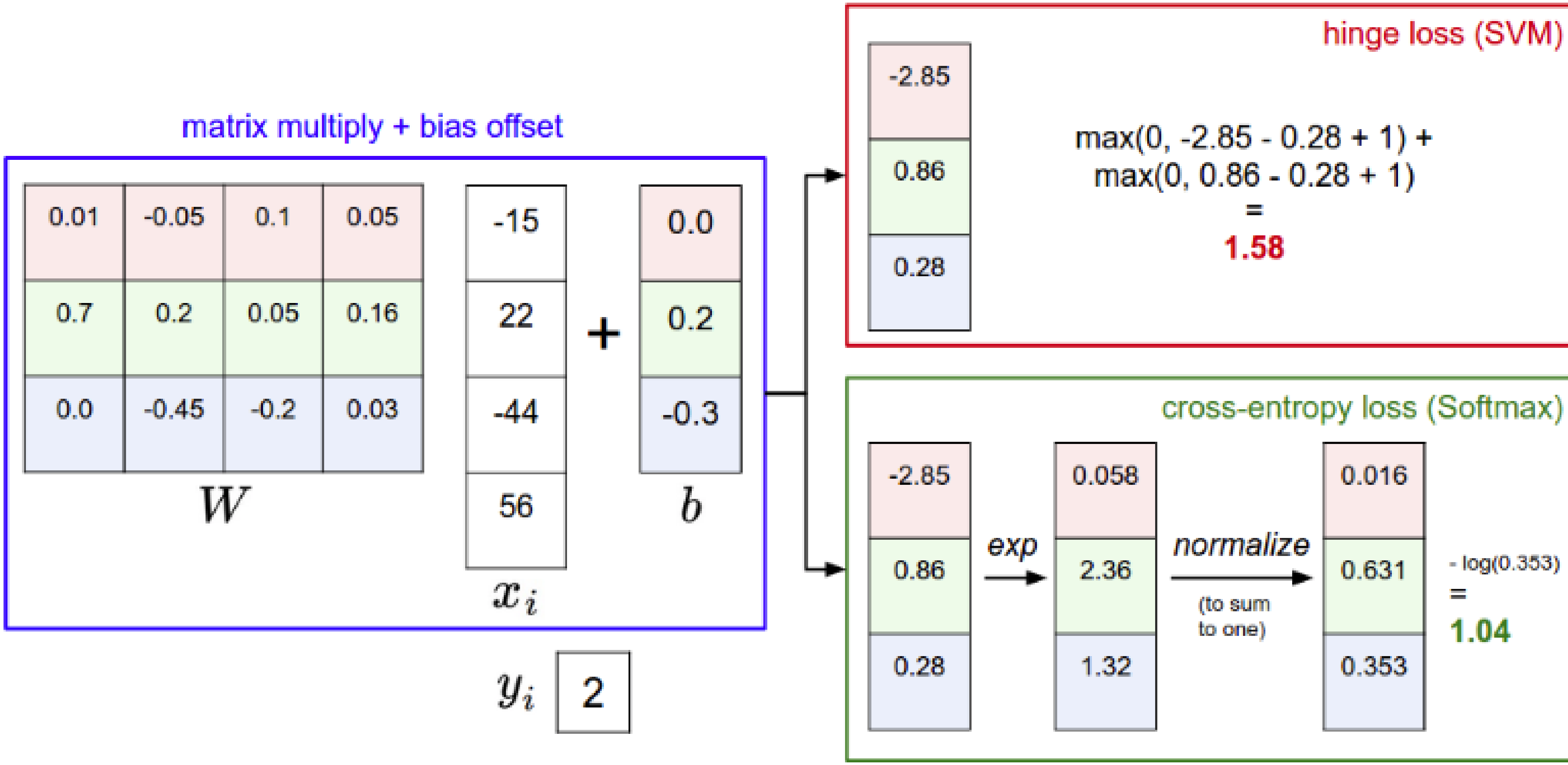
Aside: Loss function interpretation

- Probability
  - Maximum Likelihood Estimation (MLE)
  - Regularization is Maximum a posteriori (MAP) estimation

$$H(p, q) = -\sum_x p(x) \log q(x)$$

- Cross-entropy H
  - p is true distribution (1 for the correct class), q is estimated
  - Softmax classifier minimizes cross-entropy
  - Minimizes the KL divergence (Kullback-Leibler) between the distribution: distance between p and q

SVM vs. Softmax



Example of the difference between the SVM and Softmax classifiers for one datapoint. In both cases we compute the same score vector **f** (e.g. by matrix multiplication in this section). The difference is in the interpretation of the scores in **f**: The SVM interprets these as class scores and its loss function encourages the correct class (class 2, in blue) to have a score higher by a margin than the other class scores. The Softmax classifier instead interprets the scores as (unnormalized) log probabilities for each class and then encourages the (normalized) log probability of the correct class to be high (equivalently the negative of it to be low). The final loss for this example is 1.58 for the SVM and 1.04 for the Softmax classifier, but note that these numbers are not comparable; They are only meaningful in relation to loss computed within the same classifier and with the same data.

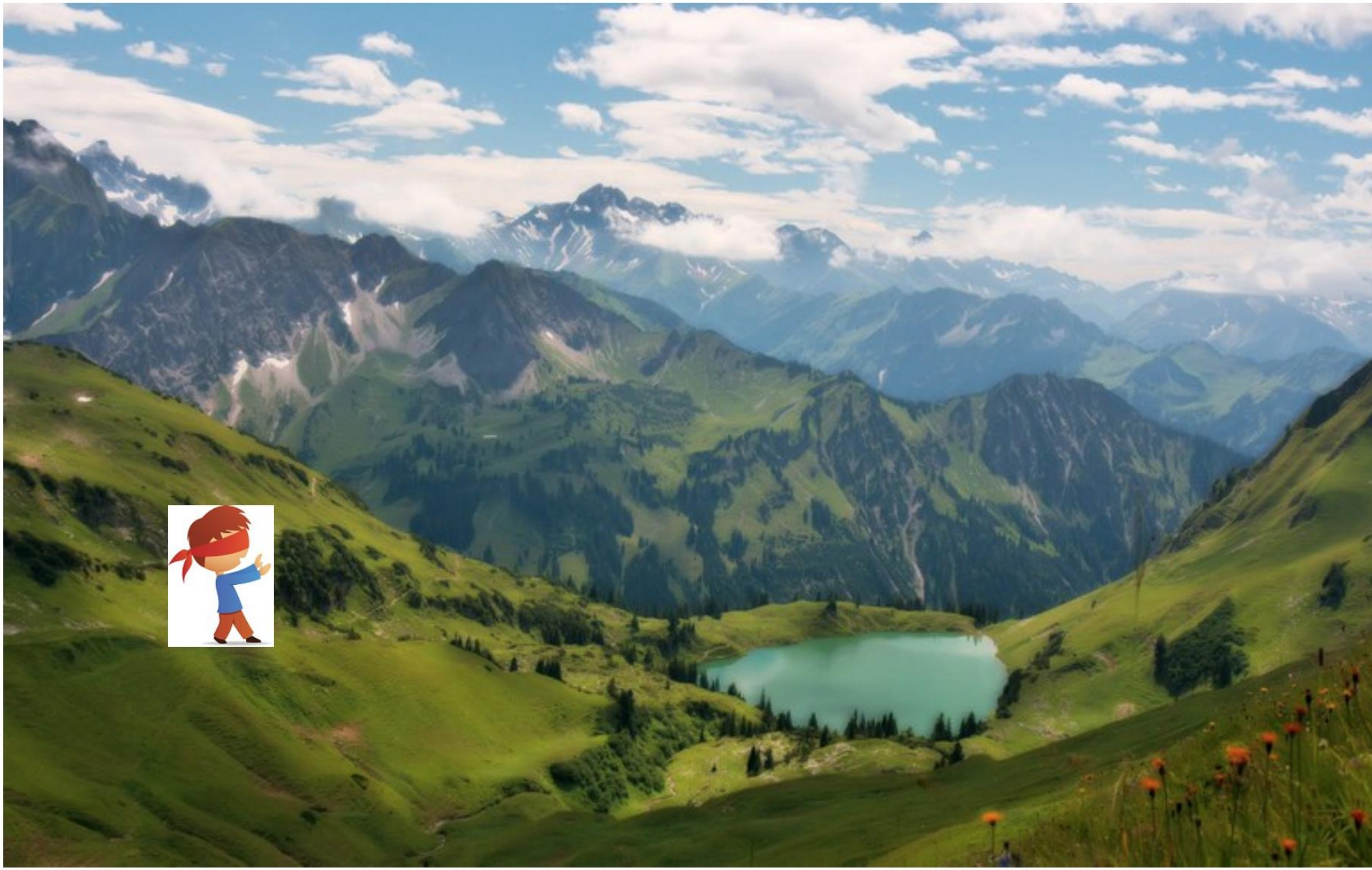
Summary

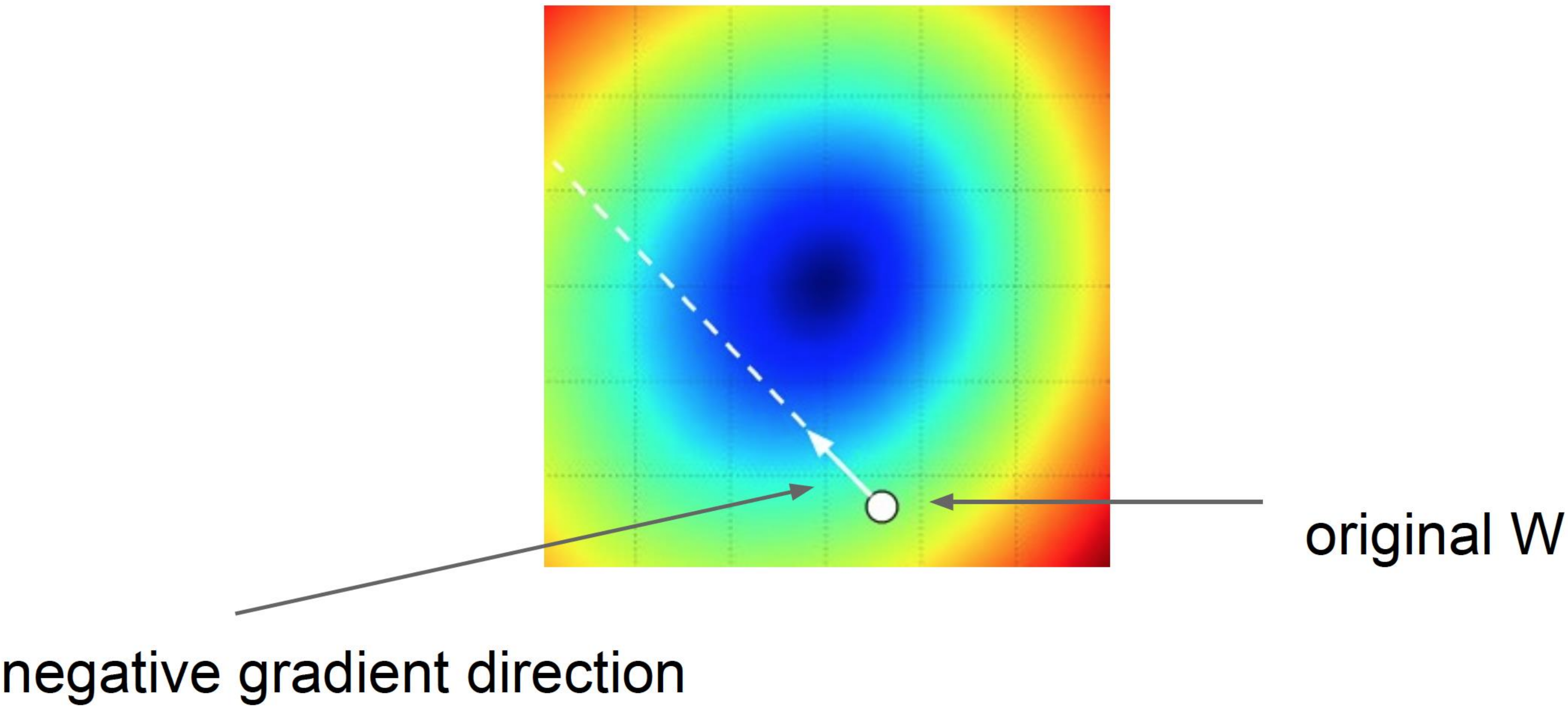
- Have score function and loss function
  - Will generalize the score function
- Find W and b to minimize loss
  - SVM vs. Softmax
    - Comparable in performance
    - SVM satisfies margins, softmax optimizes probabilities

$$L = \frac{1}{N} \sum_i \sum_{j \neq y_i} \left[\max(0, f(x_i; W)_j - f(x_i; W)_{y_i} + \Delta) \right] + \lambda \sum_k \sum_l W_{k,l}^2$$

$$L = \frac{1}{N} \sum_i -\log \left(\frac{e^{f_{y_i}}}{\sum_j e^{f_j}} \right) + \lambda \sum_k \sum_l W_{k,l}^2$$

Gradient Descent





Step size: learning rate
Too big: will miss the minimum
Too small: slow convergence

Analytic Gradient

$$L_i = \sum_{j \neq y_i} \left[\max(0, w_j^T x_i - w_{y_i}^T x_i + 1) \right]$$

$$\nabla_{w_j} L_i = 1(w_j^T x_i - w_{y_i}^T x_i + \Delta > 0) x_i$$

$$\nabla_{w_{y_i}} L_i = -\left(\sum_{j \neq y_i} 1(w_j^T x_i - w_{y_i}^T x_i + \Delta > 0) \right) x_i$$

Full gradient is the sum of all L_is over all training examples x_i

In summary:

- Numerical gradient: approximate, slow, easy to write
- Analytic gradient: exact, fast, error-prone

=>

In practice: Always use analytic gradient, but check implementation with numerical gradient. This is called a **gradient check.**

Gradient Descent

```
# Vanilla Gradient Descent

while True:
  weights_grad = evaluate_gradient(loss_fun, data, weights)
  weights += - step_size * weights_grad # perform parameter update
```

Mini-batch Gradient Descent

- only use a small portion of the training set to compute the gradient.

```
# Vanilla Minibatch Gradient Descent

while True:
  data_batch = sample_training_data(data, 256) # sample 256 examples
  weights_grad = evaluate_gradient(loss_fun, data_batch, weights)
  weights += - step_size * weights_grad # perform parameter update
```

Common mini-batch sizes are ~100 examples.
e.g. Krizhevsky ILSVRC ConvNet used 256 examples

Stochastic Gradient Descent (SGD)

- use a single example at a time

1

```
# Vanilla Minibatch Gradient Descent

while True:
  data_batch = sample_training_data(data, 256) # sample 256 examples
  weights_grad = evaluate_gradient(loss_fun, data_batch, weights)
  weights += - step_size * weights_grad # perform parameter update
```

(also sometimes called **on-line** Gradient Descent)

Summary

- Always use mini-batch gradient descent
- Incorrectly refer to it as "doing SGD" as everyone else

  (or call it batch gradient descent)
- The mini-batch size is a hyperparameter, but it is not very common to cross-validate over it (usually based on practical concerns, e.g. space/time efficiency)

The dynamics of Gradient Descent

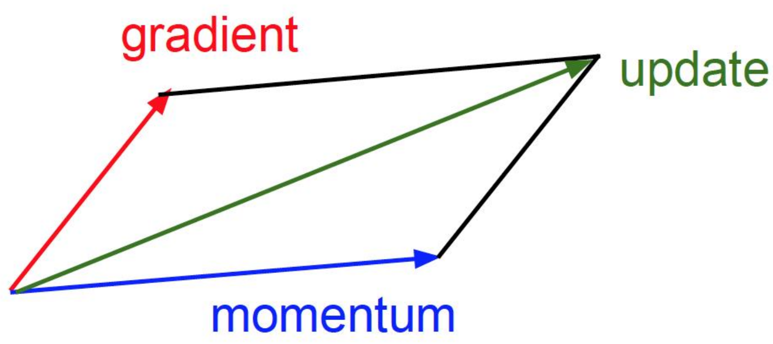
pull some weights up and some down

$$L = \boxed{\frac{1}{N}\sum_i \sum_{j \neq y_i} \left[\max(0, f(x_i; W)_j - f(x_i; W)_{y_i} + \Delta)\right]} + \boxed{\lambda \sum_k \sum_l W_{k,l}^2}$$

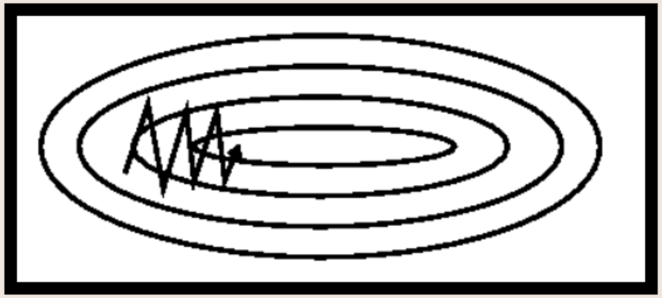
$$L = \boxed{\frac{1}{N}\sum_i -\log\left(\frac{e^{f_{y_i}}}{\sum_j e^{f_j}}\right)} + \boxed{\lambda \sum_k \sum_l W_{k,l}^2}$$

always pull the weights down

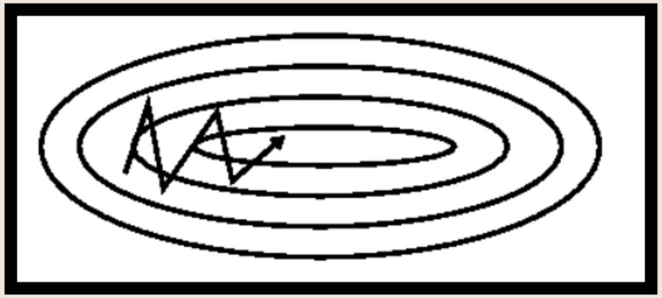
Momentum Update




```
weights_grad = evaluate_gradient(loss_fun, data, weights)
vel = vel * 0.9 - step_size * weights_grad
weights += vel
```



(Fig. 2a)



(Fig. 2b)

Many other ways to perform optimization…

- Second order methods that use the Hessian (or its approximation): BFGS, **LBFGS**, etc.

- Currently, the lesson from the trenches is that well-tuned SGD+Momentum is very hard to beat for CNNs.

Where are we?

- Classifiers: SVM vs. Softmax
- Gradient descent to optimize loss functions
  - Batch gradient descent, stochastic gradient descent
  - Momentum
  - Numerical gradients (slow, approximate), analytic gradients (fast, error-prone)

Derivatives

- Given f(x), where x is vector of inputs
  - Compute gradient of f at x: $\nabla f(x)$

Examples

$$f(x,y) = xy \qquad \rightarrow \qquad \frac{\partial f}{\partial x} = y \qquad \frac{\partial f}{\partial y} = x$$

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x+h) = f(x) + h\,\frac{df(x)}{dx}$$

$$f(x,y) = xy \qquad \rightarrow \qquad \frac{\partial f}{\partial x} = y \qquad \frac{\partial f}{\partial y} = x$$

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x+h) = f(x) + h\frac{df(x)}{dx}$$

Example: x = 4, y = -3. => f(x,y) = -12

$$\boxed{\frac{\partial f}{\partial x} = -3} \qquad \boxed{\frac{\partial f}{\partial y} = 4}$$

partial derivatives

$$\boxed{\nabla f = [\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}]}$$

gradient

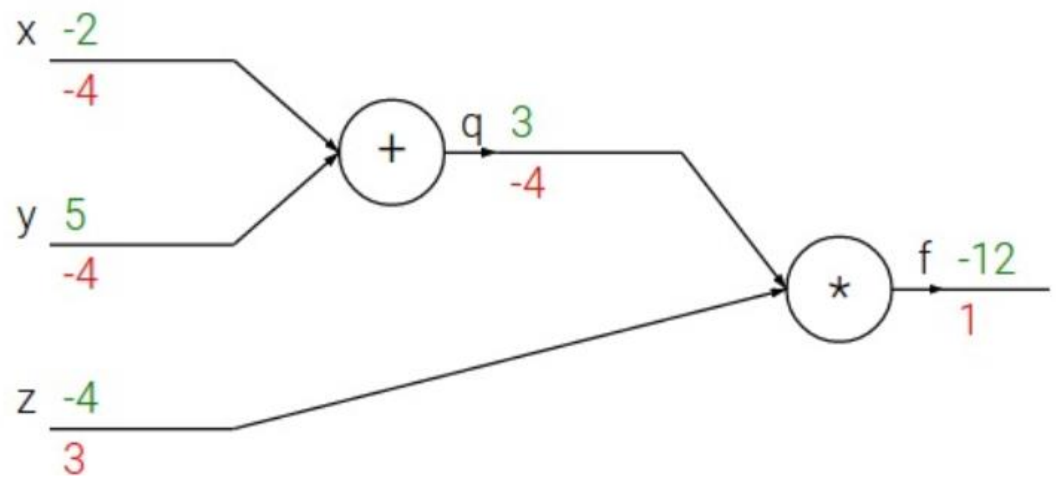
Compound expressions: $f(x, y, z) = (x + y)z$

$$q = x + y \qquad \frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

$$f = qz \qquad \frac{\partial f}{\partial q} = z, \frac{\partial f}{\partial z} = q$$

Chain rule:

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial q} \frac{\partial q}{\partial x}$$



Now onto Deep Learning

[Monroe 2014, xkcd]

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Posted on October 20, 2014 by Rob Hess, Clayton Mellina, and Friends

← Previous

Introducing: Flickr PARK or BIRD

Zion National Park Utah by Les Haines (cc) BY

OR

Secretary Bird by Bill Gracey (cc) BY-NC-ND

Slide: Flickr

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To play, drag an image from the examples or from your desktop.

EXAMPLE PHOTOS

Photo credits

PARK or BIRD

Want to know if your photo is from a U.S. national park? Want to know if it contains a bird? Just drag it into the box to the left, and we'll tell you. We'll use the GPS embedded in your photo (if it's there) to see whether it's from a park, and we'll use our super-cool computer vision skills to try to see whether it's a bird (which is a hard problem, but we do a pretty good job at it).

To try it out, just drag any photo from your desktop into the upload box, or try dragging any of our example images. We'll give you your answers below!

Want to know more about PARK or BIRD, including why the heck we did this? Just click here for more info →

PARK? BIRD?

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PARK or BIRD

Want to know if your photo is from a U.S. national park? Want to know if it contains a bird? Just drag it into the box to the left, and we'll tell you. We'll use the GPS embedded in your photo (if it's there) to see whether it's from a park, and we'll use our super-cool computer vision skills to try to see whether it's a bird (which is a hard problem, but we do a pretty good job at it).

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EXAMPLE PHOTOS

Photo credits

PARK?

YES

Ah yes, Bryce Canyon is truly beautiful.

BIRD?

NO

Beautiful clouds, but I don't see any birds flying up there.

Slide: Flickr

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PARK or BIRD

Want to know if your photo is from a U.S. national park? Want to know if it contains a bird? Just drag it into the box to the left, and we'll tell you. We'll use the GPS embedded in your photo (if it's there) to see whether it's from a park, and we'll use our super-cool computer vision skills to try to see whether it's a bird (which is a hard problem, but we do a pretty good job at it).

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Want to know more about PARK or BIRD, including why the heck we did this? Just click here for more info → i

EXAMPLE PHOTOS

Photo credits

PARK?

YES

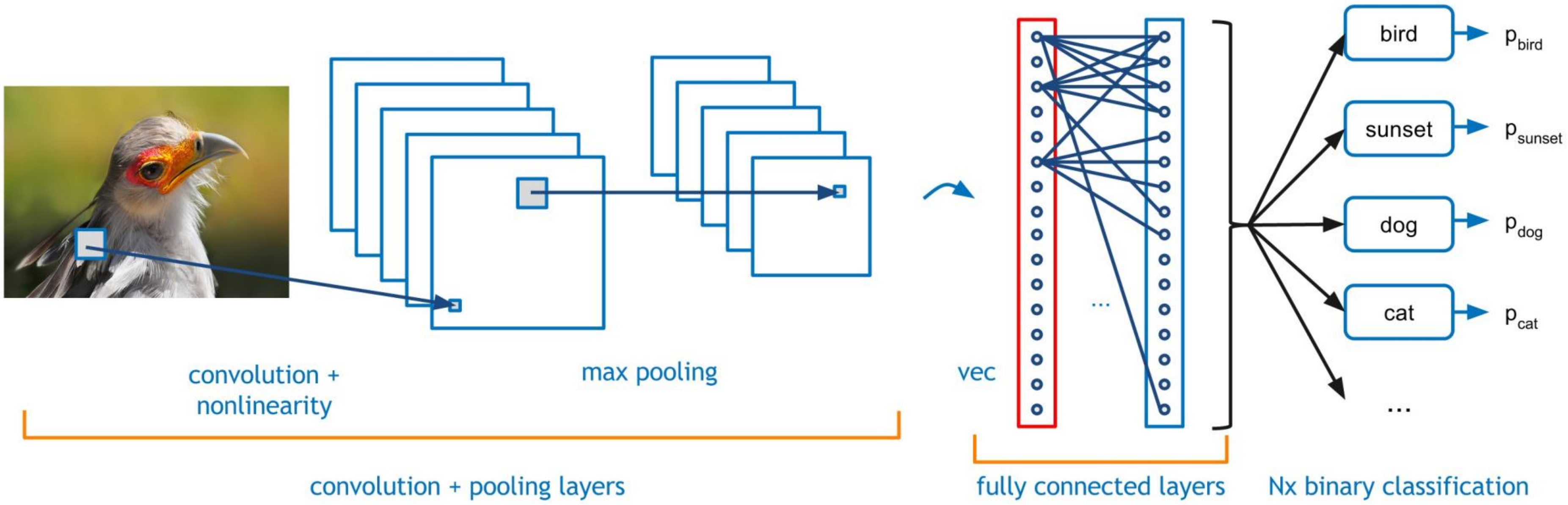
Hey, yeah! I went to Everglades once!

BIRD?

YES

Hey! Nice bird shot!

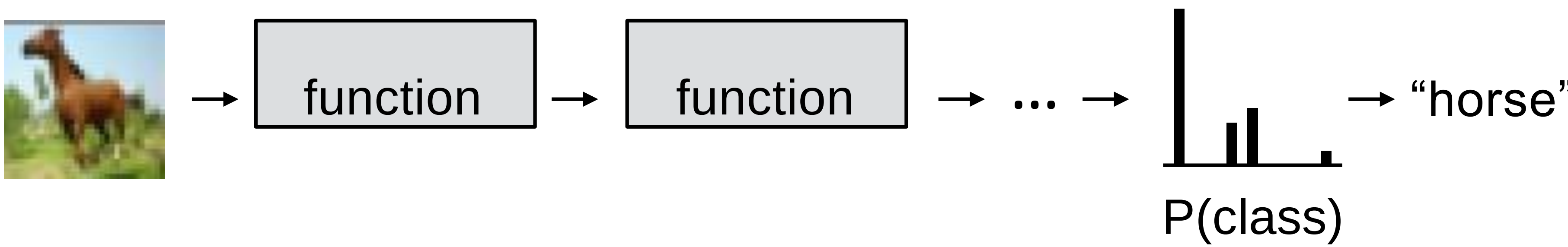
Slide: Flickr



In the next week, we'll learn what this is, how to compute it, and how to learn it

Slide: Flickr

What is a Convolutional Neural Network (CNN)?

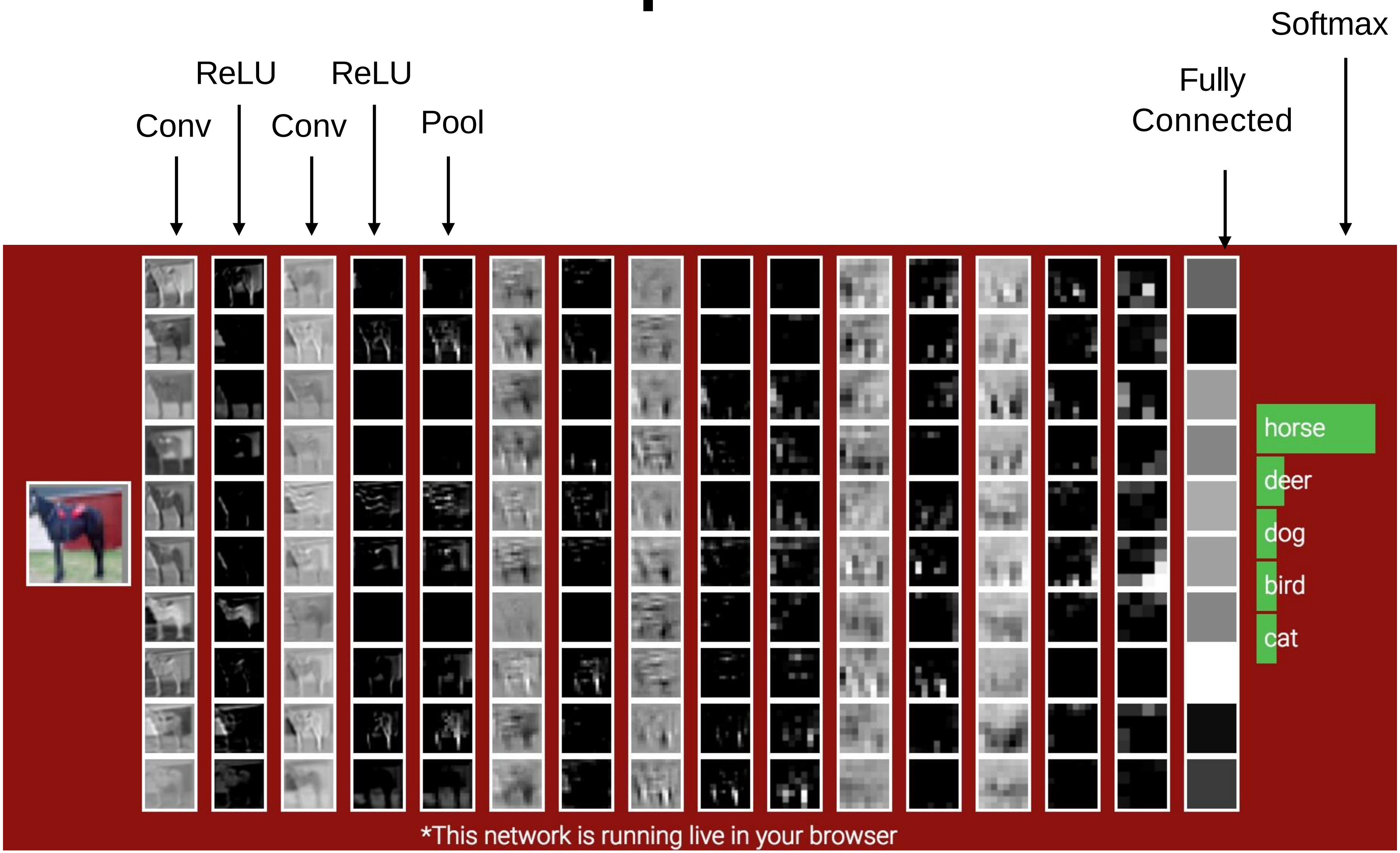


Key questions:

- What kinds of of functions should we use?

- How do we learn the parameters for those functions?

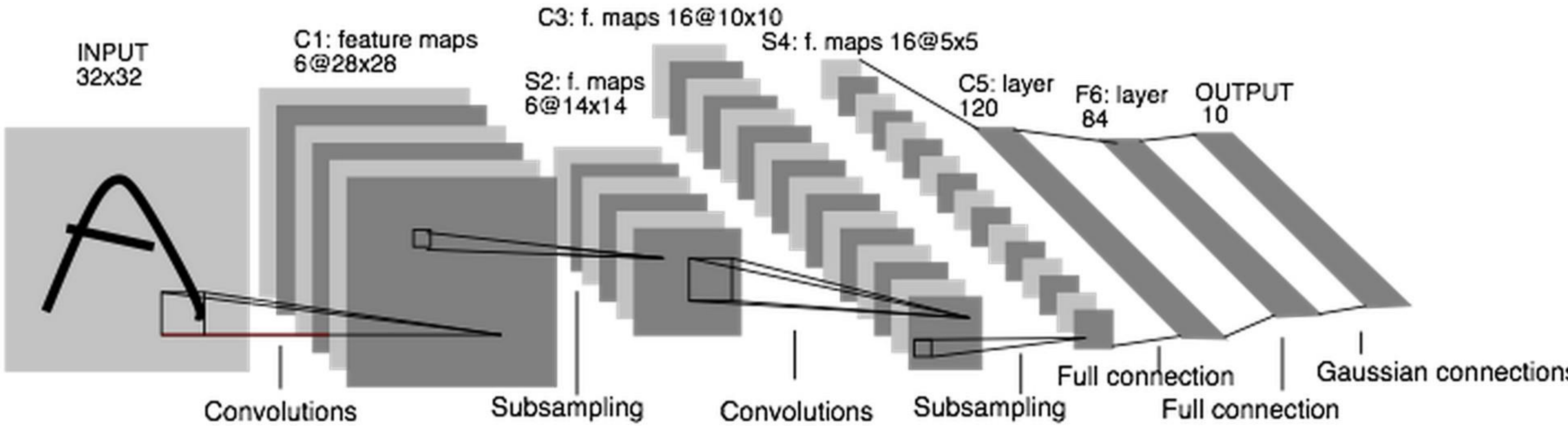
Example CNN



[Andrej Karpathy]

CNNs in 1989: “LeNet”

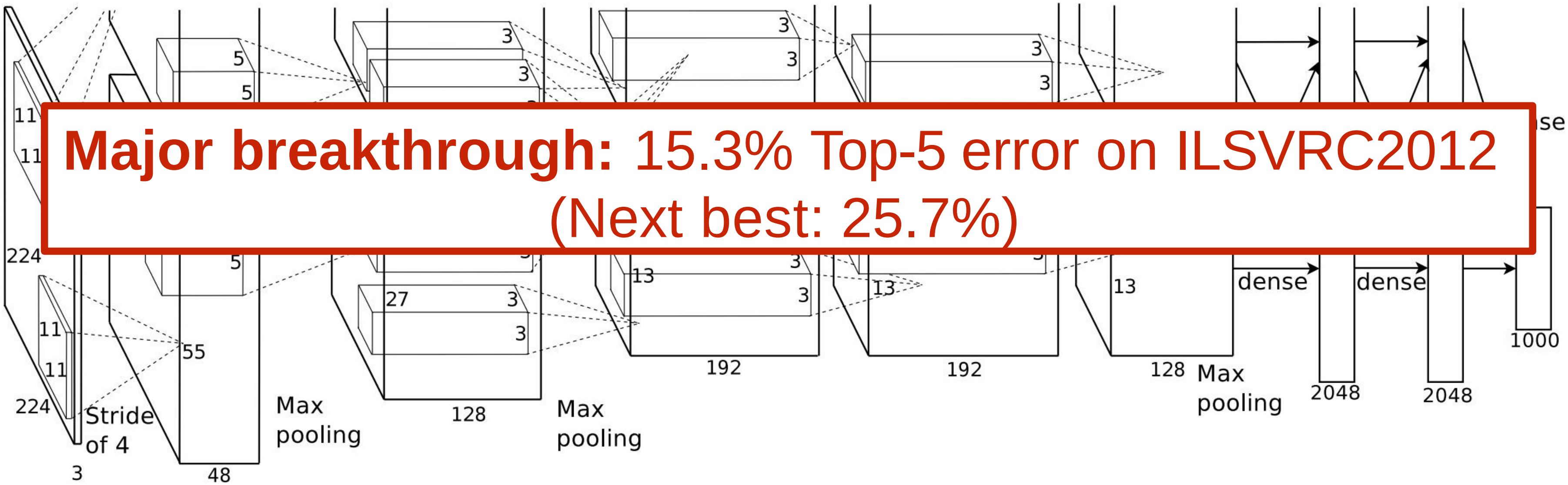
CNNs were *not* invented overnight



LeNet: a classifier for handwritten digits. [LeCun 1989]

CNNs in 2012: “SuperVision” (aka “AlexNet”)

“AlexNet” — Won the ILSVRC2012 Challenge



Major breakthrough: 15.3% Top-5 error on ILSVRC2012
(Next best: 25.7%)

Figure 2: An illustration of the architecture of our CNN, explicitly showing the delineation of responsibilities between the two GPUs. One GPU runs the layer-parts at the top of the figure while the other runs the layer-parts at the bottom. The GPUs communicate only at certain layers. The network’s input is 150,528-dimensional, and the number of neurons in the network’s remaining layers is given by 253,440–186,624–64,896–64,896–43,264–4096–4096–1000.

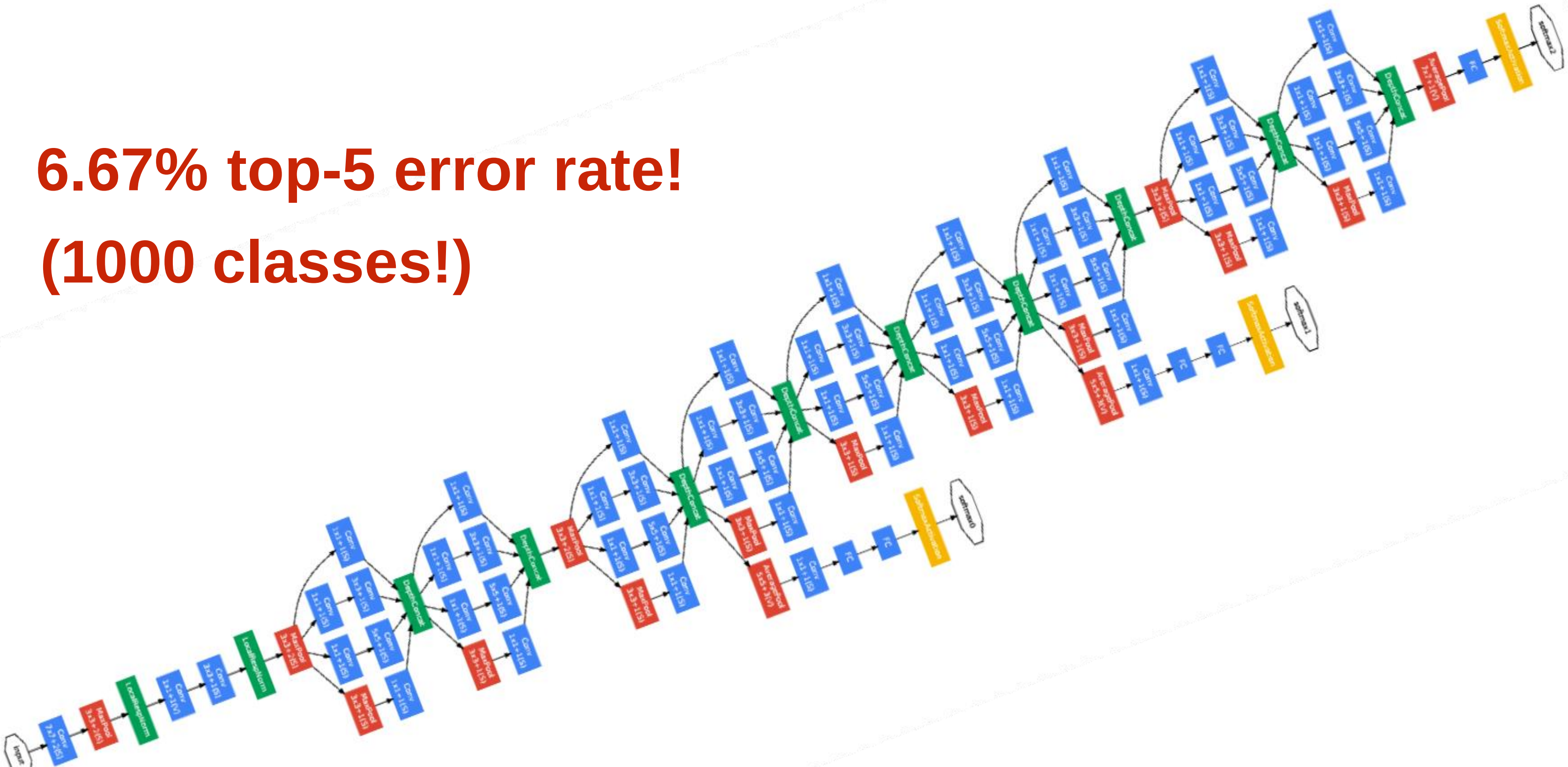
[Krizhevsky, Sutskever, Hinton. NIPS 2012]

CNNs in 2014: “GoogLeNet”

“GoogLeNet” — Won the ILSVRC2014 Challenge

6.67% top-5 error rate!

(1000 classes!)



[Szegedy et al, arXiv 2014]

CNNs in 2014: “VGGNet”

“VGGNet” — Second Place in the ILSVRC2014 Challenge

ConvNet Configuration					
A	A-LRN	B	C	D	E
11 weight layers	11 weight layers	13 weight layers	16 weight layers	16 weight layers	19 weight layers
input (224 × 224 RGB image)					
conv3-64	conv3-64 **LRN**	conv3-64 **conv3-64**	conv3-64 conv3-64	conv3-64 conv3-64	conv3-64 conv3-64
maxpool					
conv3-128	conv3-128	conv3-128 **conv3-128**	conv3-128 conv3-128	conv3-128 conv3-128	conv3-128 conv3-128
maxpool					
conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256 **conv1-256**	conv3-256 conv3-256 **conv3-256**	conv3-256 conv3-256 conv3-256 **conv3-256**
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 **conv1-512**	conv3-512 conv3-512 **conv3-512**	conv3-512 conv3-512 conv3-512 **conv3-512**
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 **conv1-512**	conv3-512 conv3-512 **conv3-512**	conv3-512 conv3-512 conv3-512 **conv3-512**
maxpool					
FC-4096					
FC-4096					
FC-1000					
soft-max					

No fancy picture, sorry

7.3% top-5 error rate

(and 1st place in the detection challenge)

[Simonyan et al, arXiv 2014]

CNNs in 2015: “ResNet”

AlexNet, 8 layers
(ILSVRC 2012)

VGG, 19 layers
(ILSVRC 2014)

GoogLeNet, 22 layers
(ILSVRC 2014)

ResNet, 152 layers
(ILSVRC 2015)

Note: Despite its massive depth, ResNet has a lower runtime complexity than VGG

https://youtu.be/1PGLj-uKT1w?t=4m40s

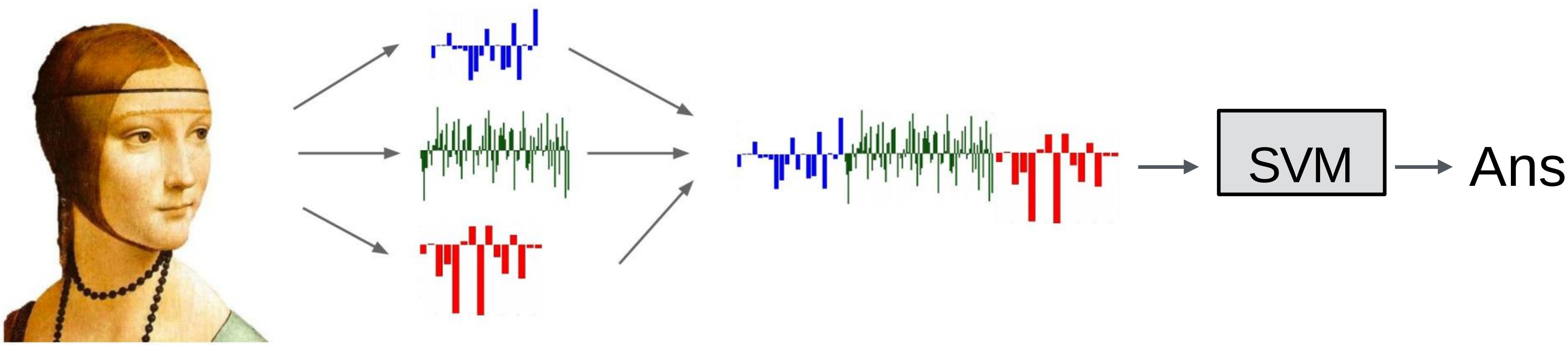
[Kaiming et al, “Deep Residual Learning for Image Recognition” 2015]

CNNs in 2015: “ResNet”

- **1st places in all five main tracks**
  - ImageNet Classification: “*Ultra-deep*” (quote Yann) 152-layer nets
  - ImageNet Detection: 16% better than 2nd
  - ImageNet Localization: 27% better than 2nd
  - COCO Detection: 11% better than 2nd
  - COCO Segmentation: 12% better than 2nd

[Kaiming et al, “Deep Residual Learning for Image Recognition” 2015]

Aside: Before Deep Learning



Input Pixels

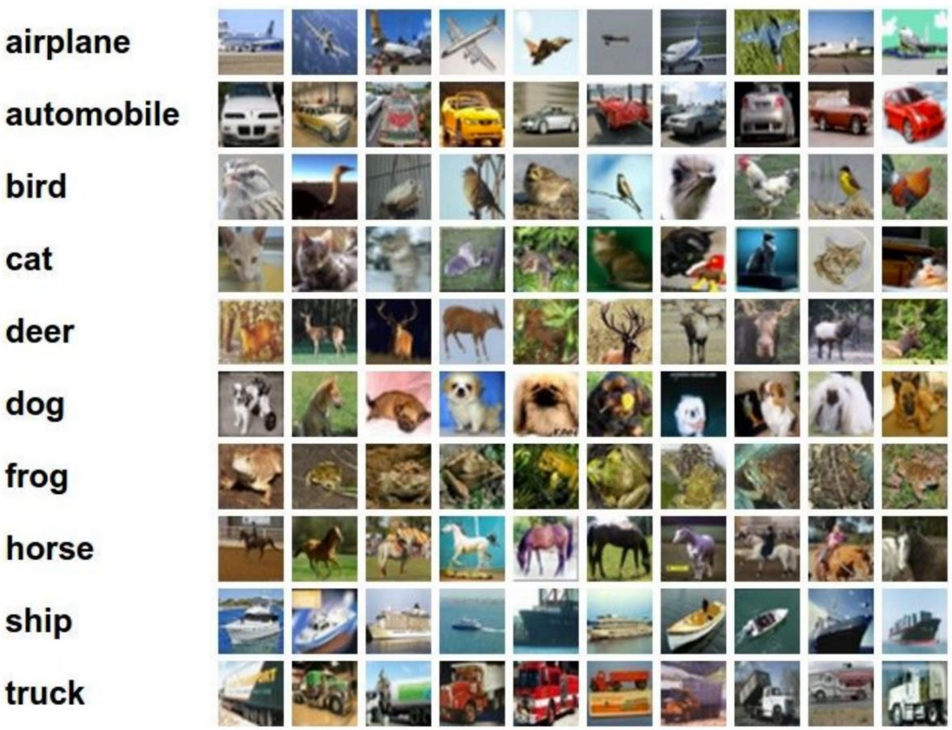
Extract Features

Concatenate into a vector ***x***

Linear Classifier

Figure: Karpathy 2016

Why use features?
Why not pixels?



$$f(x_i, W, b) = Wx_i + b$$

Q: What would be a very hard set of classes for a linear classifier to distinguish?

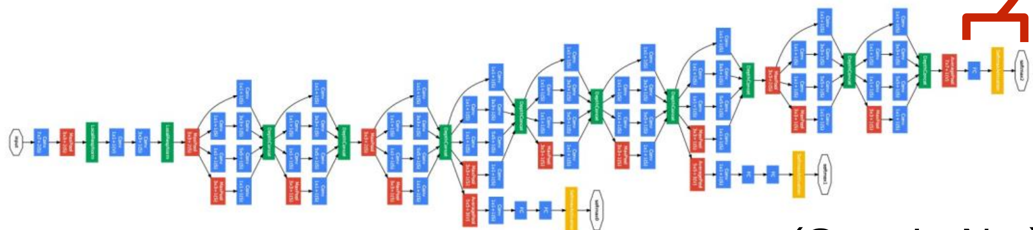
(assuming x = pixels)

Slide from Karpathy 2016

The last layer of (most) CNNs are linear classifiers

This piece is just a linear classifier

→



(GoogLeNet)

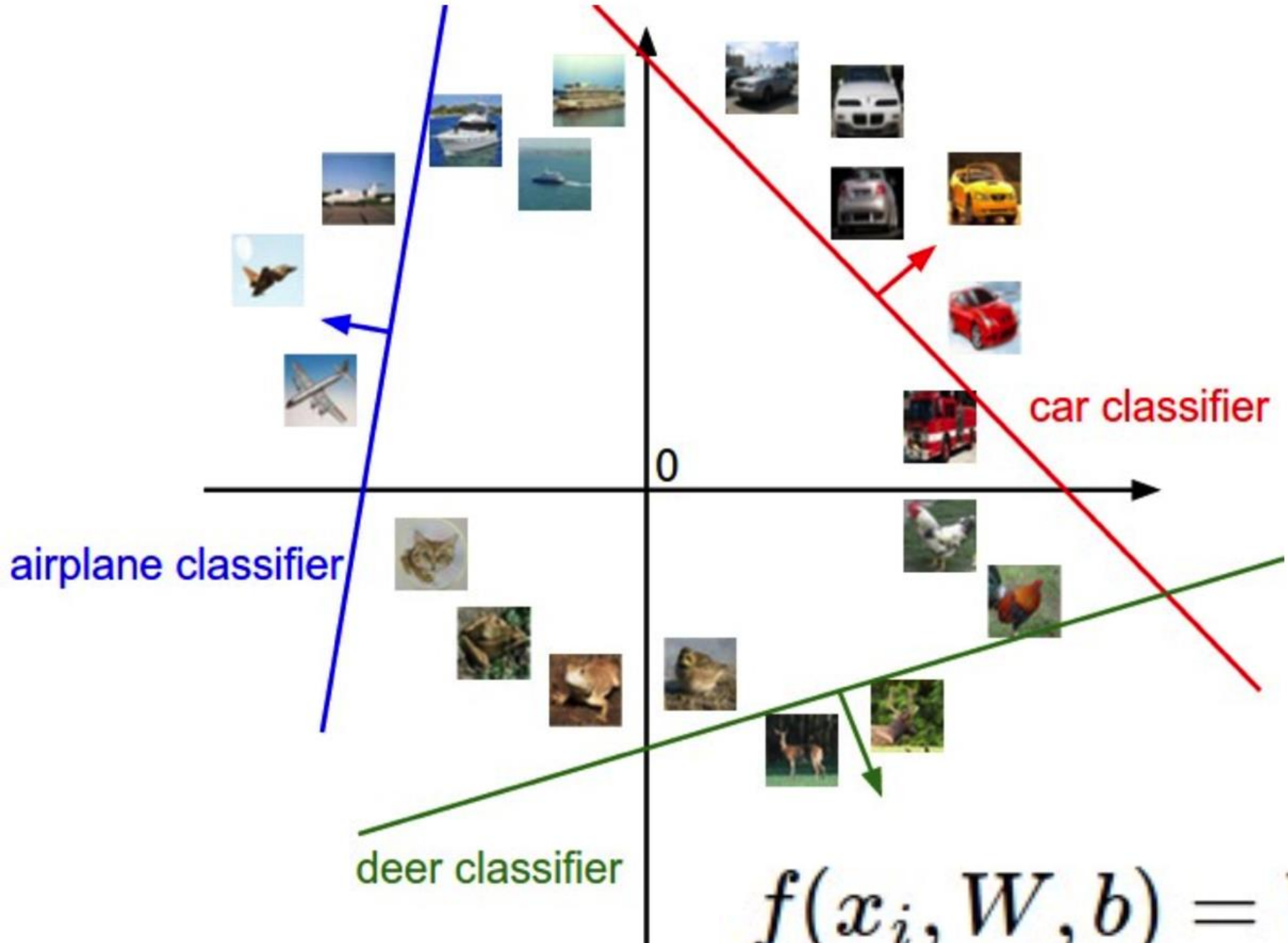
→ Ans

Input Pixels

Perform everything with a big neural network, trained end-to-end

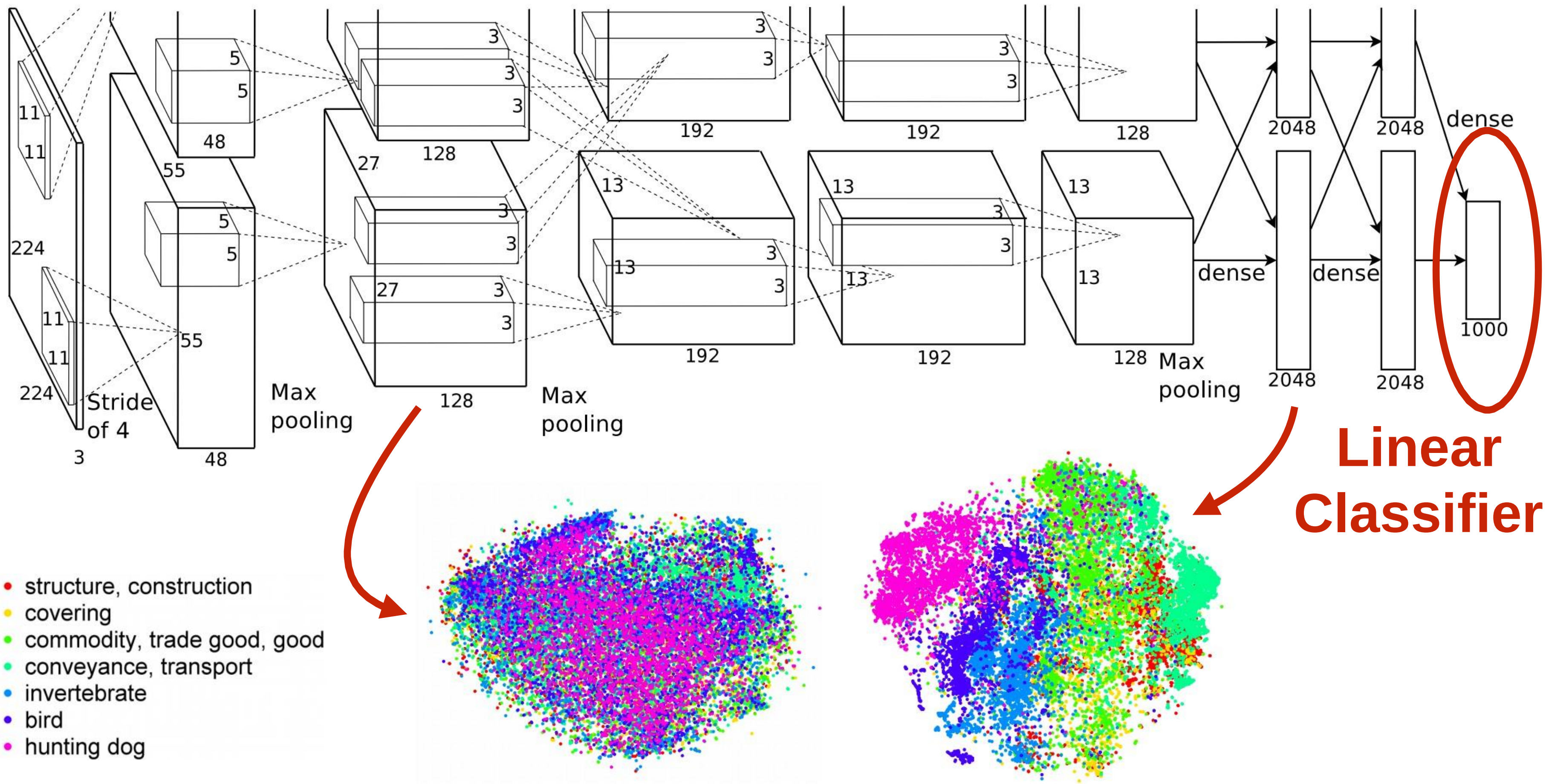
Key: perform enough processing so that by the time you get to the end of the network, the classes are linearly separable

Linearly separable classes



$$f(x_i, W, b) = Wx_i + b$$

Example: Visualizing AlexNet in 2D with t-SNE



(c) DeCAF_1 (d) DeCAF_6

(2D visualization using t-SNE)

[Donahue, "DeCAF: DeCAF: A Deep Convolutional …", arXiv 2013]

Questions?