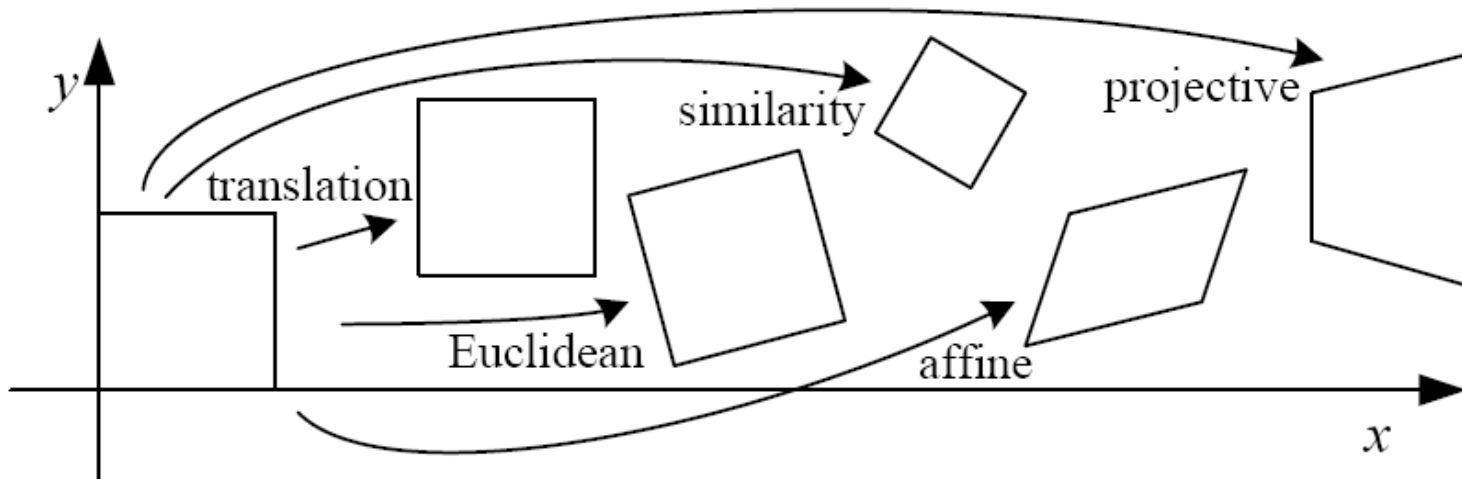


CS5670: Computer Vision

Noah Snavely

Lecture 6a: Transformations



Reading

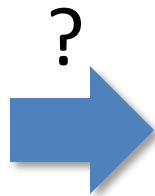
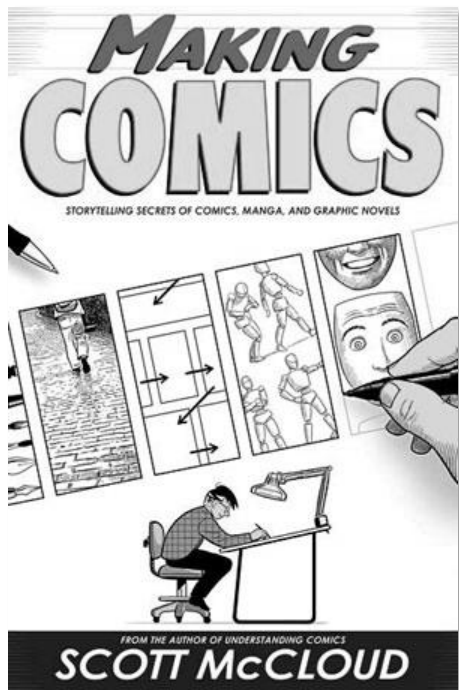
- Szeliski: Chapter 3.6

Image alignment



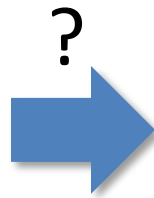
Why don't these image line up exactly?

What is the geometric relationship between these two images?



Answer: Similarity transformation (translation, rotation, uniform scale)

What is the geometric relationship between these two images?



What is the geometric relationship between these two images?

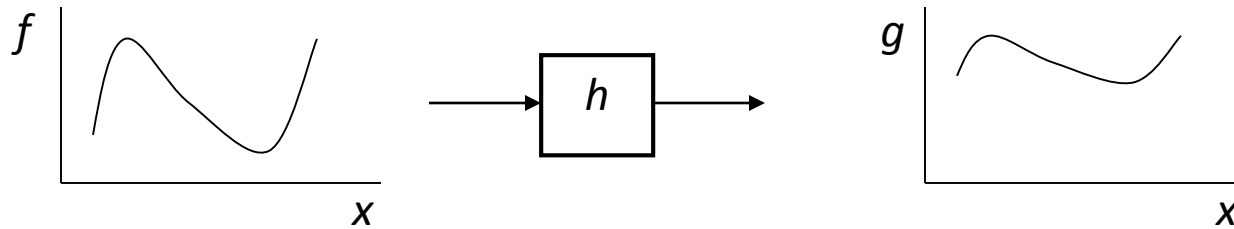


Very important for creating mosaics!

Image Warping

- image filtering: change *range* of image

- $g(x) = h(f(x))$



- image warping: change *domain* of image

- $g(x) = f(h(x))$

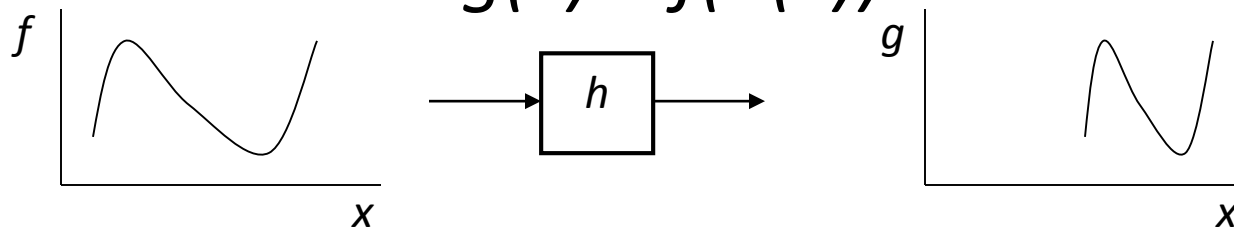
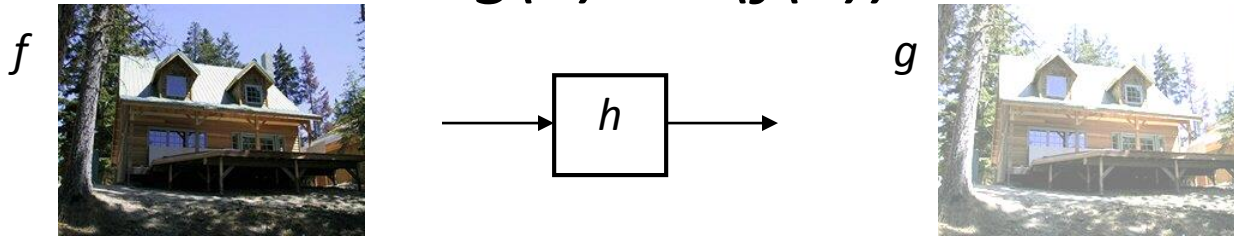


Image Warping

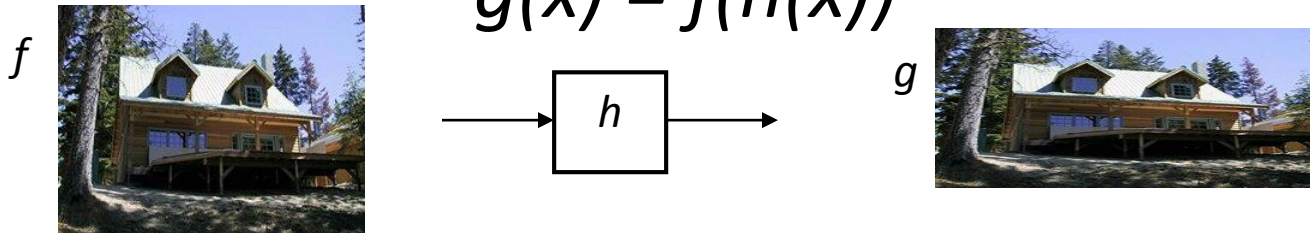
- image filtering: change *range* of image

- $g(x) = h(f(x))$



- image warping: change *domain* of image

- $g(x) = f(h(x))$



Parametric (global) warping

- Examples of parametric warps:



translation



rotation

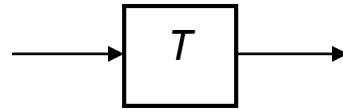


aspect

Parametric (global) warping



$\mathbf{p} = (x, y)$



$\mathbf{p}' = (x', y')$

- Transformation T is a coordinate-changing machine:

$$\mathbf{p}' = T(\mathbf{p})$$

- What does it mean that T is global?

- Is the same for any point \mathbf{p}
- can be described by just a few numbers (parameters)

- Let's consider *linear* xforms (can be represented by a 2D matrix):

$$\mathbf{p}' = \mathbf{T}\mathbf{p} \quad \begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{T} \begin{bmatrix} x \\ y \end{bmatrix}$$

Common linear transformations

- Uniform scaling by s :



(0,0) ●



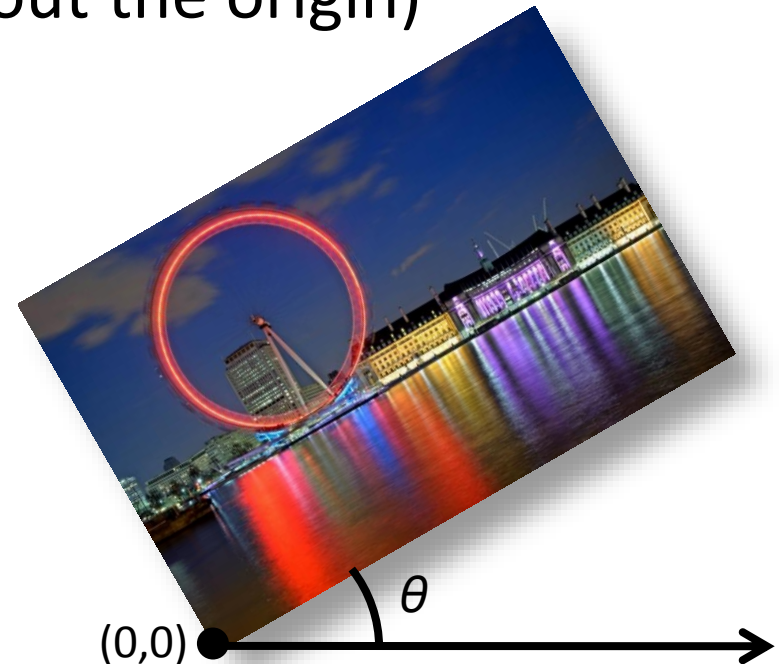
(0,0) ●

$$\mathbf{S} = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix}$$

What is the inverse?

Common linear transformations

- Rotation by angle θ (about the origin)



$$\mathbf{R} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

What is the inverse?

For rotations:

$$\mathbf{R}^{-1} = \mathbf{R}^T$$

2x2 Matrices

- What types of transformations can be represented with a 2x2 matrix?

2D mirror about Y axis?

$$\begin{aligned}x' &= -x \\ y' &= y\end{aligned} \quad \mathbf{T} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

2D mirror across line $y = x$?

$$\begin{aligned}x' &= y \\ y' &= x\end{aligned} \quad \mathbf{T} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

2x2 Matrices

- What types of transformations can be represented with a 2x2 matrix?

2D Translation?

$$x' = x + t_x \quad \text{NO!}$$

$$y' = y + t_y$$

Translation is not a linear operation on 2D coordinates

All 2D Linear Transformations

- Linear transformations are combinations of ...

- Scale,
- Rotation,
- Shear, and
- Mirror

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

- Properties of linear transformations:

- Origin maps to origin
- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved
- Closed under composition

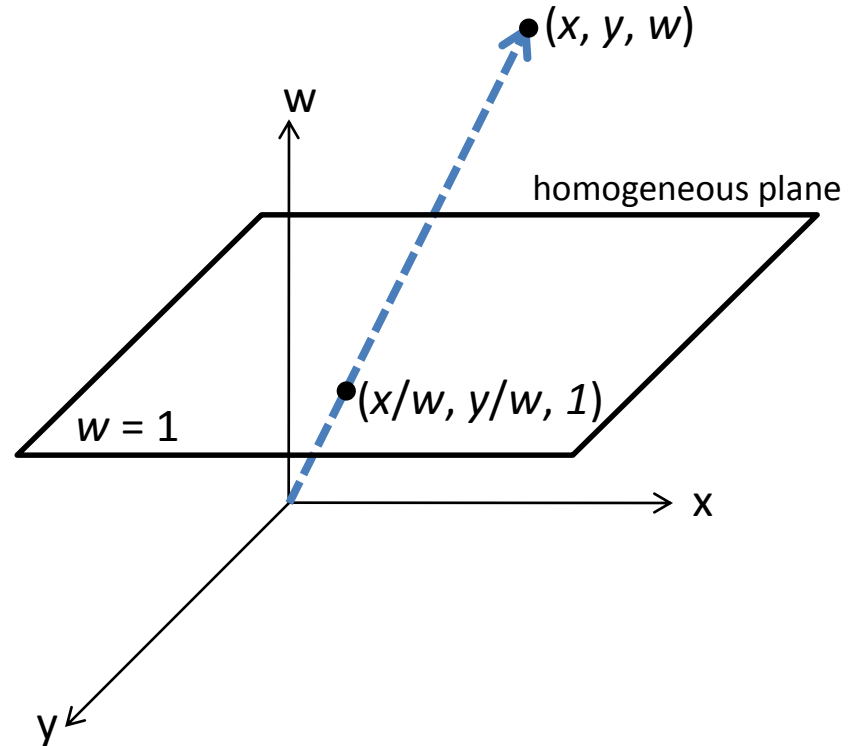
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} \begin{bmatrix} i & j \\ k & l \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Homogeneous coordinates

Trick: add one more coordinate:

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image
coordinates



Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

Translation

- Solution: homogeneous coordinates to the rescue

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$$

Affine transformations

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$



any transformation with
last row $[0 \ 0 \ 1]$ we call an
affine transformation

$$\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix}$$

Basic affine transformations

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

2D *in-plane* rotation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & sh_x & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Shear

Affine Transformations

- Affine transformations are combinations of ...

- Linear transformations, and
- Translations

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

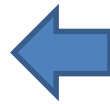
- Properties of affine transformations:
 - Origin does not necessarily map to origin
 - Lines map to lines
 - Parallel lines remain parallel
 - Ratios are preserved
 - Closed under composition

Is this an affine transformation?



Where do we go from here?

$$\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix}$$



what happens when we
mess with this row?

affine transformation

Projective Transformations aka Homographies aka Planar Perspective Maps

$$\mathbf{H} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix}$$

Called a *homography*
(or *planar perspective map*)



Homographies

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

What happens when
the denominator is 0?

\sim

$$\begin{bmatrix} \frac{ax+by+c}{gx+hy+1} \\ \frac{dx+ey+f}{gx+hy+1} \\ 1 \end{bmatrix}$$

Points at infinity

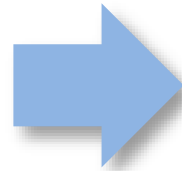
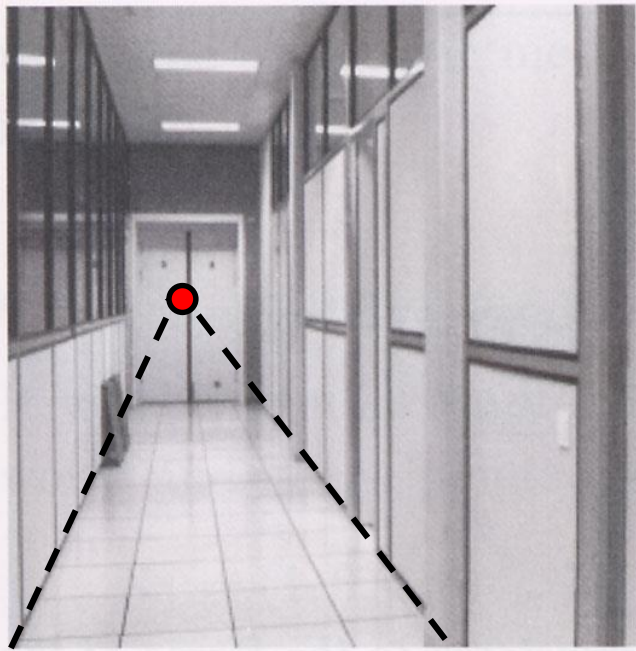
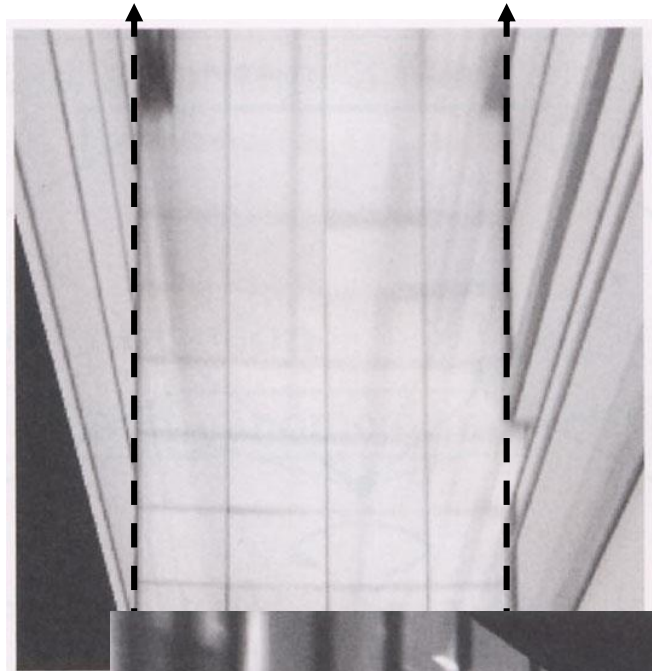


Image warping with homographies



H_1



H_2

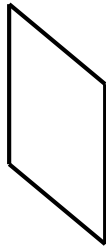
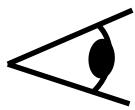
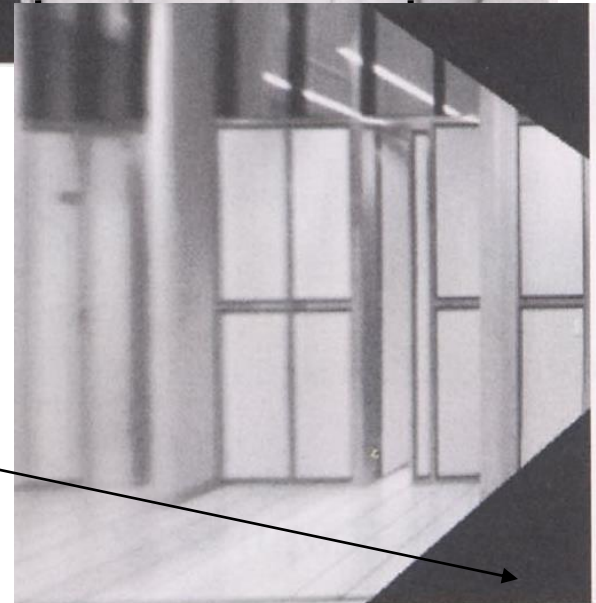


image plane in front

black area
where no pixel
maps to

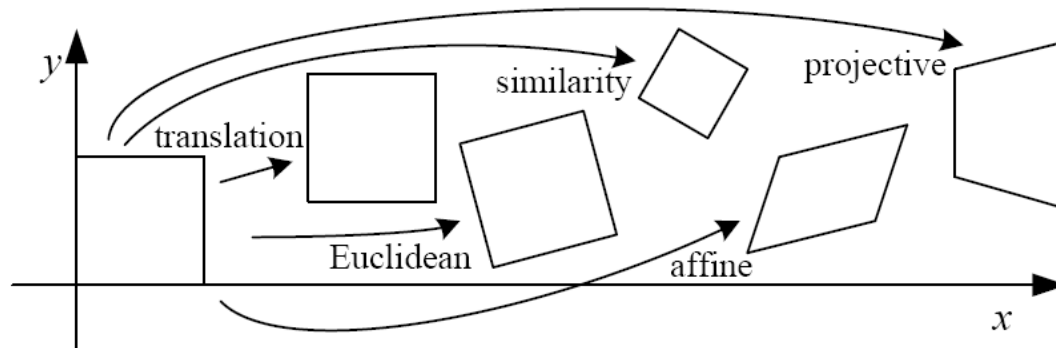
Homographies








Homographies

- Homographies ...
 - Affine transformations, and
 - Projective warps
- $$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$
- Properties of projective transformations:
 - Origin does not necessarily map to origin
 - Lines map to lines
 - Parallel lines do not necessarily remain parallel
 - Ratios are not preserved
 - Closed under composition

2D image transformations



Name	Matrix	# D.O.F.	Preserves:	Icon
translation	$\begin{bmatrix} \mathbf{I} & \mathbf{t} \end{bmatrix}_{2 \times 3}$	2	orientation + ...	
rigid (Euclidean)	$\begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix}_{2 \times 3}$	3	lengths + ...	
similarity	$\begin{bmatrix} s\mathbf{R} & \mathbf{t} \end{bmatrix}_{2 \times 3}$	4	angles + ...	
affine	$\begin{bmatrix} \mathbf{A} \end{bmatrix}_{2 \times 3}$	6	parallelism + ...	
projective	$\begin{bmatrix} \tilde{\mathbf{H}} \end{bmatrix}_{3 \times 3}$	8	straight lines	

These transformations are a nested set of groups

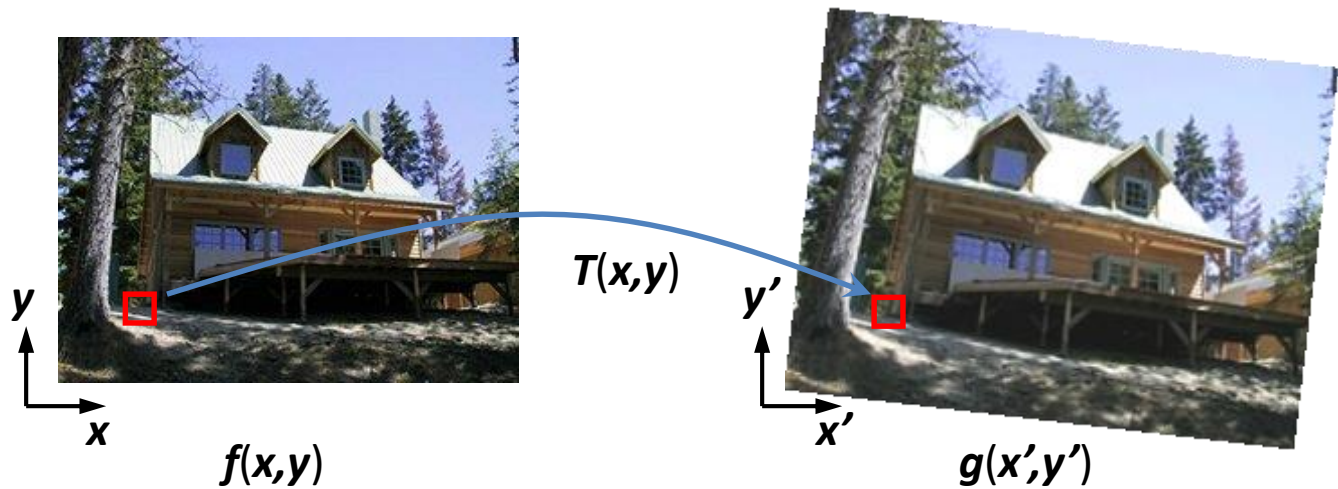
- Closed under composition and inverse is a member

Homographies



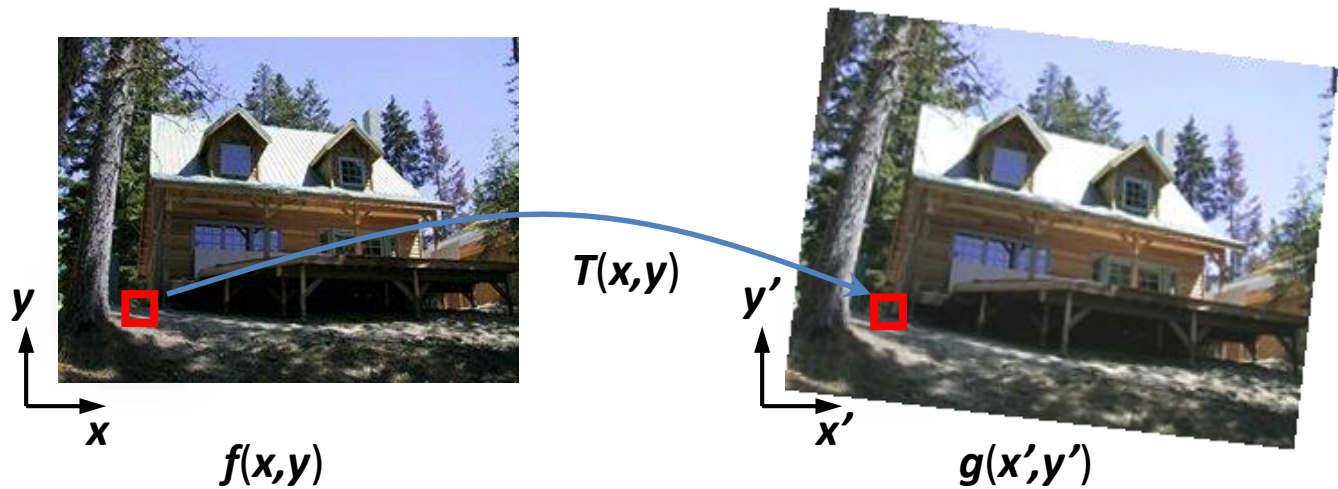
Image Warping

- Given a coordinate xform $(x',y') = T(x,y)$ and a source image $f(x,y)$, how do we compute an xformed image $g(x',y') = f(T(x,y))$?



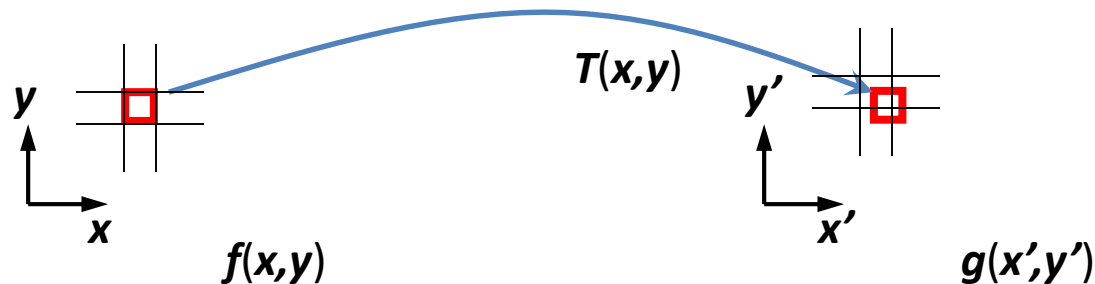
Forward Warping

- Send each pixel $f(x)$ to its corresponding location $(x',y') = T(x,y)$ in $g(x',y')$
- What if pixel lands “between” two pixels?



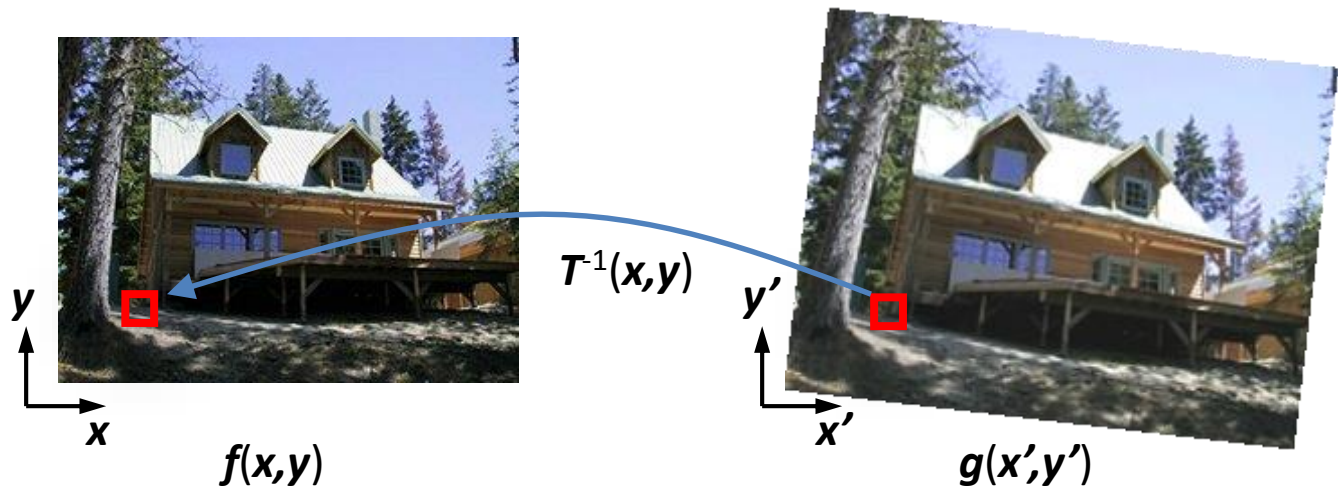
Forward Warping

- Send each pixel $f(x,y)$ to its corresponding location $x' = h(x,y)$ in $g(x',y')$
- What if pixel lands “between” two pixels?
- Answer: add “contribution” to several pixels, normalize later (*splatting*)
- Can still result in holes



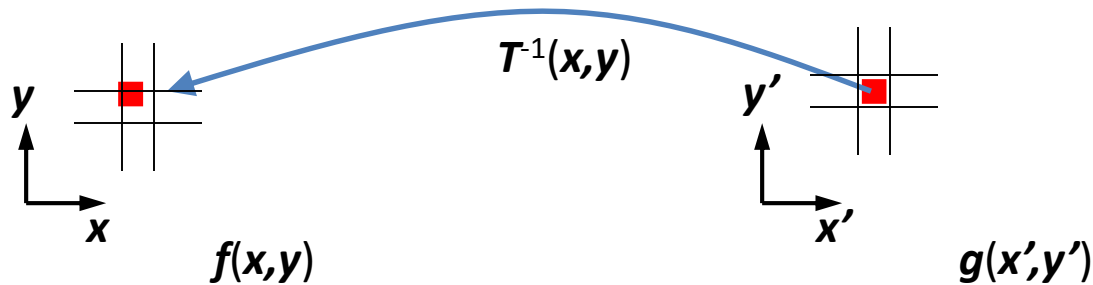
Inverse Warping

- Get each pixel $g(x',y')$ from its corresponding location $(x,y) = T^{-1}(x',y')$ in $f(x,y)$
- Requires taking the inverse of the transform
- What if pixel comes from “between” two pixels?



Inverse Warping

- Get each pixel $g(\mathbf{x}')$ from its corresponding location $\mathbf{x}' = \mathbf{h}(\mathbf{x})$ in $f(\mathbf{x})$
- What if pixel comes from “between” two pixels?
- Answer: *resample* color value from *interpolated (prefiltered)* source image



Interpolation

- Possible interpolation filters:
 - nearest neighbor
 - bilinear
 - bicubic (interpolating)
 - sinc
- Needed to prevent “jaggies” and “texture crawl”
(with prefiltering)

