

# CS5670: Computer Vision

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## Lecture 2: Edge detection

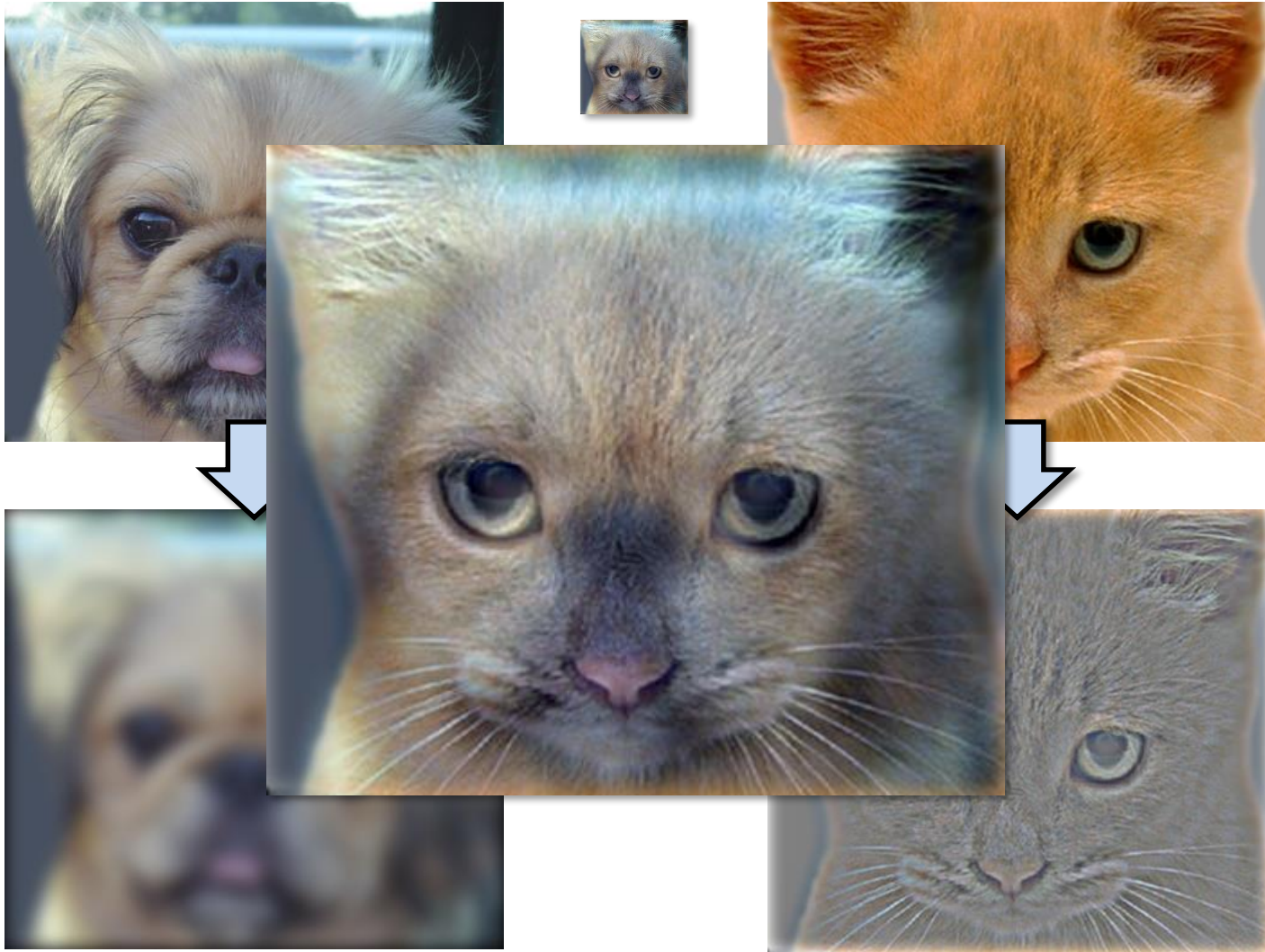
**SHADOW**

From [Sandlot Science](#)

# Announcements

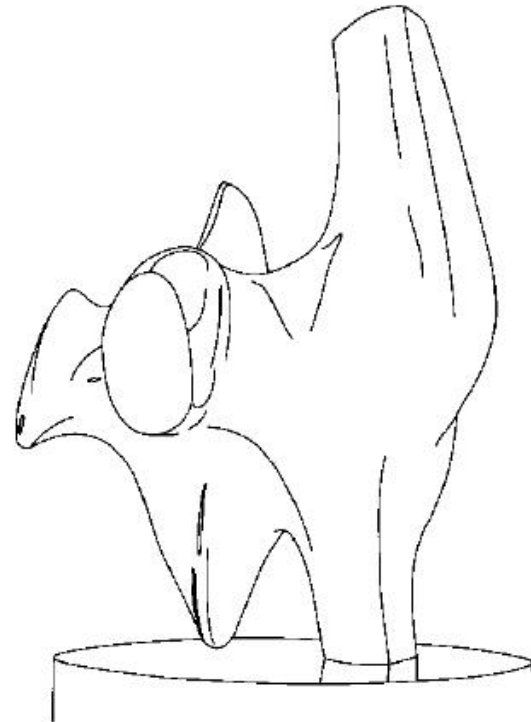
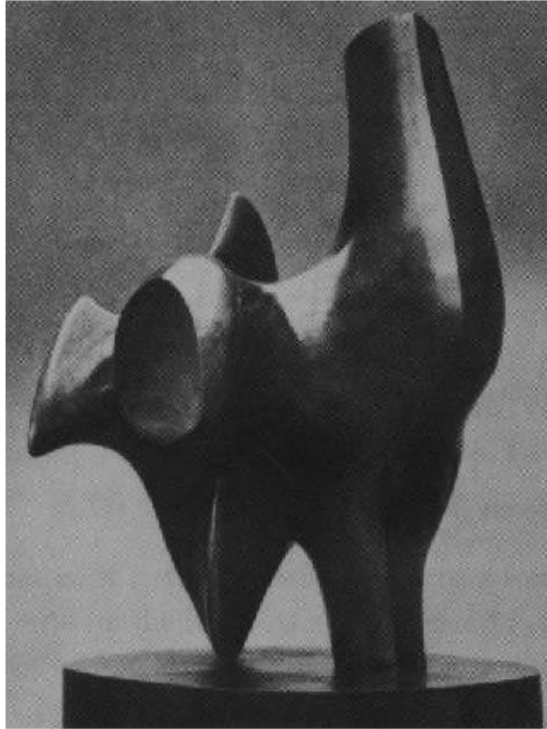
- Project 1 (Hybrid Images) is now on the course webpage (see “Projects” link)
  - Due Wednesday, Feb 15, by 11:59pm
  - Artifact due Friday, Feb 15, by 11:59pm
  - To be done individually
  - Voting system for favorite artifacts (with small amount of extra credit)
  - Files due by CMS
  - We will provide a course VM for you to run the assignments

# Project 1: Hybrid Images



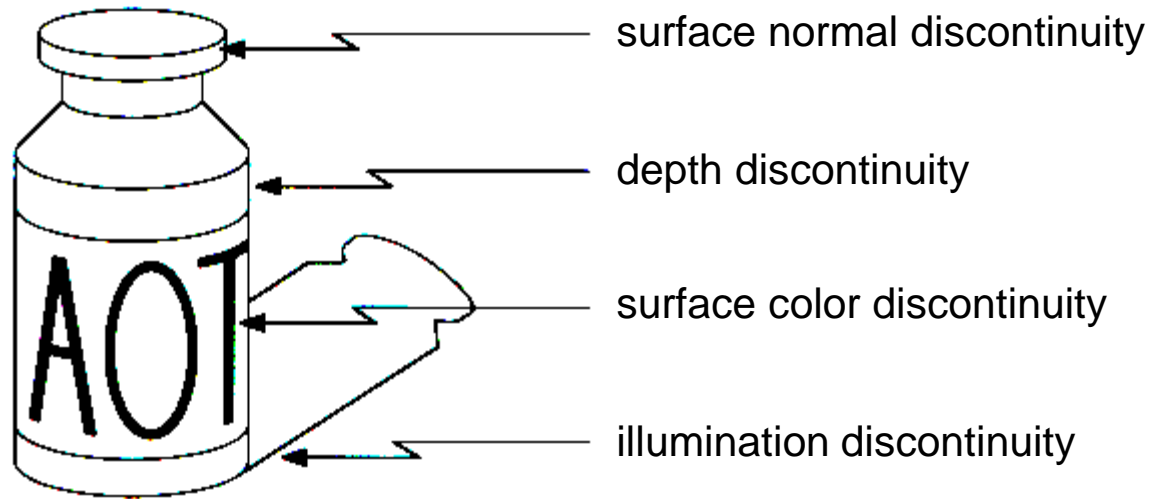
# Project 1 Demo

# Edge detection



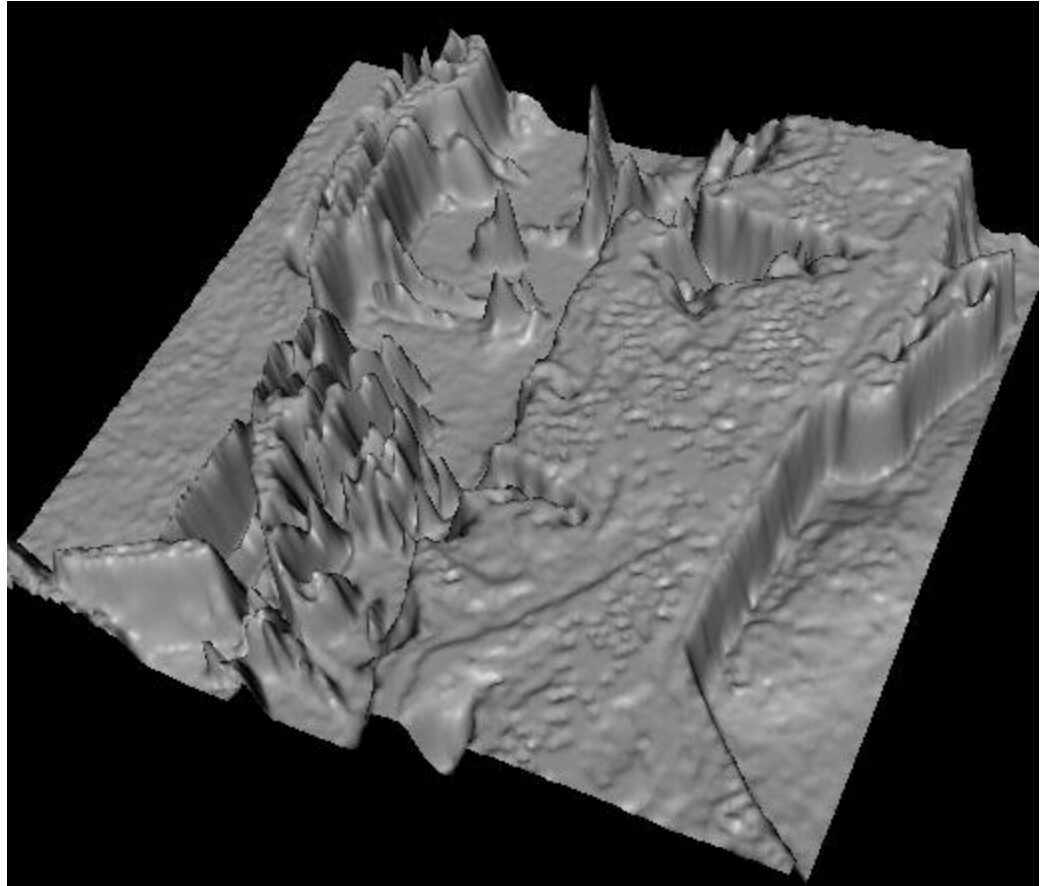
- Convert a 2D image into a set of curves
  - Extracts salient features of the scene
  - More compact than pixels

# Origin of Edges



- Edges are caused by a variety of factors

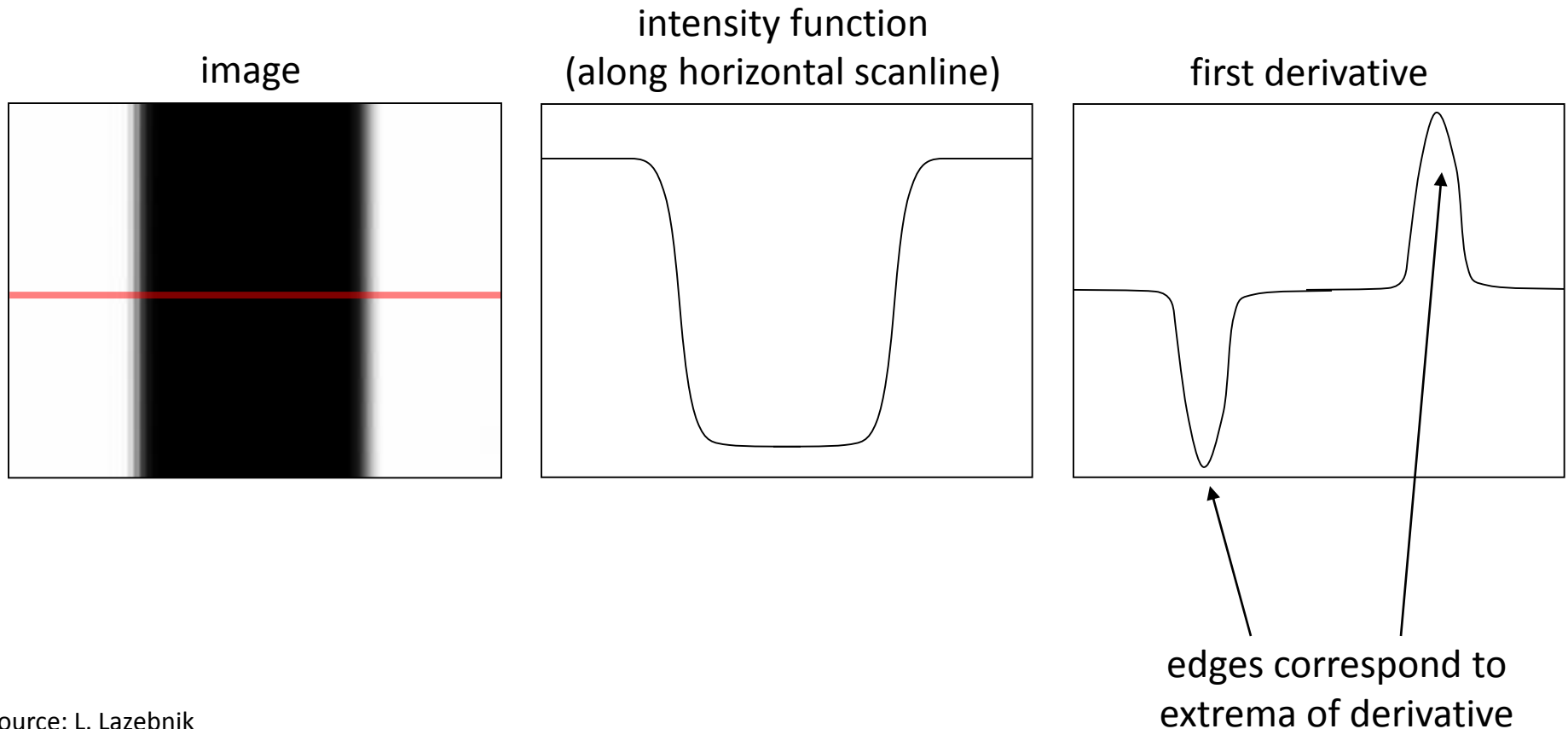
# Images as functions...



- Edges look like steep cliffs

# Characterizing edges

- An edge is a place of *rapid change* in the image intensity function





# Image derivatives

- How can we differentiate a *digital* image  $F[x,y]$ ?
  - Option 1: reconstruct a continuous image,  $f$ , then compute the derivative
  - Option 2: take discrete derivative (finite difference)

$$\frac{\partial f}{\partial x}[x, y] \approx F[x + 1, y] - F[x, y]$$

How would you implement this as a linear filter?

$$\frac{\partial f}{\partial x} : \begin{array}{|c|c|c|} \hline & & \\ \hline & & \\ \hline & & \\ \hline \end{array}$$

$H_x$

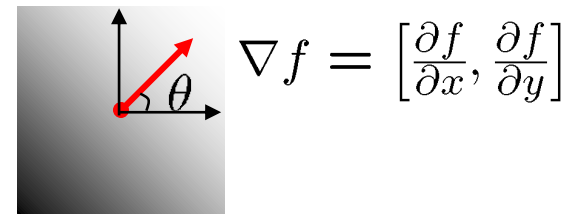
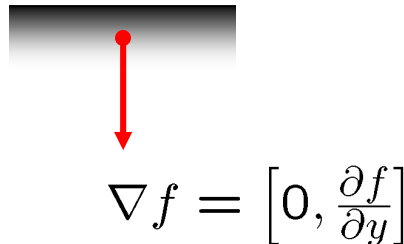
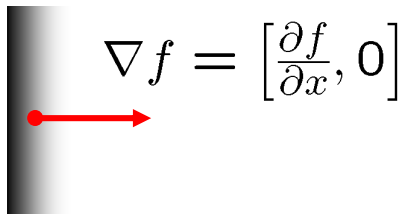
$$\frac{\partial f}{\partial y} : \begin{array}{|c|c|c|} \hline & & \\ \hline & & \\ \hline & & \\ \hline \end{array}$$

$H_y$

# Image gradient

- The *gradient* of an image:  $\nabla f = \left[ \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$

The gradient points in the direction of most rapid increase in intensity



The *edge strength* is given by the gradient magnitude:

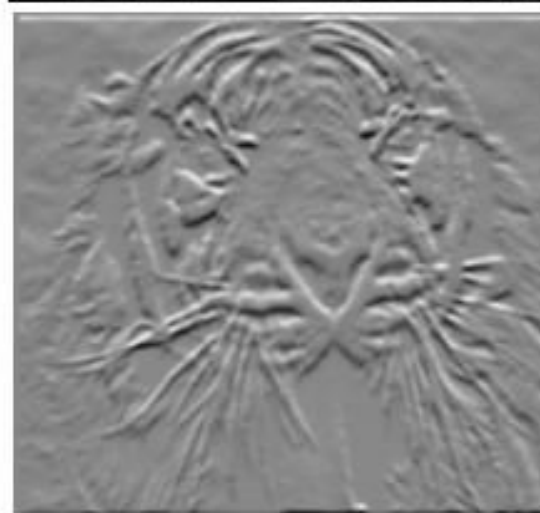
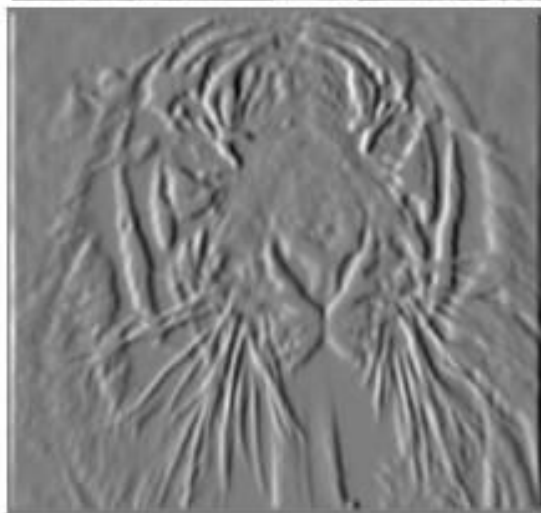
$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

The gradient direction is given by:

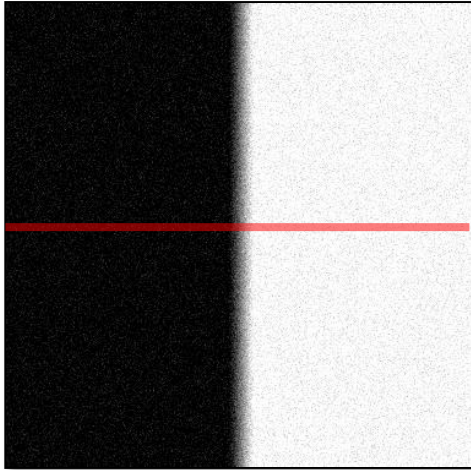
$$\theta = \tan^{-1} \left( \frac{\partial f}{\partial y} / \frac{\partial f}{\partial x} \right)$$

- how does this relate to the direction of the edge?

# Image gradient

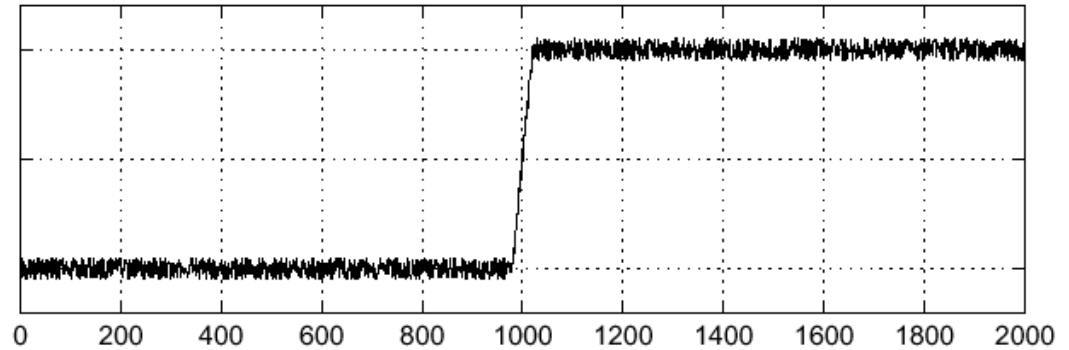


# Effects of noise

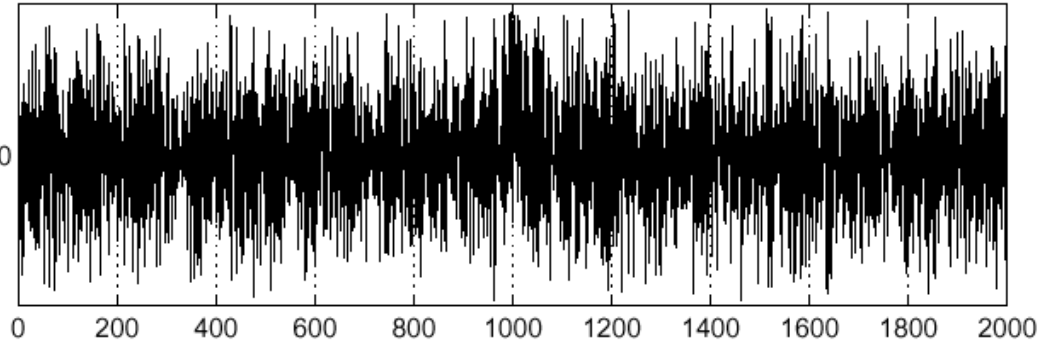


Noisy input image

$$f(x)$$

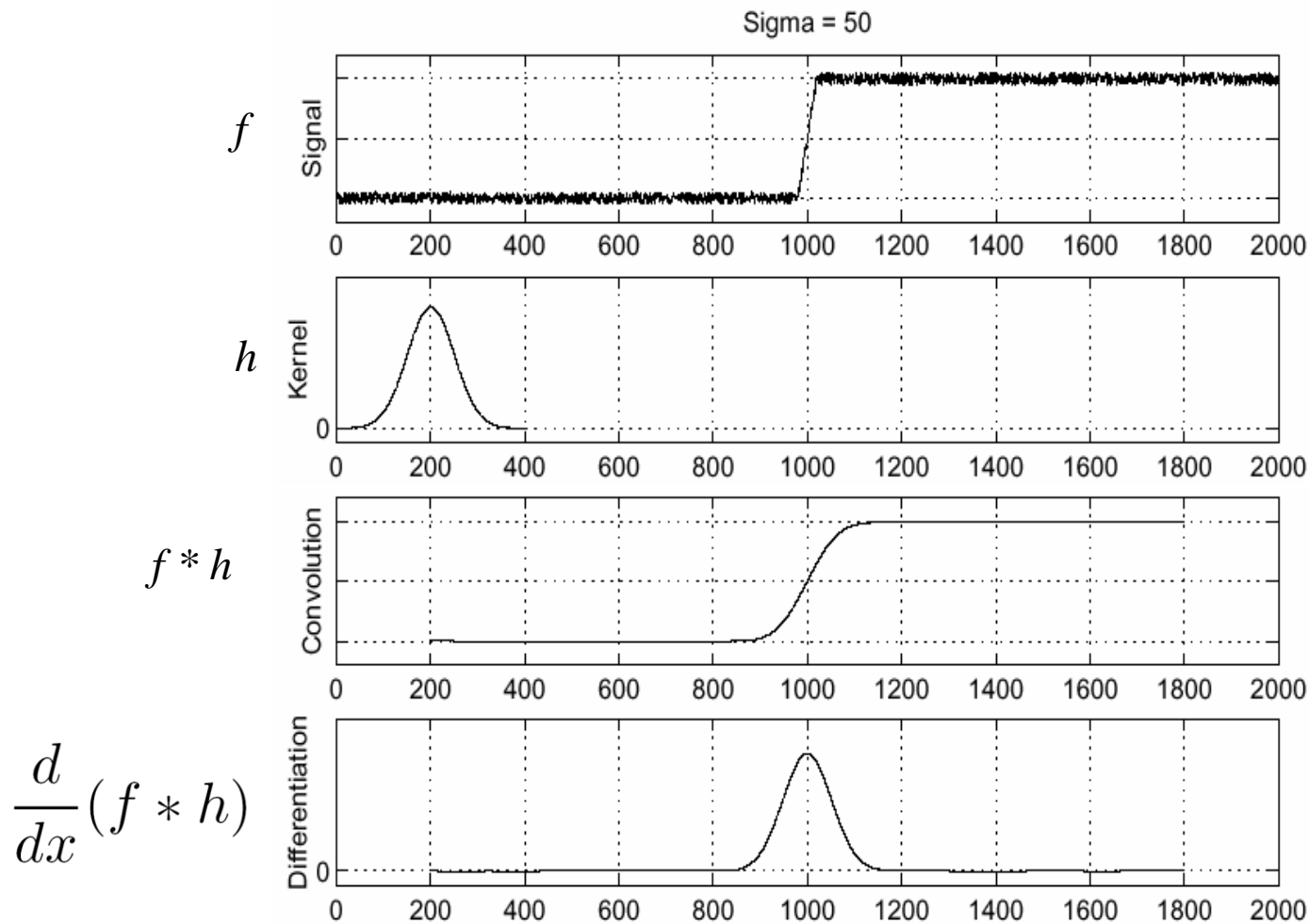


$$\frac{d}{dx} f(x)$$



Where is the edge?

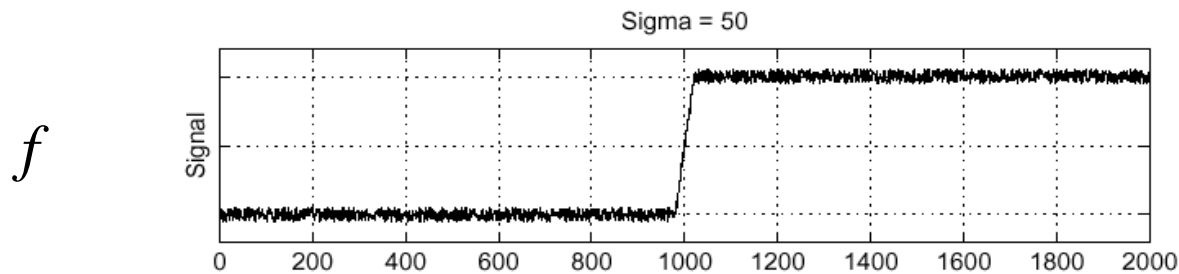
# Solution: smooth first



To find edges, look for peaks in  $\frac{d}{dx}(f * h)$

# Associative property of convolution

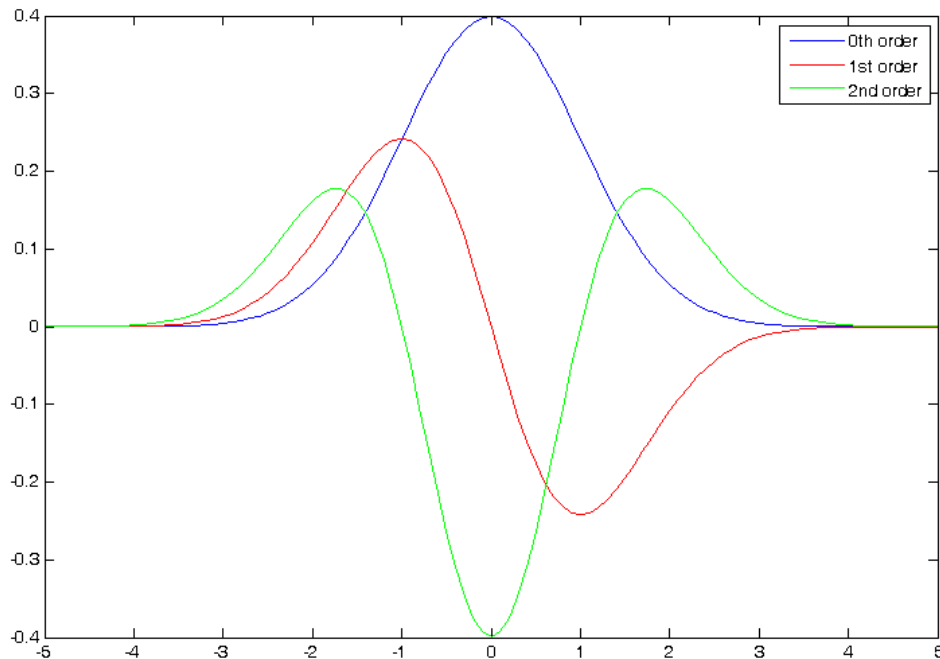
- Differentiation is convolution, and convolution is associative:  $\frac{d}{dx}(f * h) = f * \frac{d}{dx}h$
- This saves us one operation:



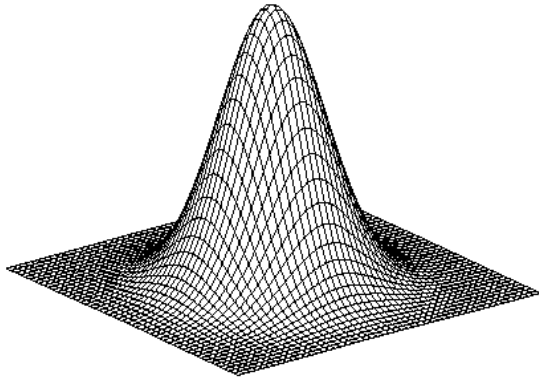
# The 1D Gaussian and its derivatives

$$G_\sigma(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}$$

$$G'_\sigma(x) = \frac{d}{dx} G_\sigma(x) = -\frac{1}{\sigma} \left(\frac{x}{\sigma}\right) G_\sigma(x)$$

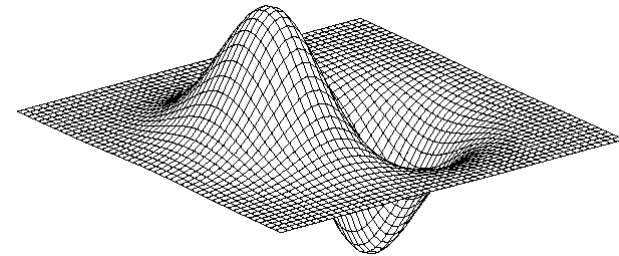


# 2D edge detection filters



Gaussian

$$h_{\sigma}(u, v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$

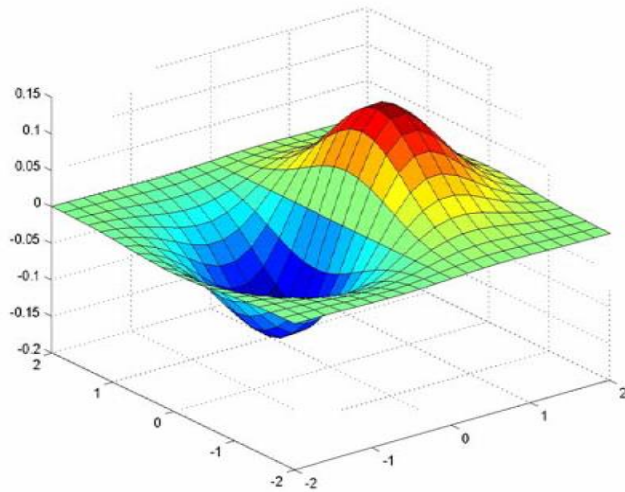


derivative of Gaussian ( $x$ )

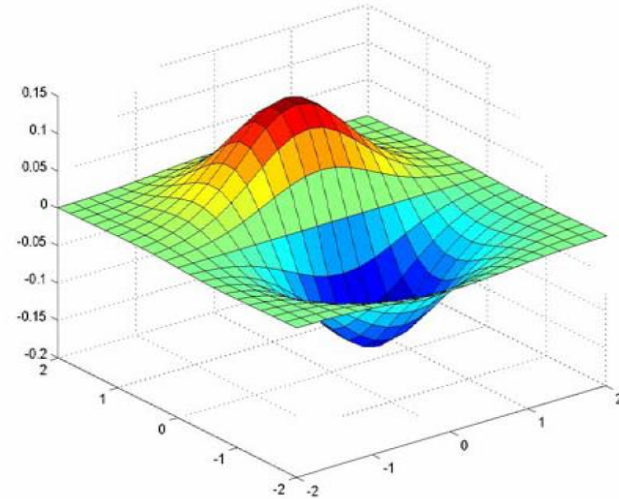
$$\frac{\partial}{\partial x} h_{\sigma}(u, v)$$



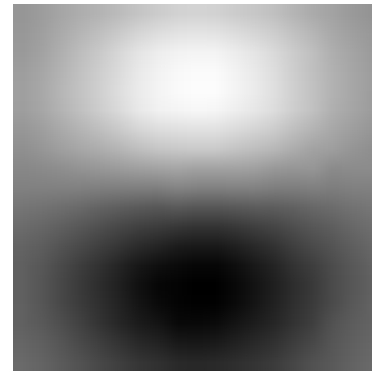
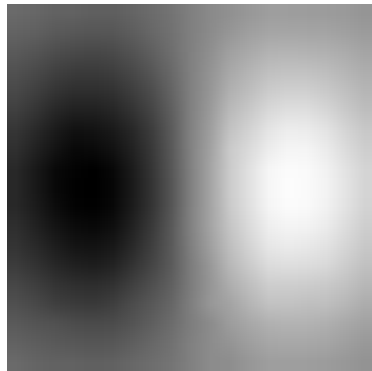
# Derivative of Gaussian filter



x-direction



y-direction



# The Sobel operator

- Common approximation of derivative of Gaussian

$$\frac{1}{8} \begin{array}{|c|c|c|} \hline -1 & 0 & 1 \\ \hline -2 & 0 & 2 \\ \hline -1 & 0 & 1 \\ \hline \end{array}$$

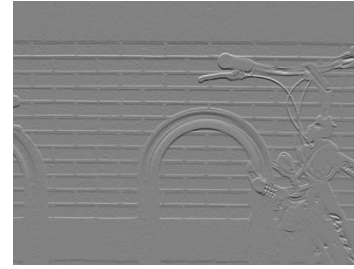
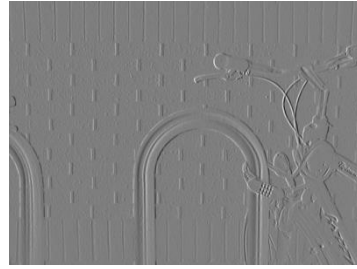
$s_x$

$$\frac{1}{8} \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 0 & 0 & 0 \\ \hline -1 & -2 & -1 \\ \hline \end{array}$$

$s_y$

- The standard defn. of the Sobel operator omits the  $1/8$  term
  - doesn't make a difference for edge detection
  - the  $1/8$  term **is** needed to get the right gradient magnitude

# Sobel operator: example



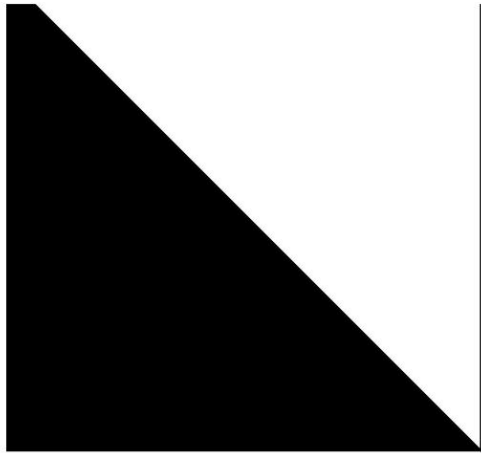
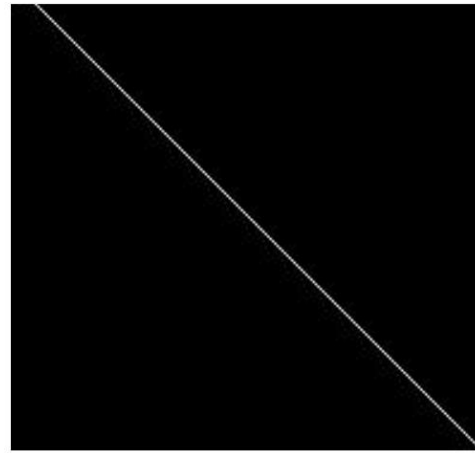


Image with Edge



Edge Location

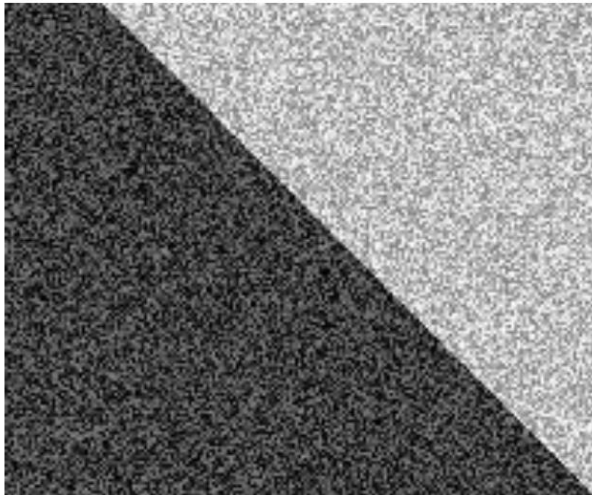
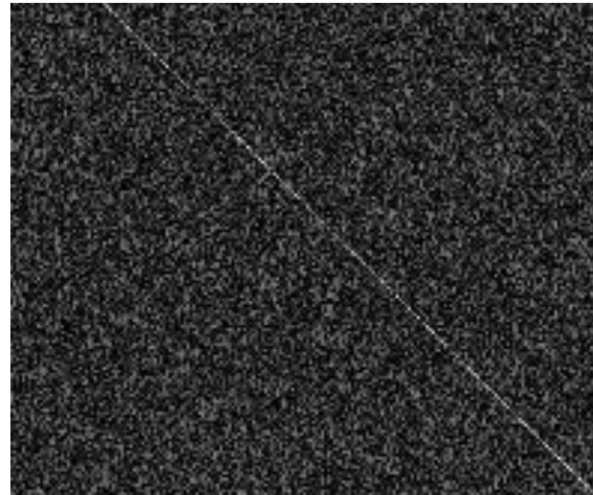
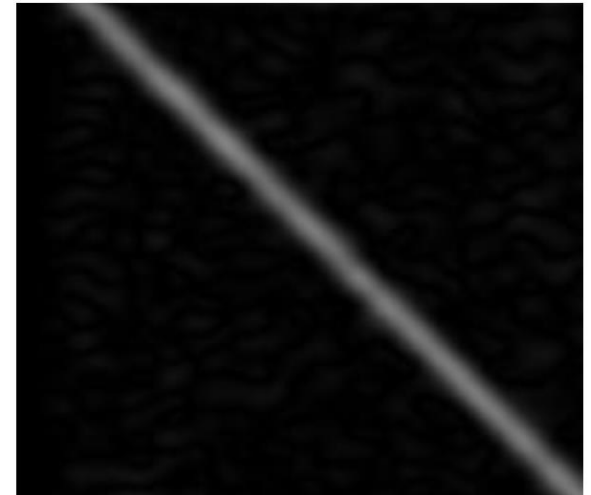


Image + Noise



Derivatives detect edge *and* noise



Smoothed derivative removes noise, but blurs edge

# Example



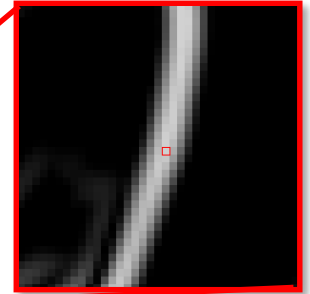
- original image (Lena)

# Finding edges



gradient magnitude

# Finding edges

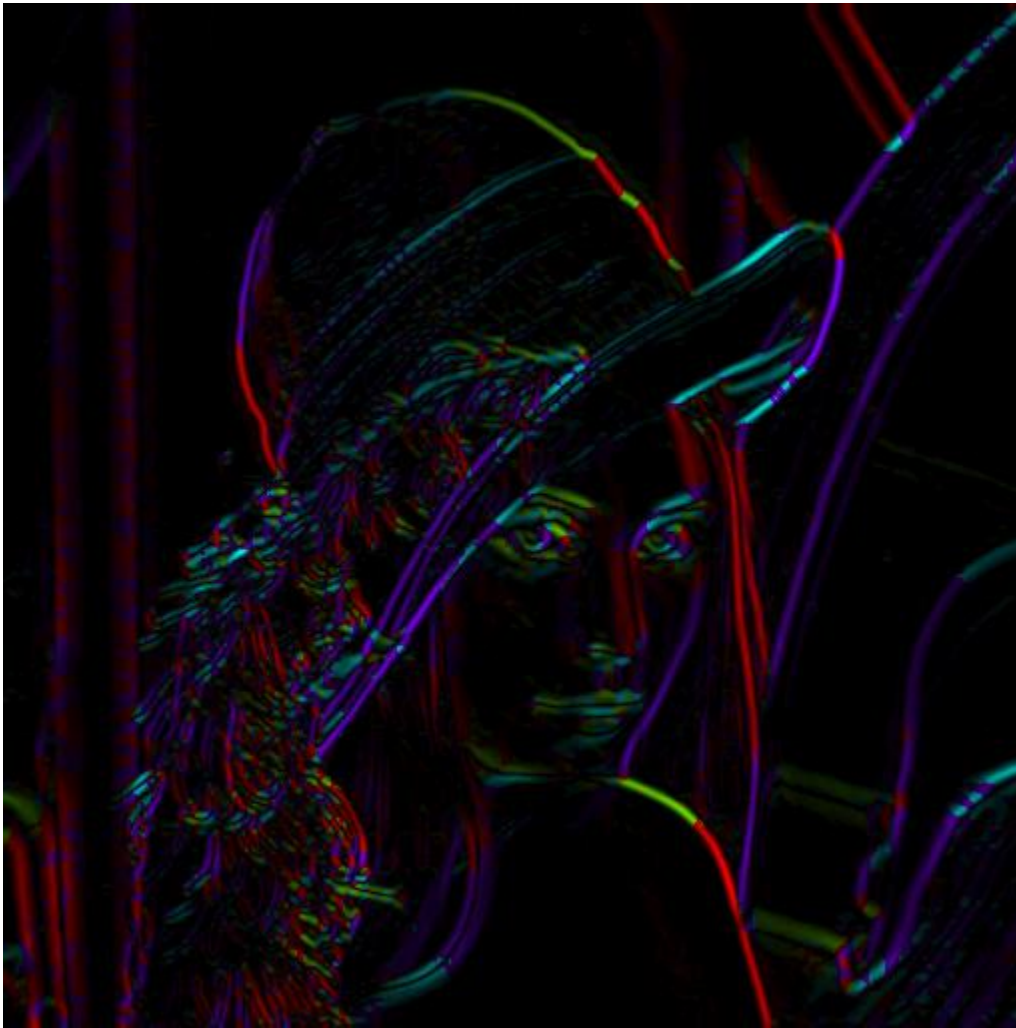


where is the edge?

thresholding

# Get Orientation at Each Pixel

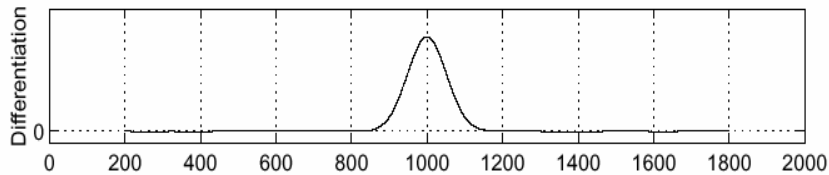
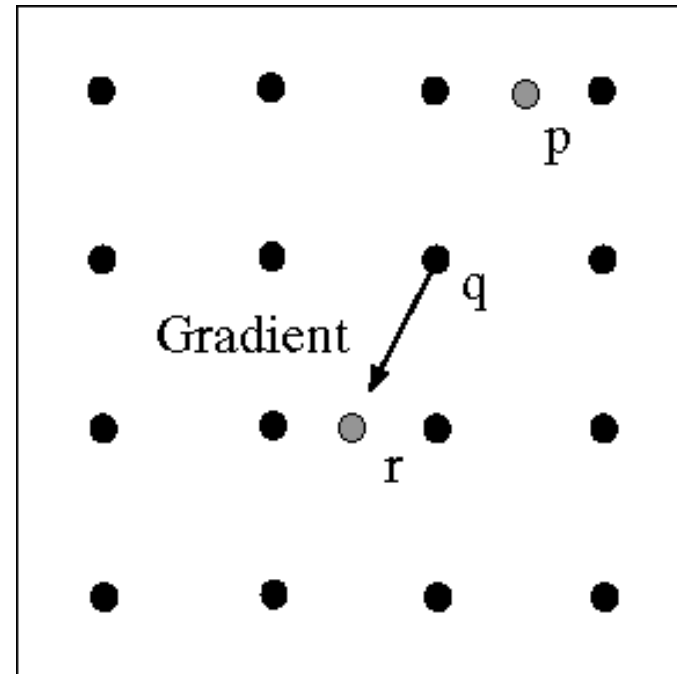
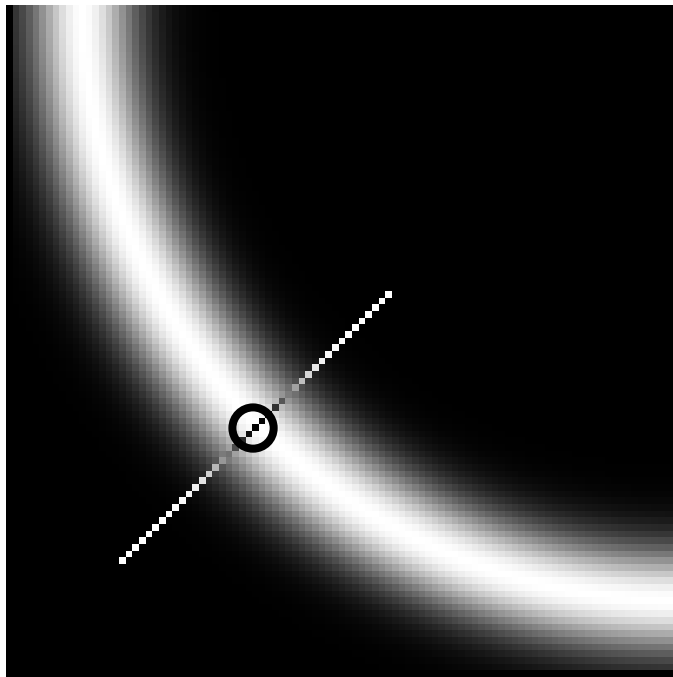
- Threshold at minimum level
- Get orientation



$$\text{theta} = \text{atan2}(\text{gy}, \text{gx})$$



# Non-maximum suppression



- Check if pixel is local maximum along gradient direction
  - requires *interpolating* pixels p and r

# Before Non-max Suppression



# After Non-max Suppression



# Finding edges



thresholding

# Finding edges



thinning

(non-maximum suppression)



# Canny edge detector

MATLAB: `edge(image, 'canny')`



1. Filter image with derivative of Gaussian
2. Find magnitude and orientation of gradient
3. Non-maximum suppression
4. Linking and thresholding (hysteresis):
  - Define two thresholds: low and high
  - Use the high threshold to start edge curves and the low threshold to continue them



# Canny edge detector

- Still one of the most widely used edge detectors in computer vision

J. Canny, [\*A Computational Approach To Edge Detection\*](#), IEEE Trans. Pattern Analysis and Machine Intelligence, 8:679-714, 1986.

- Depends on several parameters:

$\sigma$  : width of the Gaussian blur

high threshold

low threshold

# Canny edge detector



original



Canny with  $\sigma = 1$

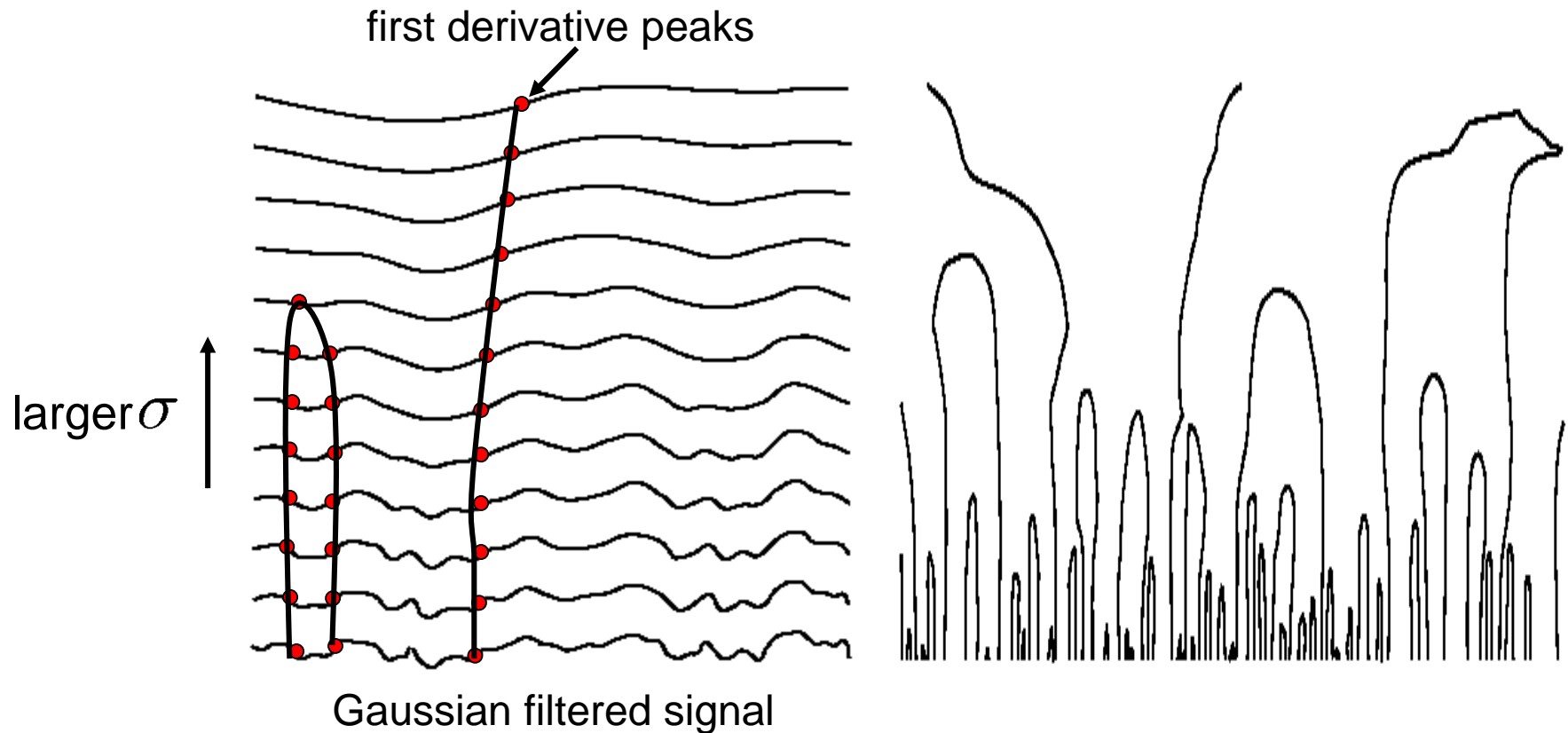


Canny with  $\sigma = 2$

- The choice of  $\sigma$  depends on desired behavior
  - large  $\sigma$  detects “large-scale” edges
  - small  $\sigma$  detects fine edges



# Scale space (Witkin 83)



- Properties of scale space (w/ Gaussian smoothing)
  - edge position may shift with increasing scale ( $\sigma$ )
  - two edges may merge with increasing scale
  - an edge may **not** split into two with increasing scale

Questions?