CS 5643 PHYSICALLY BASED ANIMATION, PROF. DOUG JAMES, TUES FEB 3, 2015

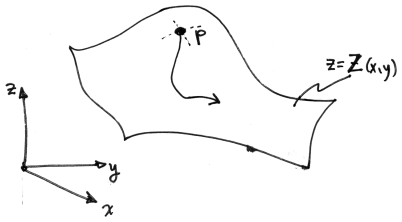
Particle systems with holonomic constraints

Due: *Thurs Feb 12, 2015 (at start of class)*

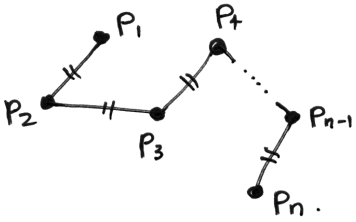
The following problems consider a 3D particle system (of n particles) subject to gravititational acceleration $\boldsymbol{g}$, and specified holonomic constraints of the form $\mathbf{C}(\mathbf{p}) = \mathbf{0}$. In each case, derive specific mathematical expressions for the following:

1. the constraint(s), $\mathbf{C}$, and their total number, N_c;
2. the constraint Jacobian, $\mathbf{J}$, and its dimensions; and
3. an expression for the constraint force $\mathbf{f}_c$ on the system in terms of the particle positions. Be as explicit as possible.

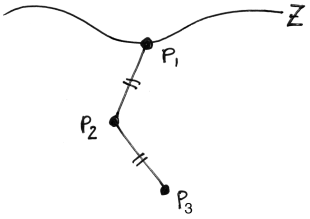
For simplicity, assume that all particles have mass m. Note that there are different answers depending on how you formulate your constraints.



Problem 1



Problem 2



Problem 3

PROBLEM 1. Particle on a height field: Consider a particle with position $\mathbf{p} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$, and a surface specified by a heightfield function, $z = Z(x, y)$. The particle is constrained to be attached to the heightfield surface at all times.

PROBLEM 2. Inextensible n-particle chain: Consider n particles $\mathbf{p}_1$, $\mathbf{p}_2$, ..., $\mathbf{p}_n$ with massless rods of length ℓ attached between each consecutive particle, i.e., the $n-1$ line segments $\overline{\mathbf{p}_1\mathbf{p}_2}$, ..., $\overline{\mathbf{p}_{n-1}\mathbf{p}_n}$. (Assume that $n \geq 2$.)

PROBLEM 3. Double pendulum on a height field: In this problem, you will combine the constraints from the previous two problems for the case of $n = 2$. Specifically, consider three particles with positions $\mathbf{p}_0$, $\mathbf{p}_1$, $\mathbf{p}_2$ with the following constraints:

- particle $\mathbf{p}_0$ is constrained to the height field from question 1, and
- there are rigid link constraints (of length ℓ) on the line segments $\overline{\mathbf{p}_0\mathbf{p}_1}$ and $\overline{\mathbf{p}_1\mathbf{p}_2}$.