15 Efficient meshes

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Follows chapter 16 in RTR 4e

Basics of efficiency for meshes

Use triangle or quad meshes

- general polygon meshes lead to too much complexity
- quad meshes are great for some applications but more constrained

Use shared-vertex triangle meshes for GPU applications

- major memory/bandwidth savings over separate triangles
- if you get separate triangles, merge them in a pre-process

Store most data at vertices

- there are ~half as many vertices as faces
- vertex data may be interpolated across faces
- in typical GPU mesh representation, vertices must be duplicated to create discontinuities

More sophistication in mesh storage

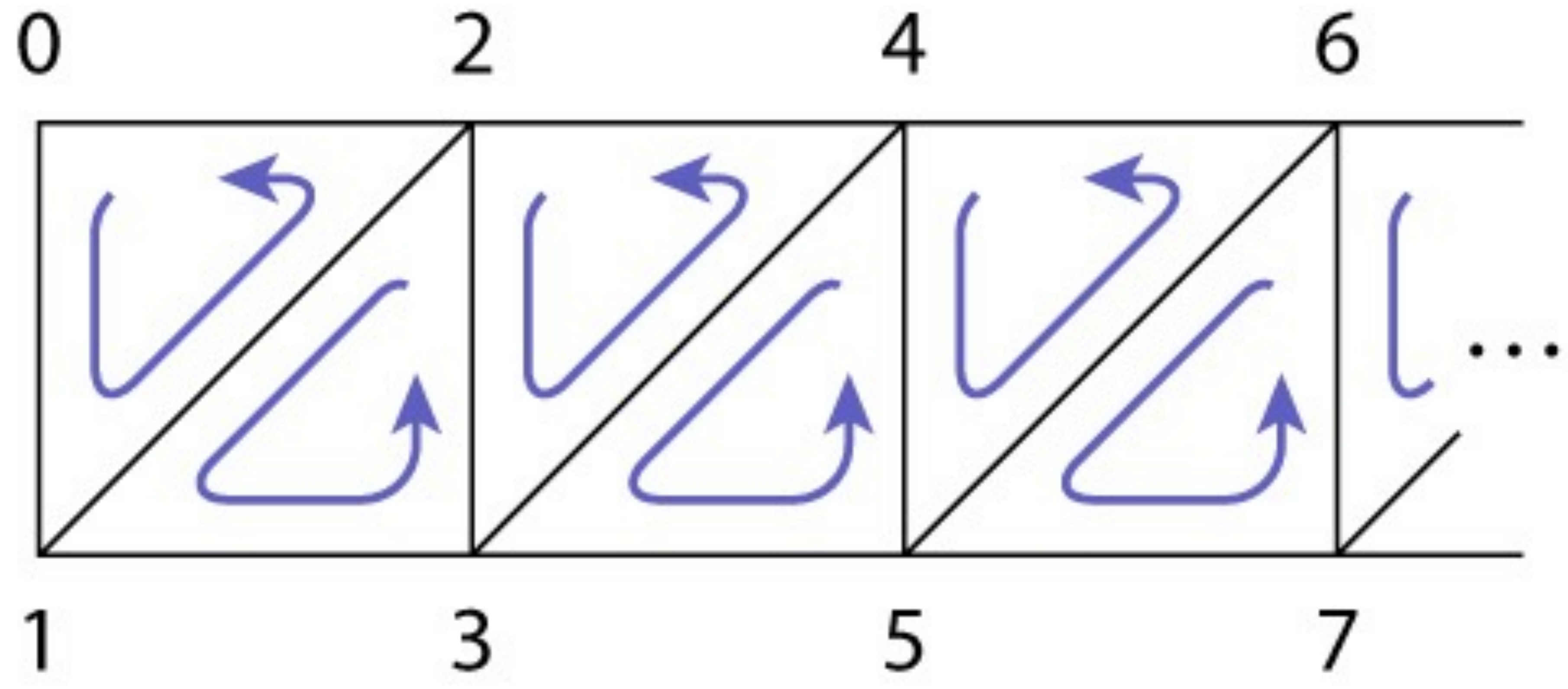
Optimizing vertex order

- strips and fans as classic examples (when per-frame bandwidth was the concern)
- modern systems don’t use these but optimize for hit rate in vertex cache

Reducing the number of triangles

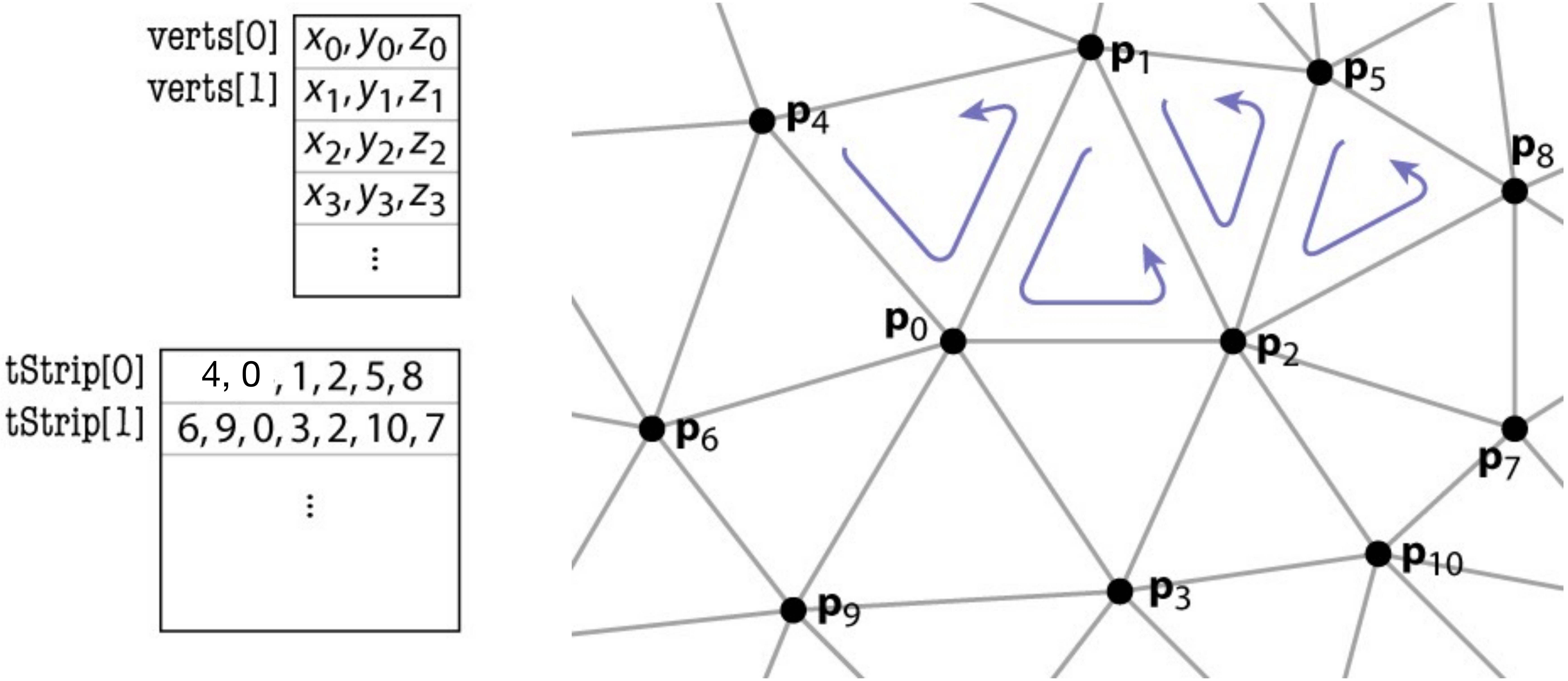
- ultimately this is needed to save more time and space
- many *levels of detail* are useful
  - simpler meshes for faraway objects
  - simpler meshes for lower-resolution screens
  - simpler meshes for lower-performance hardware or networks

Triangle strips



- **Take advantage of the mesh property**
  - each triangle is usually adjacent to the previous
  - let every vertex create a triangle by reusing the second and third vertices of the previous triangle
  - every sequence of three vertices produces a triangle (but not in the same order)
  - e. g., 0, 1, 2, 3, 4, 5, 6, 7, … leads to
    (0 1 2), (2 1 3), (2 3 4), (4 3 5), (4 5 6), (6 5 7), …
  - for long strips, this requires about one index per triangle

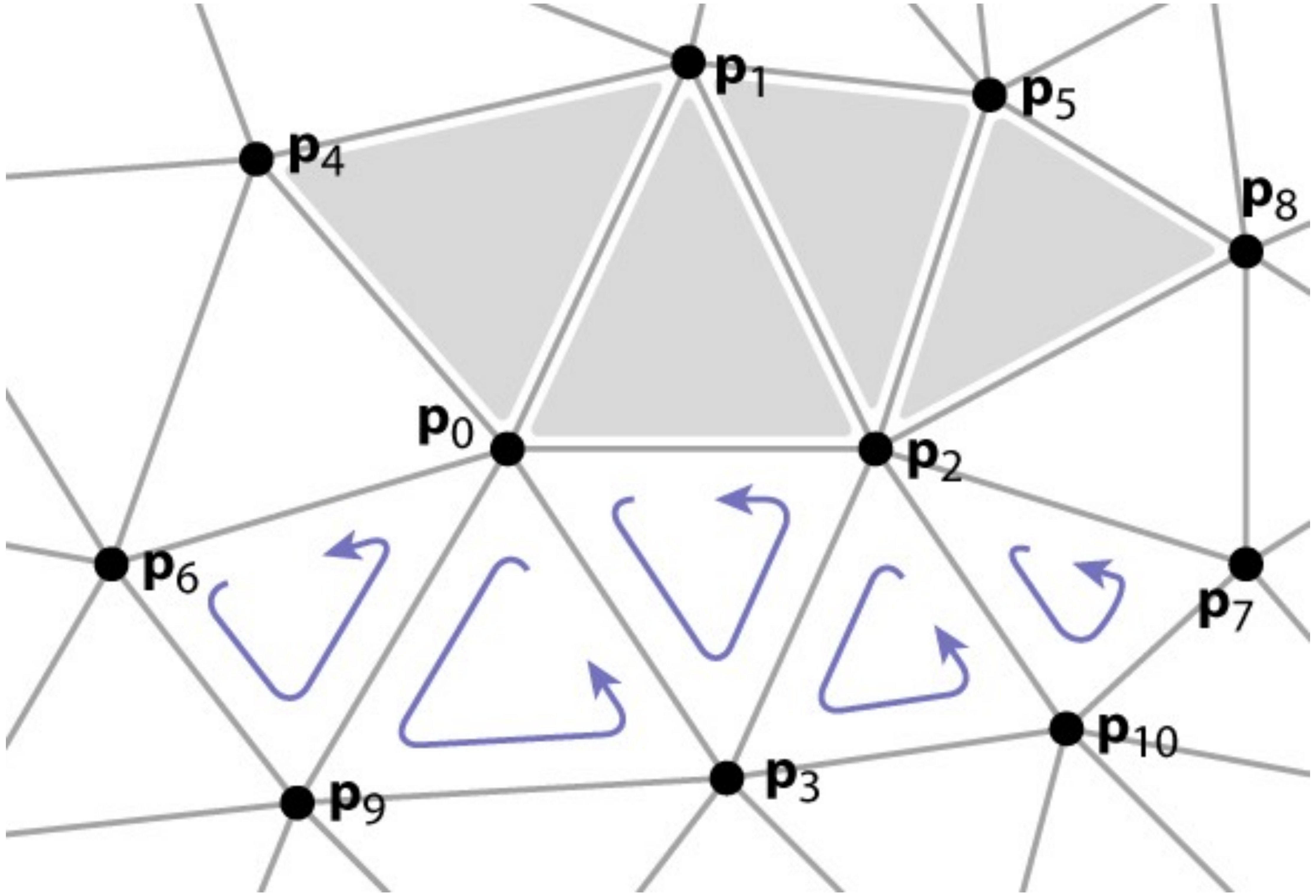
Triangle strips



Triangle strips

verts[0]	x_0, y_0, z_0
verts[1]	x_1, y_1, z_1
	x_2, y_2, z_2
	x_3, y_3, z_3
	⋮

tStrip[0]	4, 0, 1, 2, 5, 8
tStrip[1]	6, 9, 0, 3, 2, 10, 7
	⋮

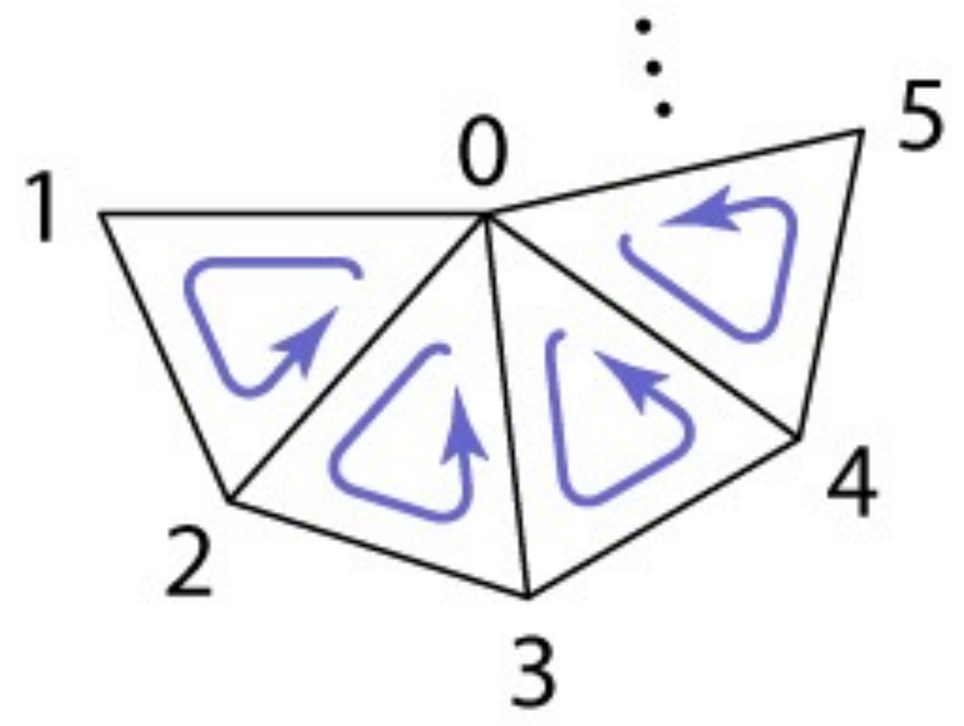


Triangle strips

- **array of vertex positions**
  - float[n_V][3]: 12 bytes per vertex
    - (3 coordinates x 4 bytes) per vertex
- **array of index lists**
  - int[n_S][***variable***]: 2 + ***n*** indices per strip
  - on average, (1 + ε) indices per triangle (assuming long strips)
    - 2 triangles per vertex (on average)
    - about 4 bytes per triangle (on average)
- **total is 20 bytes per vertex (limiting best case)**
  - factor of 3.6 over separate triangles; 1.8 over indexed mesh

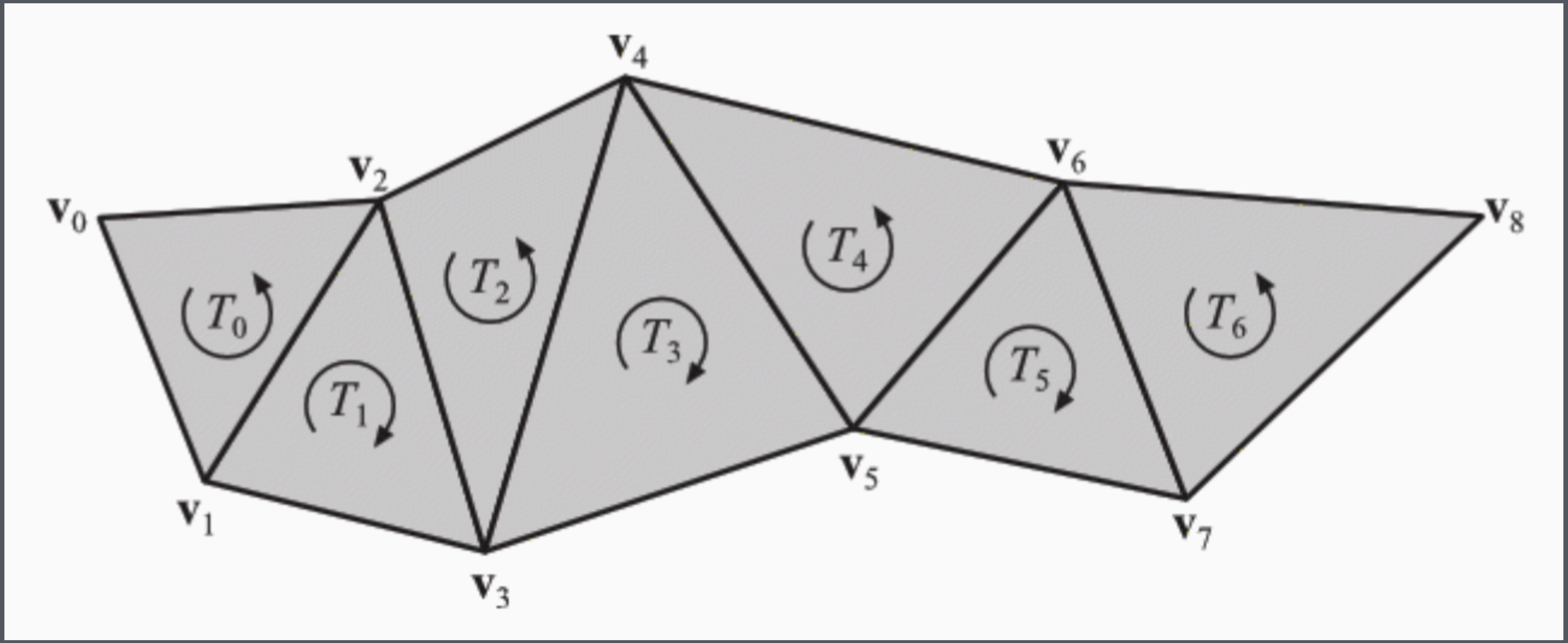
Triangle fans

- **Same idea as triangle strips, but keep oldest rather than newest**
  - every sequence of three vertices produces a triangle
  - e. g., 0, 1, 2, 3, 4, 5, … leads to
    (0 1 2), (0 2 3), (0 3 4), (0 4 5), …
  - for long fans, this requires about one index per triangle
- **Memory considerations exactly the same as triangle strip**



Vertex cache and mesh ordering

Triangle strips gain efficiency by caching the most recent two vertices



- we are essentially using a FIFO cache policy with a size of 2
- cache miss rate approaches 1 miss / triangle

Optimizing for larger caches

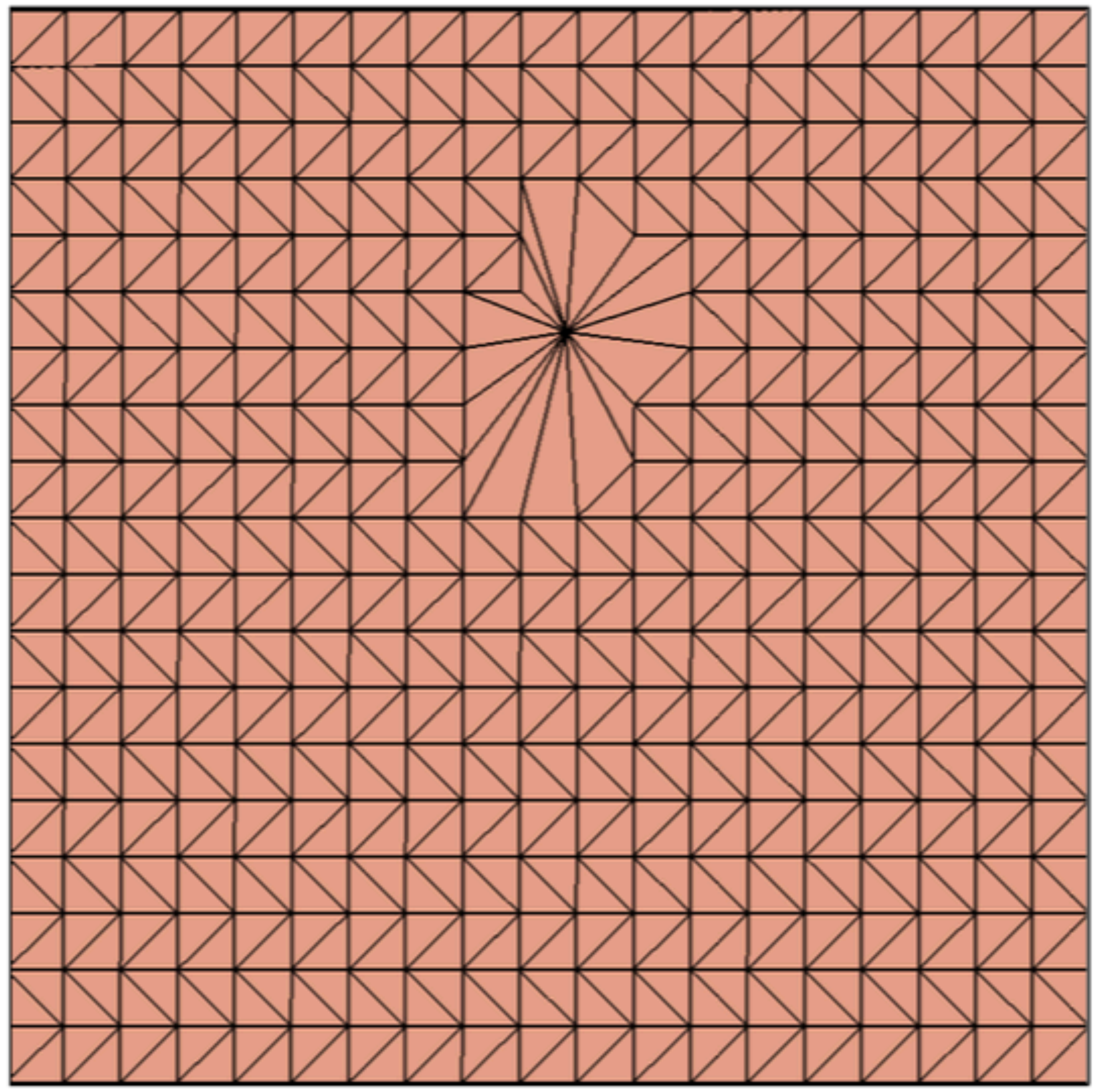
With indexed meshes, saving indices is less important

- we store lots of data at vertices; ~6 indices is the least of our worries
- just putting meshes in triangle-strip order gives you the same vertex caching behavior ("transparent" vertex caching)

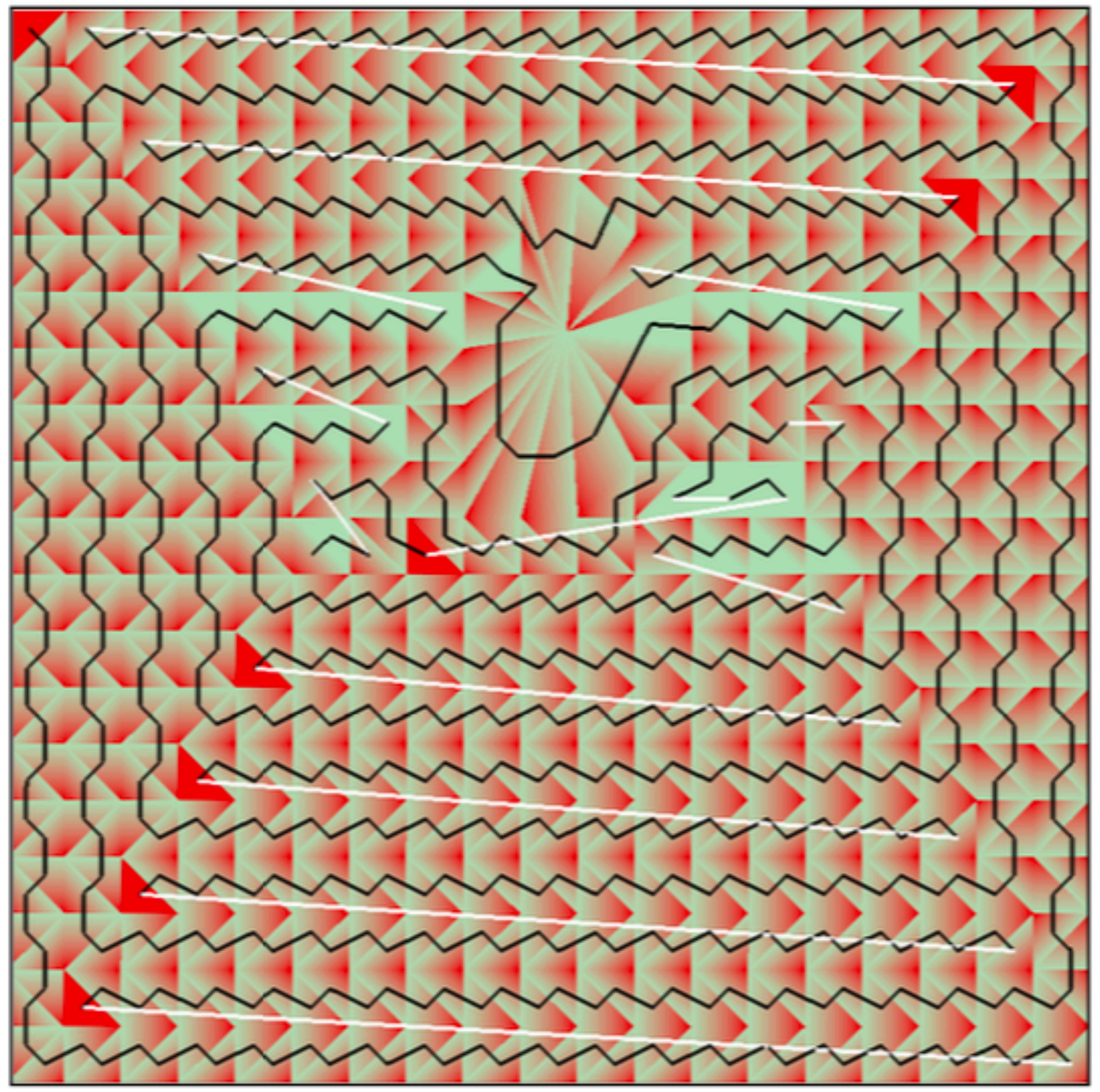
GPU pipelines are built with post-transform vertex caches

- cache the results of the vertex processing stage
- cache hits can save substantial computation
- (for parallelism newer systems process primitives in batches, but the effect is similar)

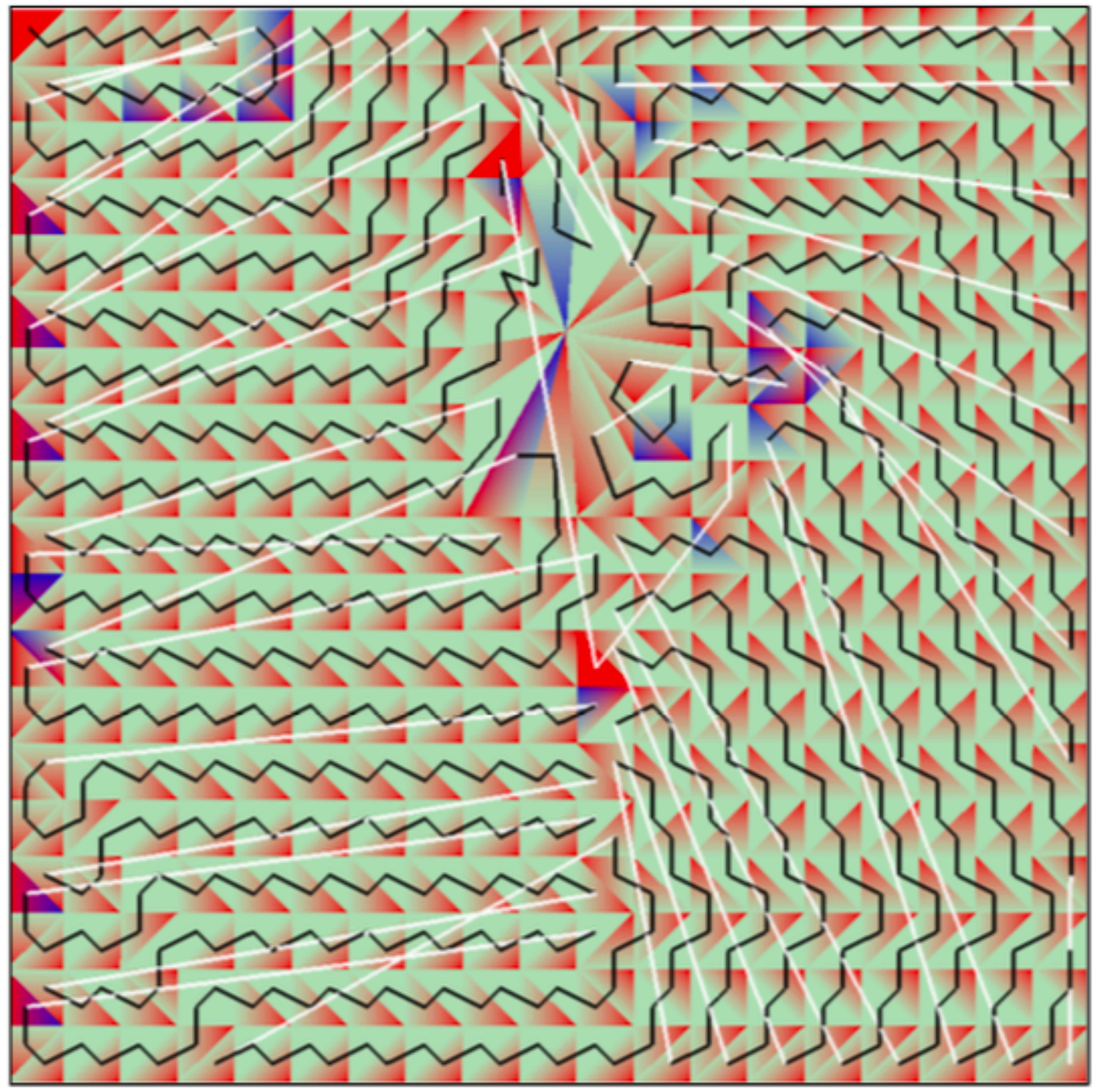
As with other applications of caches, now order of data access matters



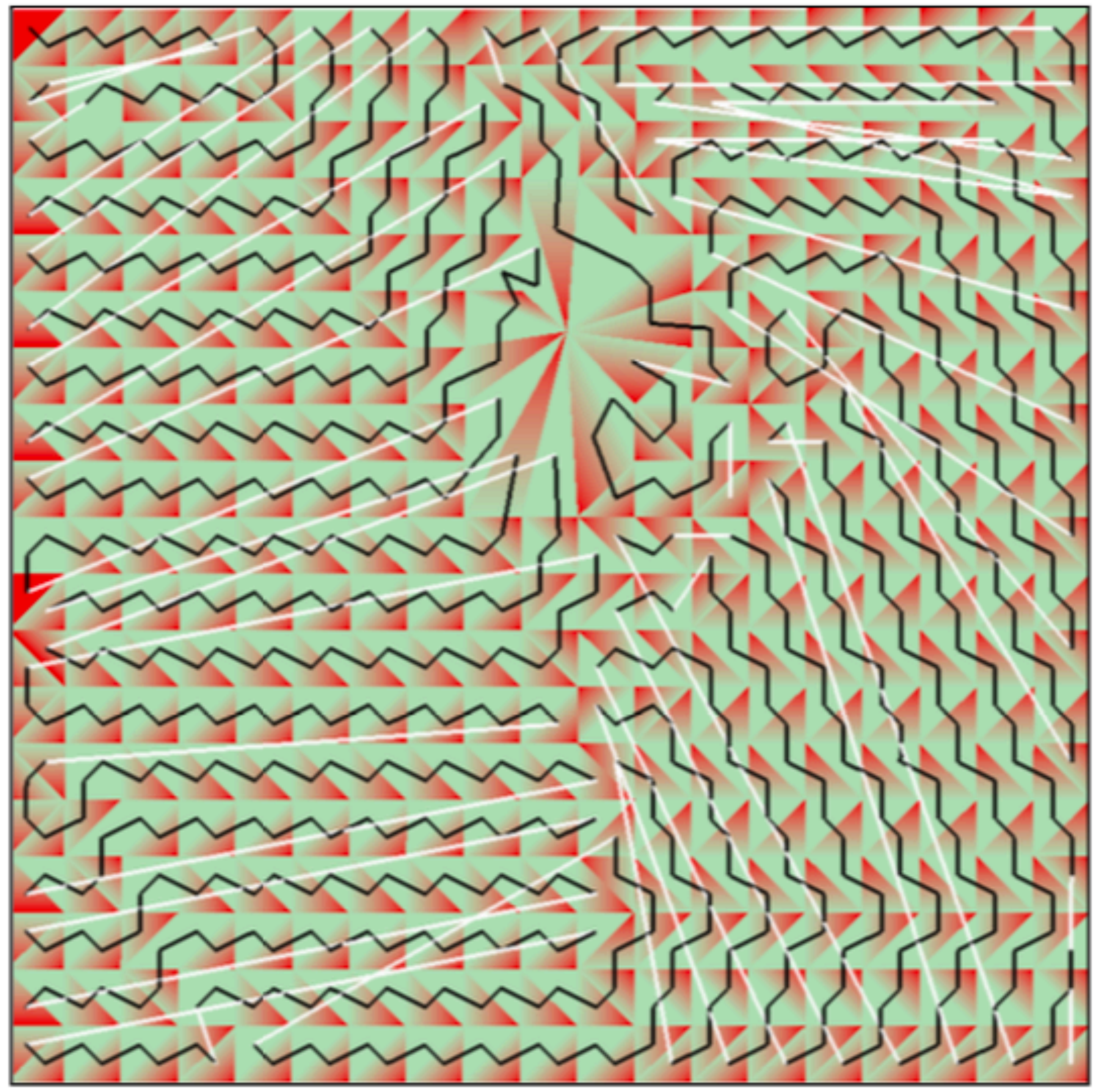
(a) Original mesh (704 faces)



(b) Traditional strips ($r = 0.99$; $b = 1.29$; $\mathcal{C} = 34.3$)



(c) Greedy strip-growing ($r = 0.62$; $b = 1.28$; $\mathcal{C} = 22.3$)



(d) Local optimization ($r = 0.60$; $b = 1.32$; $\mathcal{C} = 21.7$)

r is the average cache miss rate

[Hoppe 1999]

(a) K-Cache-Reorder Lin and Yu [2006]   (b) D3DXMesh based on [Hoppe 1999]   (c) OpenCCL Yoon and Lindstrom [2006]   (d) dfsrendseq Bogomjakov et al. [2001]   (e) Our work

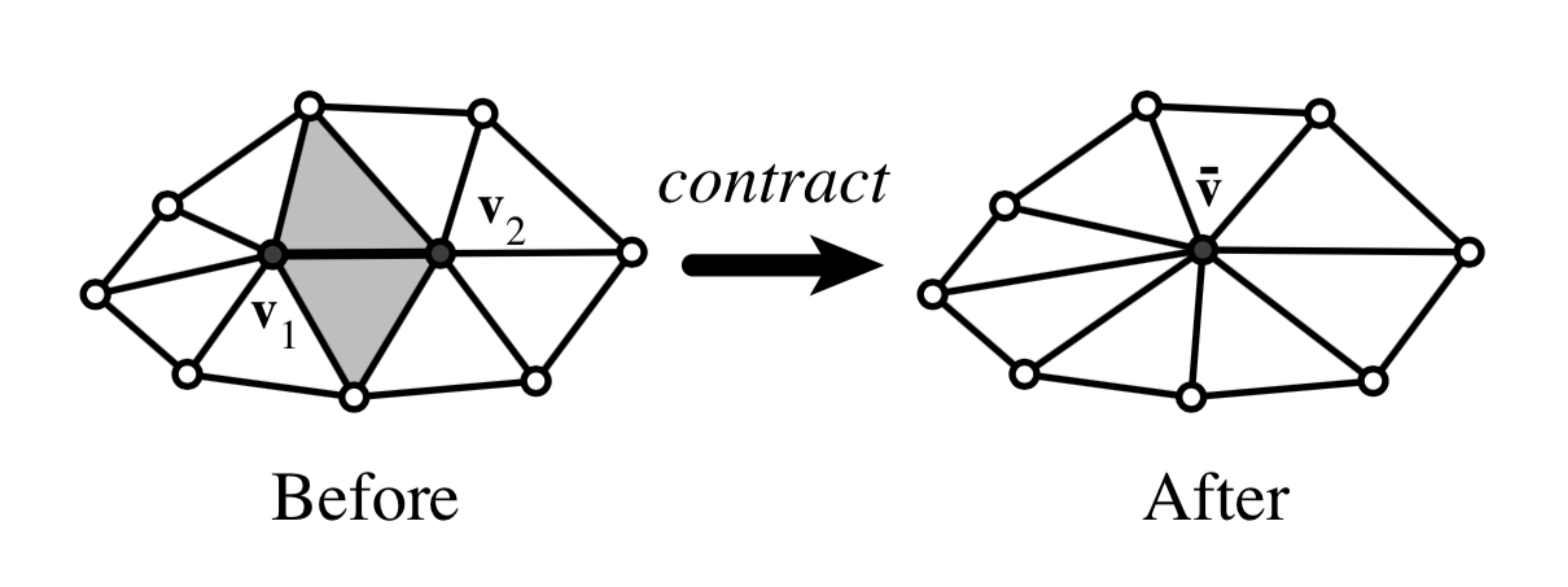
[Sander et al. 2007 “Fast Triangle Reordering for Vertex Locality and Reduced Overdraw”]

Mesh simplification

Many ways to simplify meshes

- remove chunks, retriangulate hole
- quantize vertices to centers of voxels

Particularly simple and effective is edge collapses, or edge contractions:



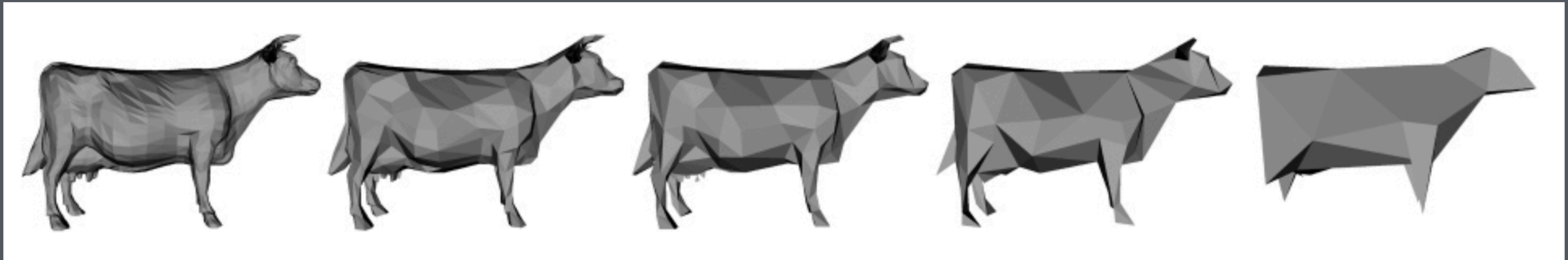
Quadric Error Metric

Edge-collapse simplification produces a sequence of meshes

- each mesh has one fewer face
- each is derived from the previous by a single edge collapse

Key question: where to put the vertex after the collapse?

- at first vertex? at second? at midpoint?
- can choose location as the solution to an optimization



Where to put the new vertex?

It depends on the mesh geometry:

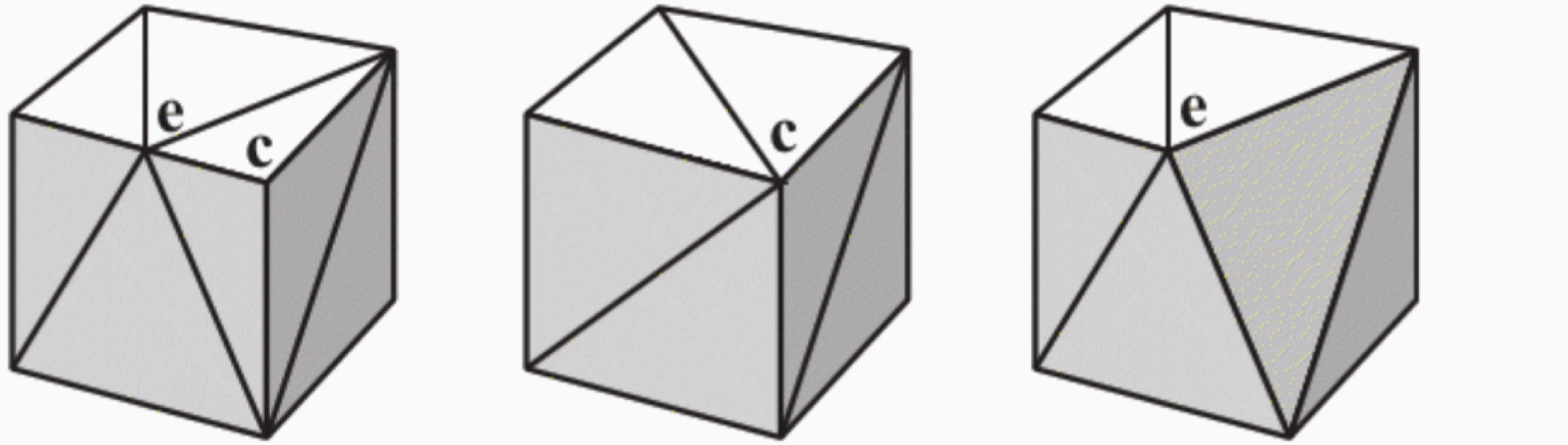


Figure 16.19. The left figure shows a cube with an extra point along one edge. The middle figure shows what happens if this point **e** is collapsed to corner **c**. The right figure shows **c** collapsed to **e**.

- one way to formalize: the new vertex should be close to the planes of the triangles around it before the edge collapse

Garland & Heckbert QEM

A particularly convenient error metric: sum of squared distances to planes

- each plane has an equation, can be represented as a 4-vector (*a*, *b*, *c*, *d*)
  with (*a*, *b*, *c*) components normalized
- distance of a vertex **v** from the plane **p** is then the inner product $\mathbf{p}^\mathsf{T}\mathbf{v}$
- squared distance from plane is in the form $\mathbf{v}^\mathsf{T}\mathbf{M}\mathbf{v}$ for a 4x4 **M** (a quadric)

$$
\begin{aligned}
\Delta(\mathbf{v}) &= \sum_{\mathbf{p}\in \text{planes}(\mathbf{v})} (\mathbf{v}^\mathsf{T}\mathbf{p})(\mathbf{p}^\mathsf{T}\mathbf{v}) \\
&= \sum_{\mathbf{p}\in \text{planes}(\mathbf{v})} \mathbf{v}^\mathsf{T}(\mathbf{p}\mathbf{p}^\mathsf{T})\mathbf{v} \\
&= \mathbf{v}^\mathsf{T}\left(\sum_{\mathbf{p}\in \text{planes}(\mathbf{v})} \mathbf{K}_\mathbf{p}\right)\mathbf{v}
\end{aligned}
$$

- and better yet, the sum-squared distance from several planes is still in the form $\mathbf{v}^\mathsf{T}\mathbf{Q}\mathbf{v}$

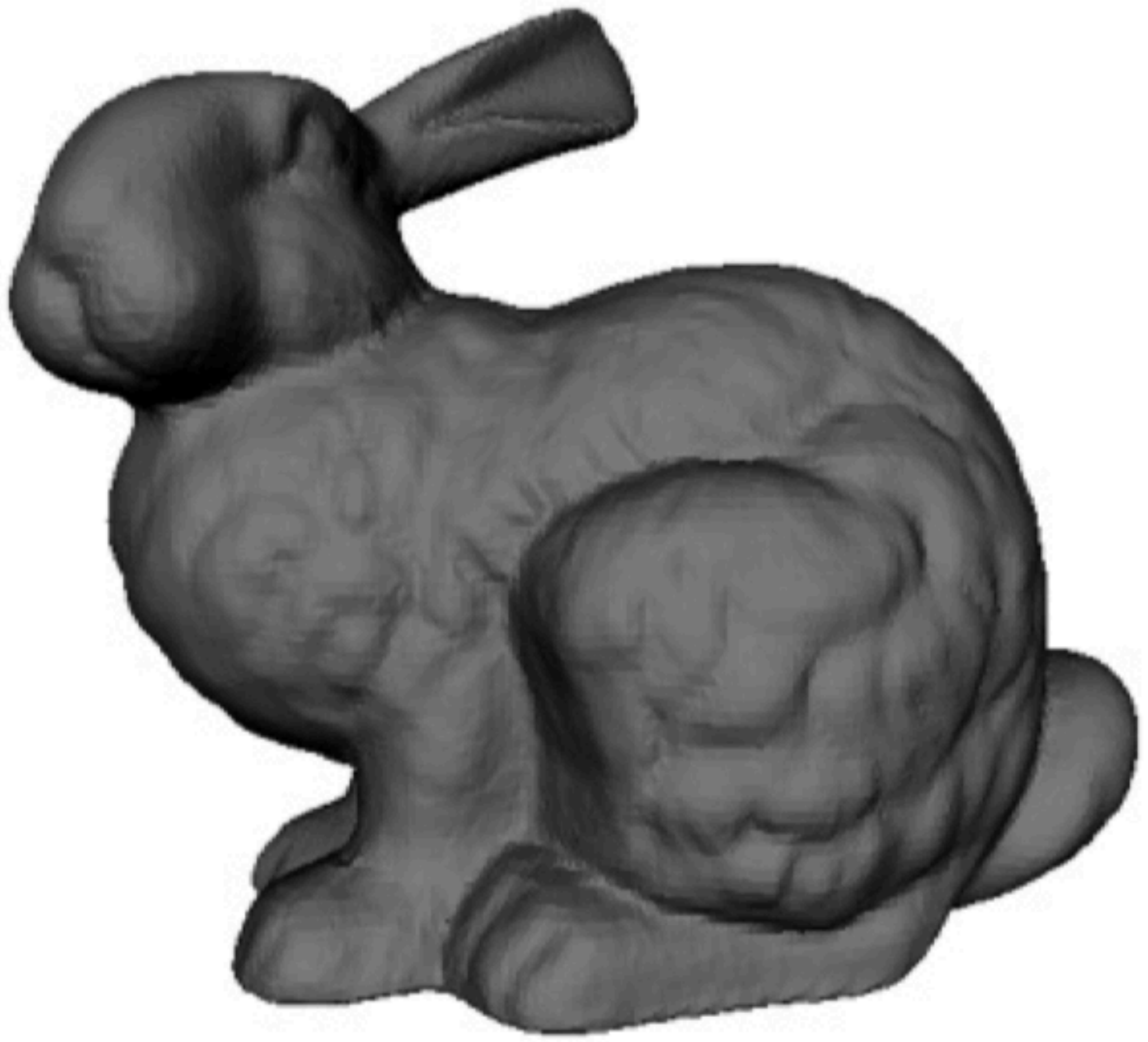
QEM simplification

With the error in the form of a quadric per vertex:

- the matrix is easy to compute from the surrounding triangles
- the error is easy to optimize. Given $\mathbf{Q}_1$ and $\mathbf{Q}_2$ belonging to a pair of vertices $\mathbf{v}_1$ and $\mathbf{v}_2$, we simply sum the errors of the two vertices:

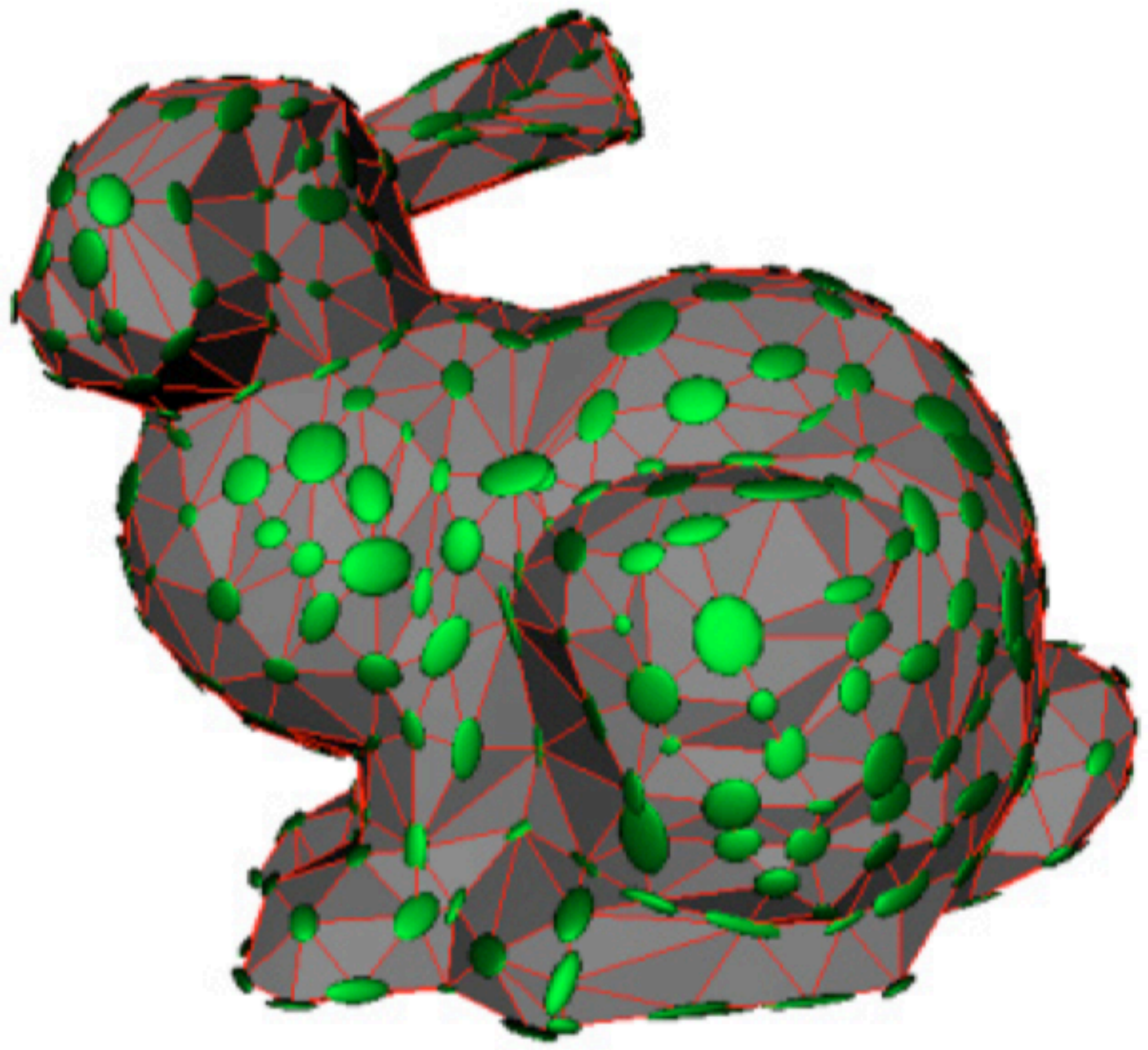
$$\begin{aligned}\Delta(\mathbf{v}) &= \Delta_1(\mathbf{v}) + \Delta_2(\mathbf{v}) \\ &= \mathbf{v}^T \mathbf{Q}_1 \mathbf{v} + \mathbf{v}^T \mathbf{Q}_2 \mathbf{v} \\ &= \mathbf{v}^T (\mathbf{Q}_1 + \mathbf{Q}_2) \mathbf{v}\end{aligned}$$

- minimizing this error is a 4x4 linear system—very fast
- algorithm
  - 0. compute **Q**s for all vertices, compute errors for all potential edge collapses.
  - 1. use priority queue to find smallest-error edge. Collapse it; update the neighboring **Q**s.
  - 2. repeat until mesh is small enough!



69k faces

1k faces

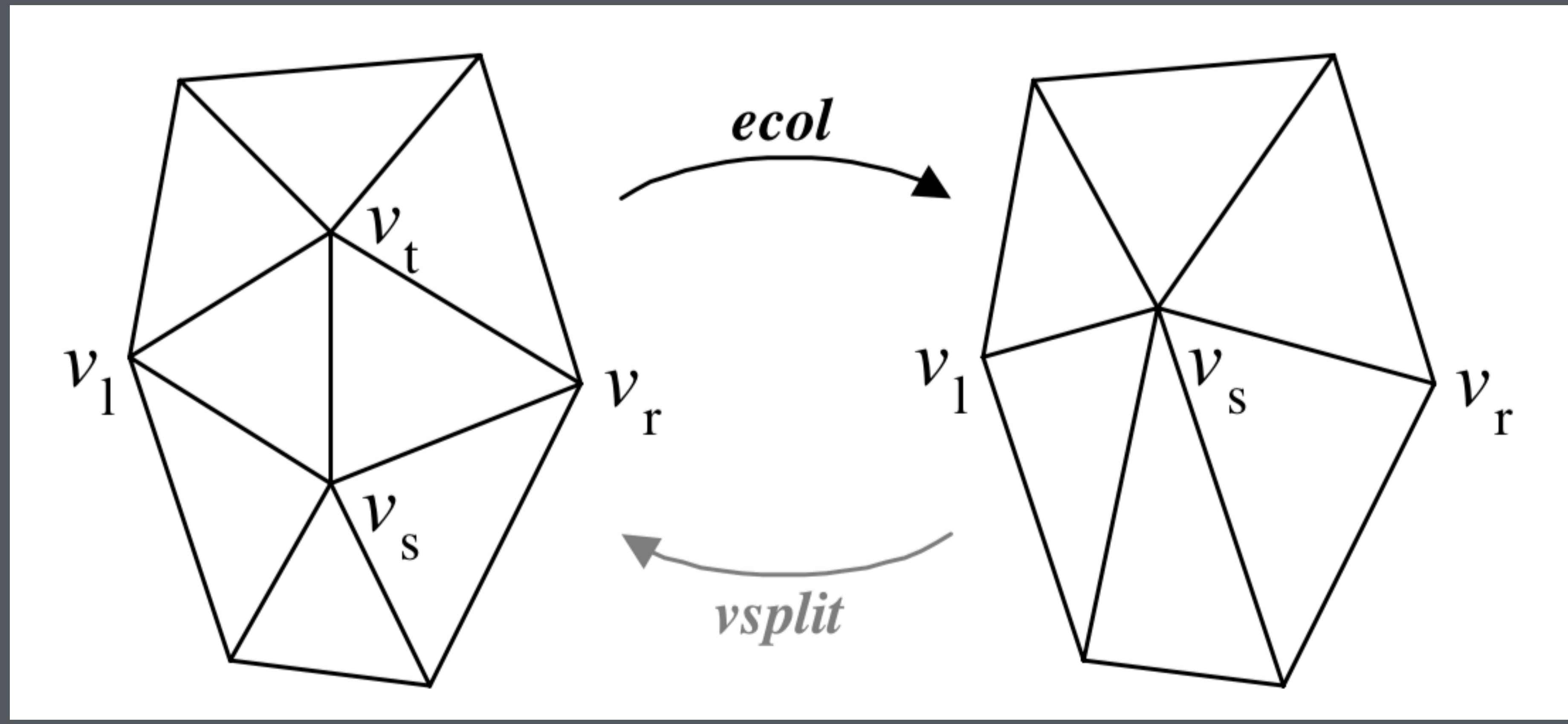


surfaces of constant cost for reducing to 999 faces

[Garland & Heckbert 1997 “Surface Simplification Using Quadric Error Metrics”]

Continuous level-of-detail: Progressive Meshes

Key observation: edge collapse is invertible



- just need to store (offsets to) the locations of the two new vertices

Thus a sequence of edge collapses, reversed, is a representation for a mesh

$$(\hat{M} = M^n) \xrightarrow{ecol_{n-1}} \ldots \xrightarrow{ecol_1} M^1 \xrightarrow{ecol_0} M^0 \,.$$

$$M^0 \xrightarrow{vsplit_0} M^1 \xrightarrow{vsplit_1} \ldots \xrightarrow{vsplit_{n-1}} (M^n = \hat{M})$$

[Hoppe 1996
"Progressive Meshes"]

Progressive Meshes

Store full representation, load various levels of detail

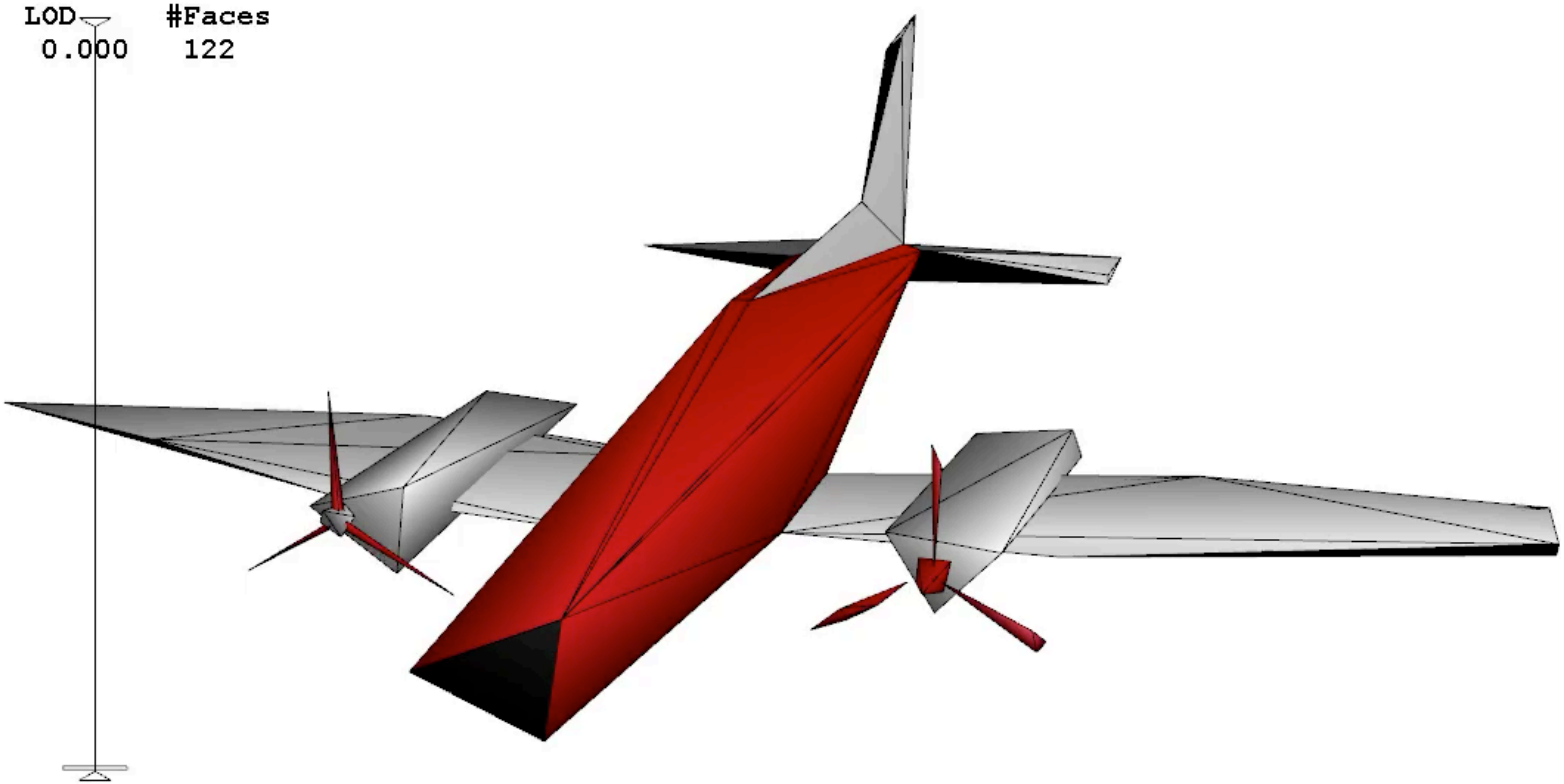
- just load or transmit a prefix of the list of edge splits
- can change level of detail smoothly depending on size/distance/salience/etc.

Can interpolate (“geomorphs”)

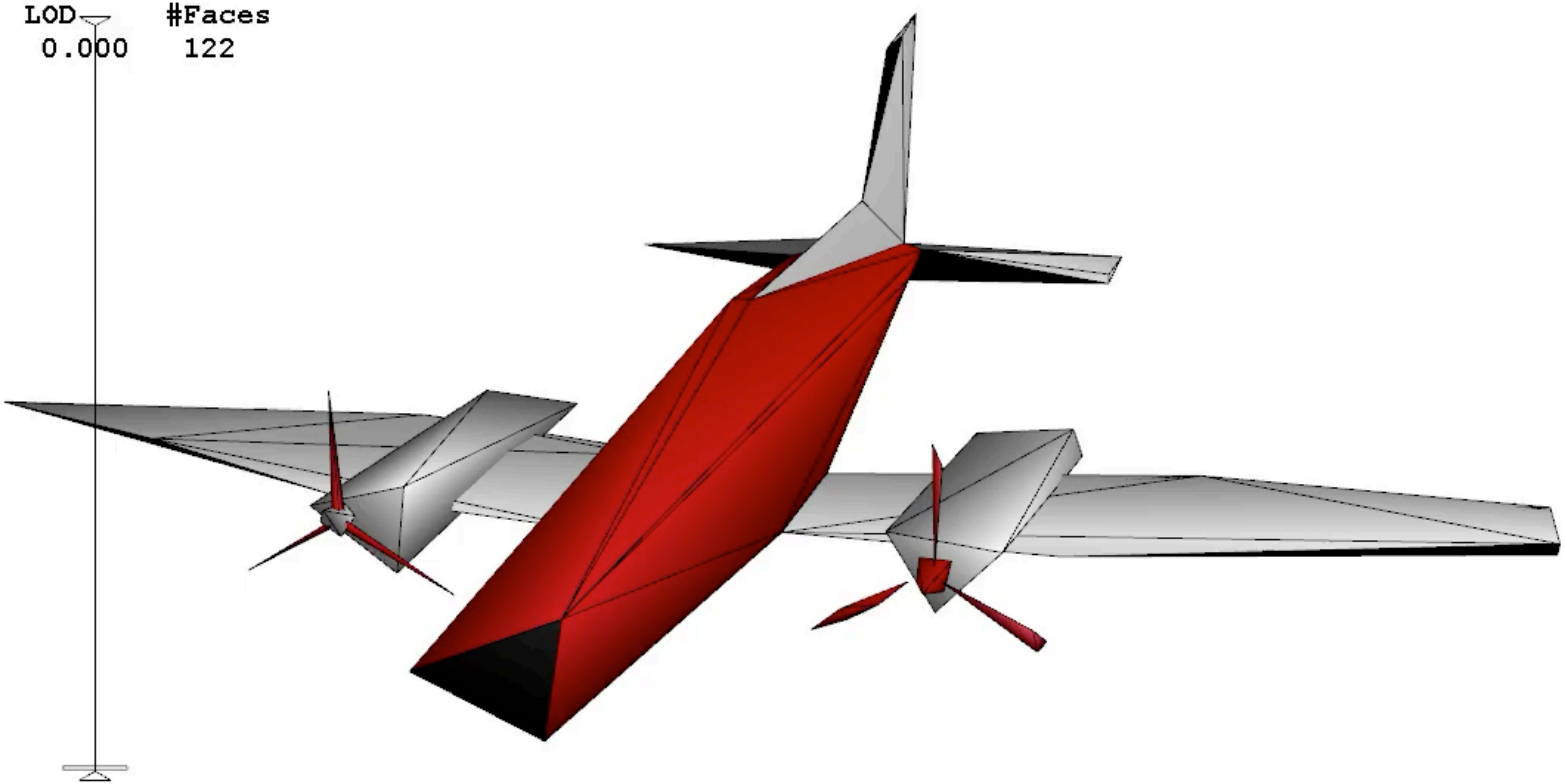
- sudden edge splits/collapses are jarring
- interpolate new vertices from merged position to new positions
- leads to truly continuous LoD

Extra details (of QEM and PM)

- boundaries, creases—want to preserve them
- merging of small pieces—otherwise can’t simplify enough
- maintenance of additional attributes—throw them in the metric too



[Hoppe 1996 “Progressive Meshes”]



[Hoppe 1996 “Progressive Meshes”]