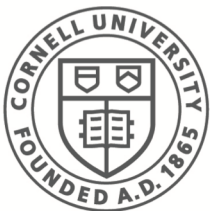


CS 5430:  
**Information Flow**  
Part I: Static Enforcement

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# Access Control

---

Access control associates restrictions with:

- Containers (CS5430)
  - access control lists, capabilities
- Values (CS5432)
  - information flow control

Example:  $x := y; \dots z := x$

- container: value in  $y$  can be leaked by reading  $z$
- value: restrictions on  $z$  include all restrictions on  $y$   
... no need to trust clients who access  $y$ .

# Flow-based Access Control (FBAC)

---

- Labels propagate with flow.
- Labels restrict allowed info flow.

Flow-Label Invariant (FLI):

$$v \rightarrow w \implies \Gamma(v) \sqsubseteq \Gamma(w)$$

$v \rightarrow w$ :  $v$  flows to  $w$ . *NB really  $\rightarrow_S$  for flow in  $S$ .*

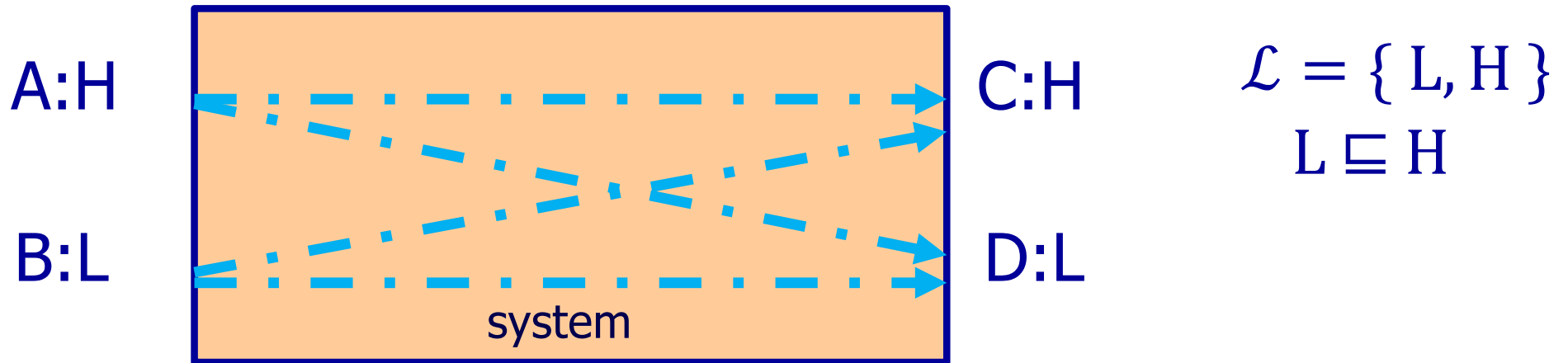
$\Gamma(v)$ : label assoc with  $v$  --- gives restrictions on use of  $v$

$\sqsubseteq$  reflexive and transitive relation on a set  $\mathcal{L}$  of labels.

$\lambda_1 \sqsubseteq \lambda_2$ :  $\lambda_2$  includes all restrictions in  $\lambda_1$

# An application of FBAC

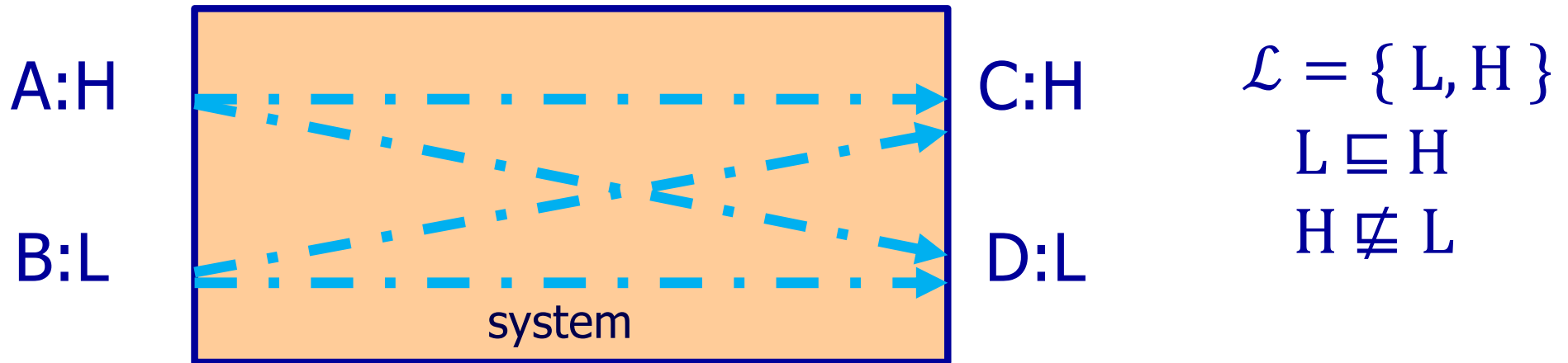
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$$v \rightarrow w \implies \Gamma(v) \sqsubseteq \Gamma(w)$$

# An application of FBAC

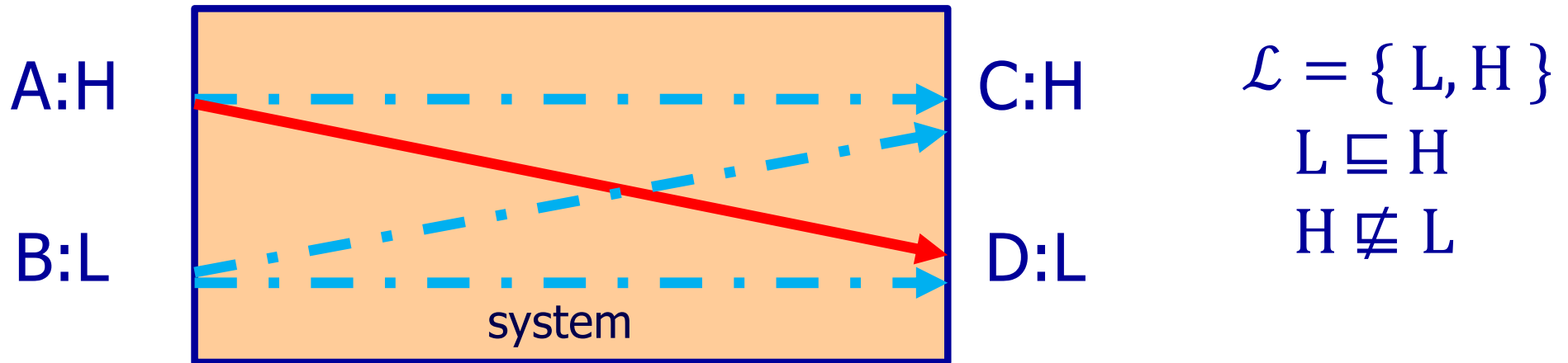
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$$v \rightarrow w \implies \Gamma(v) \sqsubseteq \Gamma(w)$$
$$= \Gamma(v) \not\sqsubseteq \Gamma(w) \implies \neg(v \rightarrow w)$$

# An application of FBAC

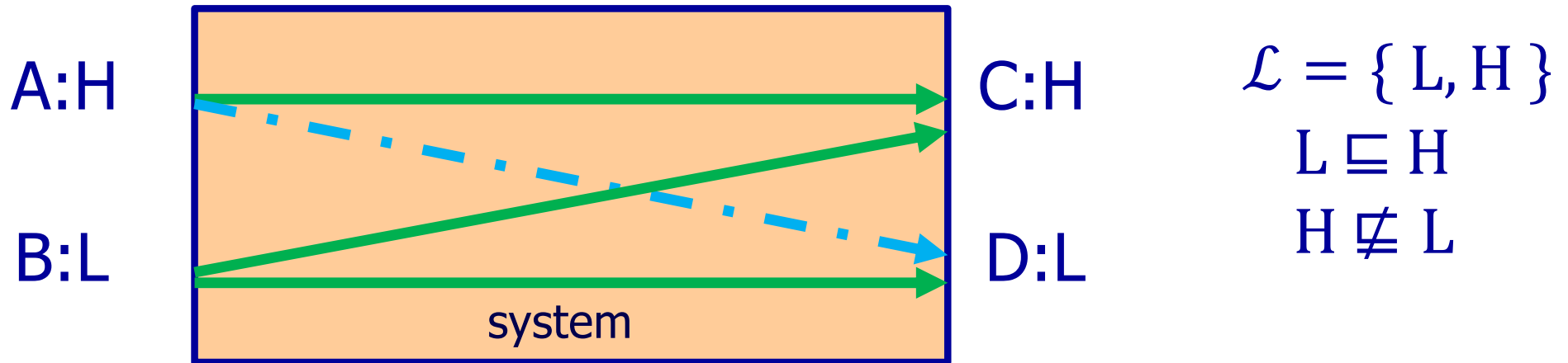
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$$\begin{aligned} v \rightarrow w &\implies \Gamma(v) \sqsubseteq \Gamma(w) \\ &= \Gamma(v) \not\sqsubseteq \Gamma(w) \implies \neg(v \rightarrow w) \end{aligned}$$

# An application of FBAC

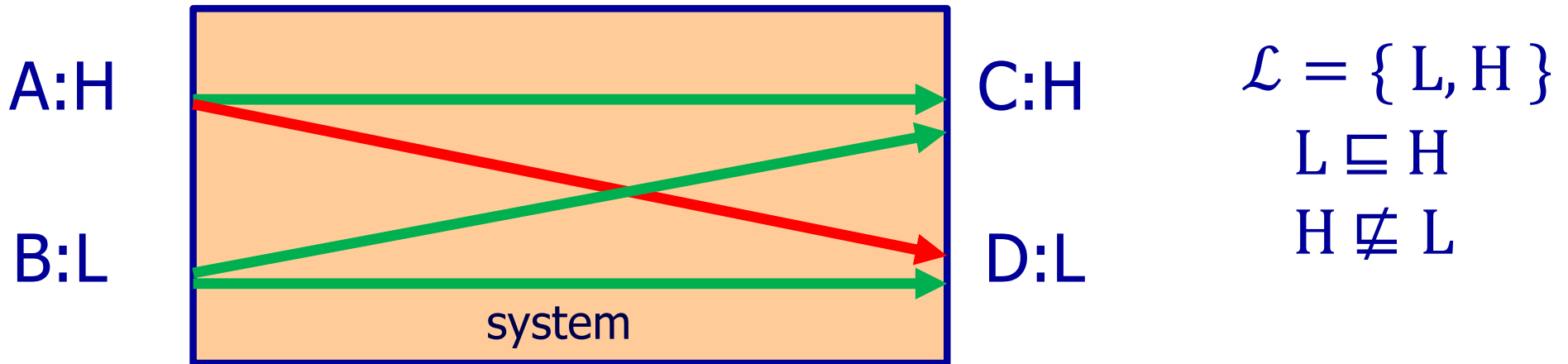
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$$v \rightarrow w \implies \Gamma(v) \sqsubseteq \Gamma(w)$$
$$= \Gamma(v) \not\sqsubseteq \Gamma(w) \implies \neg(v \rightarrow w)$$

# An application of FBAC

---



$$v \rightarrow w \implies \Gamma(v) \sqsubseteq \Gamma(w)$$
$$= \Gamma(v) \not\sqsubseteq \Gamma(w) \implies \neg(v \rightarrow w)$$

- Confidentiality: L: public and H: secret
- Integrity: L: trusted and H: untrusted



# FBAC in General

---

Possible source/destination of flows:

- ports
- people
- variables

# FBAC in Programs

---

Example:  $x := y; \dots z := x$

- $y \rightarrow x, \quad x \rightarrow z$
- $\Gamma(y) \sqsubseteq \Gamma(x), \quad \Gamma(x) \sqsubseteq \Gamma(z).$

# FBAC in Programs

---

Example:  $x := y; \dots z := x$

- $y \rightarrow x, \quad x \rightarrow z$
- $\Gamma(y) \sqsubseteq \Gamma(x), \quad \Gamma(x) \sqsubseteq \Gamma(z).$ 
  - Conclude: If  $y \rightarrow z$  then  $\Gamma(y) \sqsubseteq \Gamma(z)$  also must hold.
  - Nb.  $\rightarrow$  is not necessarily transitive.

# Agenda

---

- Formalize Flow:  $v \rightarrow v'$ 
  - Examples for intuition
  - Formal definitions
- Derive policies FBAC enforces:
  - Confidentiality
  - Integrity
- Means of enforcement
  - Static
  - Dynamic

# $v \rightarrow v'$ ? Direct Flows in Programs

---

$x := y \bmod 2$

$x := y * 0$

$z := y + 2; x := z$

$z := y + 2; x := z - y$

# $v \rightarrow v'$ ? Direct Flows in Programs

---

$x := y \bmod 2$

$y \rightarrow x$

$x := y * 0$

$\neg (y \rightarrow x)$

$z := y + 2; x := z$

$y \rightarrow x$

$z := y + 2; x := z - y$

$\neg (y \rightarrow x)$

... Illustrates intransitive flow

# $v \rightarrow v'$ ? Indirect Flows in Programs

---

**if  $y > 0$  then  $x := 1$  else  $x := 2$**                        $y \rightarrow x$

**while  $y > 0$  do  $x := x + 1; y := y - 1$  end**                       $y \rightarrow x$

# Definitions for Flow

---

$v \rightarrow w?$

Satisfied if there exist two executions

- that differ only in the initial value of  $v$  –and–
- terminate having different final values of  $w$ .



# $v \rightarrow w?$ Formal Definition

---

Let

$\text{dom}(m) = D$  for  $m \in \text{Mem}$

$\llbracket S \rrbracket: \text{Mem} \rightarrow \text{Mem} \cup \{\perp\}$

$m =_V m': (\forall v \in V: m(v) = m'(v))$

Define  $v \rightarrow w$ :

$(\exists m, m': m =_{D-\{v\}} m' \wedge \llbracket S \rrbracket m \neq \perp \wedge \llbracket S \rrbracket m' \neq \perp$   
 $\wedge \llbracket S \rrbracket m \neq_{\{w\}} \llbracket S \rrbracket m')$

# FBAC in action

---

Partition the set of all program variables:  $V_H$  and  $V_L$

- $V_H = \{v \mid \Gamma(v) = H\}$        $V_L = \{v \mid \Gamma(v) = L\}$ ,
- $L \subseteq H$ .

For all  $v_H \in V_H$ ,  $v_L \in V_L$  FBAC requires

$$\begin{aligned} & v_H \rightarrow v_L \Rightarrow \Gamma(v_H) \subseteq \Gamma(v_L) \\ = & v_H \rightarrow v_L \Rightarrow H \subseteq L \\ = & v_H \rightarrow v_L \Rightarrow \text{false} \\ = & \neg (v_H \rightarrow v_L) \end{aligned}$$

# FBAC in action

---

$$\begin{aligned} & \neg (v_H \rightarrow v_L) \\ &= \neg (\exists m, m': m =_{D-\{v\}} m' \wedge \llbracket S \rrbracket m \neq \perp \wedge \llbracket S \rrbracket m' \neq \perp \\ & \quad \wedge \llbracket S \rrbracket m \neq_{\{w\}} \llbracket S \rrbracket m') \\ &= (\forall m, m': m =_{D-\{v\}} m' \wedge \llbracket S \rrbracket m \neq \perp \wedge \llbracket S \rrbracket m' \neq \perp \\ & \quad \Rightarrow \llbracket S \rrbracket m =_{\{w\}} \llbracket S \rrbracket m') \end{aligned}$$

**Conclusion:** Changes to  $v_H$  do not cause changes to  $v_L$  in terminating executions.

- Confidentiality: H is secret; L is public
- Integrity: H is untrusted; L is trusted.

# Non-interference

---

Generalize variables  $v_H, v_L$  to sets  $V_H, V_L$ .

$$(\forall m, m': m =_{D-V_H} m' \wedge \llbracket S \rrbracket m \neq \perp \wedge \llbracket S \rrbracket m' \neq \perp \\ \Rightarrow \llbracket S \rrbracket m =_{V_L} \llbracket S \rrbracket m')$$

Changes to variables in  $V_H$  do not affect the final values of variables in  $V_L$ . Property (with **terms** often left implicit) is called :

- Termination **i**nsensitive non-interference (TINI)
- Goguen-Meseguer non-interference
- Relational non-interference (RNI)

# Additional Leaks: Termination

---

```
if  $h > 0$   
  then while true do skip end  
  else skip  
fi
```

Termination leaks value of  $h > 0$ .

Value of  $h$  flows to termination:  $h \rightarrow \perp$

# $v \rightarrow \perp?$ Formal Definition

---

$v \rightarrow \perp$ :

$$\begin{aligned} & (\exists m, m': m =_{D-\{v\}} m' \\ & \quad \wedge (\llbracket S \rrbracket m = \perp) \neq (\llbracket S \rrbracket m' = \perp)) \end{aligned}$$

Define  $\Gamma(\perp)$ : Label needed by a principal in order to ascertain whether execution has terminated.

Usually  $\Gamma(\perp) = L$ .

# Derive: Termination Sensitive NI 1/3

---

Flow-Label Invariant:

$$\begin{aligned} & (v \rightarrow w \implies \Gamma(v) \sqsubseteq \Gamma(w)) \wedge (v \rightarrow \perp \implies \Gamma(v) \sqsubseteq \Gamma(\perp)) \\ = & (v \rightarrow w \implies \Gamma(v) \sqsubseteq \Gamma(w)) \wedge (v \rightarrow \perp \implies \Gamma(v) \sqsubseteq L) \\ = & (\Gamma(v) = L) \\ & \vee (\neg(v \rightarrow \perp) \wedge (v \rightarrow w \implies \Gamma(v) \sqsubseteq \Gamma(w))) \end{aligned}$$

# Termination Sensitive NI

2/3

$$\Gamma(v) = L \vee (\neg(v \rightarrow \perp) \wedge (v \rightarrow w \Rightarrow \Gamma(v) \sqsubseteq \Gamma(w)))$$

$$\neg(v \rightarrow \perp):$$

$$= \neg(\exists m, m': m =_{D-\{v\}} m' \wedge (\llbracket S \rrbracket m = \perp) \neq (\llbracket S \rrbracket m' = \perp))$$

$$= (\forall m, m': m =_{D-\{v\}} m' \Rightarrow (\llbracket S \rrbracket m = \perp) = (\llbracket S \rrbracket m' = \perp))$$

$$\neg(v \rightarrow w): \quad \{ \text{since } \llbracket S \rrbracket m \neq \perp \Rightarrow \llbracket S \rrbracket m' \neq \perp \}$$

$$= \neg(\exists m, m': m =_{D-\{v\}} m' \wedge \llbracket S \rrbracket m \neq \perp \wedge (\llbracket S \rrbracket m \neq_{\{w\}} \llbracket S \rrbracket m'))$$

$$= (\forall m, m': m =_{D-\{v\}} m' \wedge \llbracket S \rrbracket m \neq \perp \Rightarrow (\llbracket S \rrbracket m =_{\{w\}} \llbracket S \rrbracket m'))$$



Define: TSNI

$$\begin{aligned} & (\forall m, m': m =_{D-\{V_H\}} m' \Rightarrow \\ & \quad ((\llbracket S \rrbracket m = \perp) = (\llbracket S \rrbracket m' = \perp)) \\ & \quad \wedge (\llbracket S \rrbracket m \neq \perp \Rightarrow (\llbracket S \rrbracket m =_{\{V_L\}} \llbracket S \rrbracket m'))) \end{aligned}$$

# Other Generalizations of $v \rightarrow w$

---

Let  $\text{dom}(m) = D$  for  $m \in \text{Mem}$

$\llbracket S \rrbracket: \text{Mem} \rightarrow \text{Mem}^* \cup \{\perp\}$

$m =_V m': (\forall v \in V: m(v) = m'(v))$

$(m_1 m_2 \dots m_i \dots) \approx_V (m'_1 m'_2 \dots m'_i \dots):$

$(m_1|_V m_2|_V \dots m_i|_V \dots) =^* (m'_1|_V m'_2|_V \dots m'_i|_V)$

where:  $=^*$  is equality of de-stuttered sequences.

Define  $v \rightarrow w$ :

$(\exists m, m': m =_{D-\{v\}} m' \wedge \llbracket S \rrbracket m \neq \perp \wedge \llbracket S \rrbracket m' \neq \perp$

$\wedge \neg (\llbracket S \rrbracket m \approx_{\{w\}} \llbracket S \rrbracket m'))$

# Enforcement of FBAC

---

FLI potentially imposes restrictions on statements.

- **Static Enforcement**

- Compiler ensures program is type correct.
- Type correct programs will satisfy restrictions.

- **Dynamic Enforcement**

- Insert run-time checks that halt program execution about to violate restrictions.
- Change labels to satisfy restrictions as program execution proceeds.

# Toy Language

---

$e ::= x \mid n \mid e_1 + e_2 \mid \dots$

$c ::= x := e$   
| if  $e$  then  $c_1$  else  $c_2$  fi  
| while  $e$  do  $c$  end  
|  $c_1; c_2$

Restrictions for:

# Assignment $x := e$

---

$$v \rightarrow w \implies \Gamma(v) \sqsubseteq \Gamma(w)$$

$x := y$  causes  $y \rightarrow x$

- requires  $\Gamma(y) \sqsubseteq \Gamma(x)$

$x := y+z$  causes  $y \rightarrow x$  and  $z \rightarrow x$

- requires:  $\Gamma(y+z) \sqsubseteq \Gamma(x)$
- implied by:  $\Gamma(y) \sqcup \Gamma(z) \sqsubseteq \Gamma(x)$

Restrictions for:

# Assignment $x := E$

---

$x := E$  causes  $E \rightarrow x$

define  $E \rightarrow x$ :  $(\forall v \in E: v \rightarrow x)$

define  $\Gamma(E)$ :  $(\sqcup \Gamma(v) \in E)$

where  $\lambda \sqcup \lambda'$  is smallest label satisfying

$\lambda \sqsubseteq \lambda \sqcup \lambda'$  and  $\lambda' \sqsubseteq \lambda \sqcup \lambda'$

Restrictions for:

# Assignment $x := E$

---

$x := E$  causes  $E \rightarrow x$

define  $E \rightarrow x$ :  $(\forall v \in E: v \rightarrow x)$

define  $\Gamma(E)$ :  $(\sqcup \Gamma(v) \in E)$

where  $\lambda \sqcup \lambda'$  is smallest label satisfying

$\lambda \sqsubseteq \lambda \sqcup \lambda'$  and  $\lambda' \sqsubseteq \lambda \sqcup \lambda'$

$x := E$  causes  $E \rightarrow x$

– requires  $(\sqcup \Gamma(v) \in E) \sqsubseteq \Gamma(x)$

# Restrictions for: If Statements

---

**if**  $y > 0$  **then**  $x := 1$  **else**  $x := 2$  **fi**



# Restrictions for: If Statements

---


**if**  $y > 0$  **then**  $x := 1$  **else**  $x := 2$  **fi**



# Restrictions for: If Statements

---

**if**  $y > 0$  **then**  $x := 1$  **else**  $x := 2$  **fi**




$y > 0 \rightarrow pc, \quad pc \rightarrow x,$

# Restrictions for: If Statements

---

**if**  $y > 0$  **then**  $x := 1$  **else**  $x := 2$  **fi**



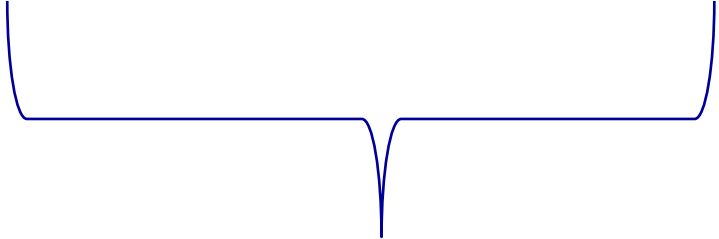
$y > 0 \rightarrow pc$ ,  $pc \rightarrow x$ ,  $y > 0 \rightarrow x$

$y > 0 \rightarrow x$  requires  $\Gamma(y > 0) \sqsubseteq \Gamma(x)$

# Restrictions for: If Statements

---

**if**  $y > 0$  **then**  $x := 1$  **else**  $x := 2$  **fi**


$$\begin{aligned} ctx &= \Gamma(y > 0) \\ &= \Gamma(y) \sqcup \Gamma(0) \\ &= \Gamma(y) \end{aligned}$$

# Restrictions for: If Statements

---

**if**  $B$  **then**  $x := E$  **else** ... **fi**  
 $B \rightarrow x, \quad E \rightarrow x$

# Restrictions for: If Statements

---

**if**  $B$  **then**  $x := E$  **else** ... **fi**

$B \rightarrow x, \quad E \rightarrow x$

requires:  $\Gamma(B) \sqsubseteq \Gamma(x), \quad \Gamma(E) \sqsubseteq \Gamma(x)$

# Restrictions for: If Statements

---

**if**  $B$  **then**  $x := E$  **else** ... **fi**

$B \rightarrow x, \quad E \rightarrow x$

requires:  $\Gamma(B) \sqsubseteq \Gamma(x), \quad \Gamma(E) \sqsubseteq \Gamma(x)$

implied by:

$\text{ctx} = \Gamma(B)$

$\text{ctx} \sqcup \Gamma(E) \sqsubseteq \Gamma(x)$

# Restrictions for: Nested If Statements

---

```
if z > 0
  then y := 23
       if y > 0
          then x := 1
          else u := 2
       fi
  else
    w := 3
fi
```



# Restrictions for: Nested If Statements

---

if  $z > 0$

then  $y := 23$

if  $y > 0$

then  $x := 1$  ---  $\text{ctx} = \Gamma(y)$

else  $u := 2$  ---  $\text{ctx} = \Gamma(y)$

fi

else

$w := 3$

fi

# Restrictions for: Nested if Statements

---

if  $z > 0$

then

$y := 23$  ---  $ctx = \Gamma(z)$

if  $y > 0$

then  $x := 1$  ---  $ctx = \Gamma(y) \sqcup \Gamma(z)$

else  $u := 2$  ---  $ctx = \Gamma(y) \sqcup \Gamma(z)$

fi

else

$w := 3$  ---  $ctx = \Gamma(z)$

fi

# A Type System

---

- Fixed label assignment  $\Gamma$
- Goal:
  - Type correctness implies Flow-Label invariant will hold throughout executions.  
$$v \rightarrow w \implies \Gamma(v) \sqsubseteq \Gamma(w)$$
  - Flow-Label invariant implies RNI will hold throughout executions.

# Type Systems: Big Picture

---

“Program S is type correct” is a theorem in a logic (say)  $\text{secL}$ .

- Logic is decidable.
  - Compiler’s type checker “proves” these theorems.
- Logic is sound with respect to:
  - “Program S satisfies FLI invariant”

# Formulas of secL

---

Formulas of secL are called judgements.

Formulas of secL are given as sequents:

- $\Gamma, ctx \vdash Expr: \lambda$  for expression  $Expr$ , label  $\lambda$
- $\Gamma, ctx \vdash S$  for statement  $S$

Inference rules give premises and conclusion

$$\frac{P_1, P_2, \dots, P_n}{C}$$

# Rules for Expressions

---

- Constant:  $\frac{}{\Gamma, \text{ctx} \vdash n : L}$
- Variable:  $\frac{\Gamma(x)=\lambda}{\Gamma, \text{ctx} \vdash x : \lambda}$
- Expression:  $\frac{\Gamma, \text{ctx} \vdash e : \lambda \quad \Gamma, \text{ctx} \vdash e' : \lambda'}{\Gamma, \text{ctx} \vdash e+e' : \lambda \sqcup \lambda'}$

# A Proof

(1/3)

---

Given  $\Gamma(x) = L$  and  $\Gamma(y) = H$ :

$$\frac{\Gamma(x) = L}{\Gamma, \text{ctx} \vdash x : L}$$

# A Proof

(2/3)

---

Given  $\Gamma(x) = L$  and  $\Gamma(y) = H$ :

$$\frac{\Gamma(x) = L}{\Gamma, \text{ctx} \vdash x : L} \quad \frac{\Gamma(y) = H}{\Gamma, \text{ctx} \vdash y : H}$$



# A Proof

(3/3)

---

Given  $\Gamma(x) = L$  and  $\Gamma(y) = H$ :

$$\frac{\frac{\Gamma(x) = L}{\Gamma, \text{ctx} \vdash x : L} \quad \frac{\Gamma(y) = H}{\Gamma, \text{ctx} \vdash y : H}}{\Gamma, \text{ctx} \vdash x + y : L \sqcup H}$$

Conclusion:  $x+y : H$  (since  $L \sqcup H = H$ )

# Assignment Rule

---

$x := E$

- causes:  $E \rightarrow x$
- requires:  $\Gamma(E) \sqsubseteq \Gamma(x)$

$$\text{Assign: } \frac{\Gamma, \text{ctx} \vdash E : \lambda, \lambda \sqcup \text{ctx} \sqsubseteq \Gamma(x)}{\Gamma, \text{ctx} \vdash x := E}$$

# if Rule

---

$$\frac{\Gamma, \text{ctx} \vdash e : \lambda, \quad \Gamma, \lambda \sqcup \text{ctx} \vdash C_1, \quad \Gamma, \lambda \sqcup \text{ctx} \vdash C_2}{\Gamma, \text{ctx} \vdash \mathbf{if\ e\ then\ } C_1 \mathbf{\ else\ } C_2 \mathbf{\ fi}}$$

# if Rule Example Proof

---

1. Constant:

$$\frac{}{\Gamma, (L \sqcup \text{ctx}) \vdash 1:L}$$

2. Assign:

$$\frac{\Gamma, (L \sqcup \text{ctx}) \vdash 1:L, \quad L \sqcup (L \sqcup \text{ctx}) \sqsubseteq \Gamma(x)}{\Gamma, (L \sqcup \text{ctx}) \vdash x:=1}$$

3. if

$$\frac{\Gamma, \text{ctx} \vdash y>0 : L \quad \Gamma, L \sqcup \text{ctx} \vdash x:=1 \quad \Gamma, L \sqcup \text{ctx} \vdash x:=2}{\Gamma, \text{ctx} \vdash \text{if } y>0 \text{ then } x:=1 \text{ else } x:=2 \text{ fi}}$$

# while Rule

---

while:

$$\frac{\Gamma, \text{ctx} \vdash e : \lambda \quad \Gamma, \lambda \sqcup \text{ctx} \vdash C}{\Gamma, \text{ctx} \vdash \mathbf{while\ e\ do\ c\ end}}$$

; (sequence) rule

---

$$\frac{\Gamma, \text{ctx} \vdash C_1, \quad \Gamma, \text{ctx} \vdash C_2}{\Gamma, \text{ctx} \vdash C_1 ; C_2}$$

# secL Type System Retrospective

---

- Soundness

- Type correct programs satisfy
  - Flow-Label Invariant:  $v \rightarrow w \implies \Gamma(v) \sqsubseteq \Gamma(w)$
  - Relational non-interference (RNI)
- If program doesn't satisfy
  - Flow-Label invariant or
  - RNI

then program won't be type correct.

# secL Type System Retrospective

---

- (in)Completeness

- The type system is incomplete.
- If a program is not type correct then that program might still satisfy Flow-Label invariant and RNI.

$$\frac{\Gamma, \text{ctx} \vdash y * 0 : H, \quad H \sqcup L \sqsubseteq \Gamma(x)}{\Gamma, L \vdash x := y * 0}$$

If  $\Gamma(x) = L \dots$

- Type checking fails



# secL Type System Retrospective

---

- (in)Completeness
  - The type system is incomplete.
  - If a program is not type correct then that program might still satisfy Flow-Label invariant and RNI.

$$\frac{\Gamma, \text{ctx} \vdash y * 0 : H, \quad H \sqcup L \sqsubseteq \Gamma(x)}{\Gamma, L \vdash x := y * 0}$$

If  $\Gamma(x) = L \dots$

- Type checking fails
- FLI invariant valid
- RNI satisfied.

# Eliminate Incompleteness?

---

Sequence rule

$$\frac{\Gamma, \text{ctx} \vdash C_1, \quad \Gamma, \text{ctx} \vdash C_2}{\Gamma, \text{ctx} \vdash C_1 ; C_2}$$

Consider:

**if**  $h > 0$  **then**  $C$ ;  $v_L := 2$  **else skip fi**

- Satisfies RNI (=termination insensitive) if  $C$  diverges.
- Sequence rule must predict that  $C_1$  diverges.
  - Predicting divergence requires solving the halting problem.

# Program with Termination Channel

---

**while**  $v_H > 0$  **do skip end;**  $v_L := 2$

- Program is secL type correct.
- Program satisfies RNI.
- Program does not satisfy termination sensitive non interference (TSNI):  $v_H \rightarrow \perp$

# Type system for TSNI

---

Prevent channel arising from infinite loops.

- Allow only L terms in **while** guards.
  - Loop termination does not depend of H values.

$$\frac{\Gamma, \text{ctx} \vdash e:L \quad \Gamma, \text{ctx} \vdash C}{\Gamma, \text{ctx} \vdash \mathbf{while\ } e \mathbf{\ do\ } C \mathbf{\ end}}$$

- Type correct programs now exhibit TSNI.
- What about loops involving H terms?