# CS 5154: Software Testing 

## Active Clause Coverage

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## Recall the four software models in this course



A: $\{0,1,>1\}$
B: $\{600,700,800\}$
C: \{cs, ece, is, sds\}


Syntax


## We need criteria that are not as costly as CoC

- The general idea is quite simple:


## Test each clause independently from the other clauses

- But, getting the details right is hard
- e.g., what exactly does "independently" mean ?
- The book presents this idea as "making clauses active" ...


## Active Clauses

- A weakness of Clause Coverage: values do not always make a difference
- Values $((5<10) \vee$ true $) \wedge(1>=1 * 1)$ for $((a<b) \vee D) \wedge\left(m>=n^{*} o\right)$
- Only the last clause counts!
- To really test the results of a clause, the clause should be the determining factor in what the predicate evaluates to

Determination
A clause $c_{i}$ in predicate $p$, called the major clause, determines $p$ if and only if the values of the remaining minor clauses $\mathrm{c}_{\mathrm{j}}$ are such that changing $c_{i}$ changes the value of $p$

- Making $c_{i}$ determine $p$ is said to make the clause active
- Condition under which $c_{i}$ determines $p$

$$
\forall c_{j} \in c_{p} \backslash c_{i} \exists \text { assignment }\left(c_{j}\right) \text { st. } p\left(c_{i}=\text { true }\right) \neq p\left(c_{i}=\text { fobs }\right)
$$

$$
\text { Where a ssifrment }\left(c_{j}\right) \text { is } c_{j}=\text { pone or } c_{j}=\text { false }
$$

## Acfine chase cúnteria <br> The essence of testing with determination

1. Pick one clause in predicate $p$ to be the major clause $c_{i}$
2. Find conditions under which $c_{i}$ determines $p$
3. Find a test that makes $c_{i}$ true and a test that makes $c_{i}$ false
4. Repeat steps 1 to 3 for all other clauses in $p$
5. Eliminate redundant tests

## Examples: determining predicates

$$
P=A \vee B
$$

if $B=$ true, $p$ is always true.
so if $B=$ false, $A$ determines $p$.
if $A=$ false, $B$ determines $p$.

## $P=A \wedge B$

if $B=$ false, $p$ is always false.
so if $B=$ true, $A$ determines $p$.
if $A=$ true, $B$ determines $p$.

|  | $\mathbf{a}$ | $\mathbf{b}$ | $\mathbf{a} \vee \mathrm{b}$ |
| :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | T | T | T |
| $\mathbf{2}$ | T | F | T |
| 3 | F | T | T |
| $\mathbf{4}$ | F | F | F |


|  | $\mathbf{a}$ | $\mathbf{b}$ | $\mathbf{a} \wedge \mathbf{b}$ |
| :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | T | T | T |
| $\mathbf{2}$ | T | F | F |
| 3 | F | T | F |
| $\mathbf{4}$ | F | F | F |

## More examples: determining predicates

## $P=A \oplus B$

if $B=$ true, $A$ determines $p$.
if $B=$ false, $\boldsymbol{A}$ determines $p$.
so, $A$ determines $p$ for any $B$.

|  | $\mathbf{a}$ | $\mathbf{b}$ | $\mathbf{a} \oplus \mathbf{b}$ |
| :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | T | T | F |
| $\mathbf{2}$ | T | F | T |
| 3 | F | T | T |
| $\mathbf{4}$ | F | F | F |

## $P=A \leftrightarrow B$

if $B=$ true, $A$ determines $p$.
if $B=$ false, $A$ determines $p$.
so, $A$ determines $p$ for any $B$.

|  | $\mathbf{a}$ | $\mathbf{b}$ | $\mathbf{a} \leftrightarrow \mathbf{b}$ |
| :---: | :---: | :---: | :---: |
| 1 | $\mathbf{T}$ | $\mathbf{T}$ | T |
| 2 | T | F | F |
| $\mathbf{3}$ | F | $\mathbf{T}$ | F |
| 4 | F | F | T |

## Testing with determination - )

- Goal : Find tests for each clause when that clause determines the value of the predicate
- This goal is formalized in a family of criteria that have subtle, but very important, differences


## Active Clause Coverage

- Step 1: For each $p$ in $P$ and each major clause $c_{i}$ in $C p$, choose minor clauses $c_{j, j!=i}$, so that $c_{i}$ determines $p$.

Active Clause Coverage (ACC) : TR has two requirements for each $c_{i}: c_{i}$ evaluates to true and $c_{i}$ evaluates to false.

- ACC is a form of Multiple Condition Decision Coverage (MCDC)
- MCDC is required by the FAA for safety-critical software


## Example on Active Clause Coverage



## A formulaic way of determining predicates?

- Finding values for minor clauses $c_{j}$ is easy for simple predicates
- How to find values for more complicated predicates?
- We need some "formula" that is easy to apply


## A definitional way: when does $c$ determine $p$ ?

- Let $p_{c=t r u e}$ be predicate $p$ with every occurrence of $c$ replaced by true
- Let $p_{c=f a l s e}$ be predicate $p$ with every occurrence of $c$ replaced by false
- To find values for the minor clauses, connect $p_{c=t r u e}$ and $p_{c=f a l s e}$ with XOR

$$
p_{c}=p_{c=t r u e} \oplus p_{c=\text { false }}
$$

- After solving, $p_{c}$ describes exactly the values needed for $c$ to determine $p$


## An example using the definitional way

- Let $\mathrm{p}=\mathrm{a} \vee(\mathrm{b} \wedge \mathrm{c})$. What values of b and c will cause a to determine p ?

$$
\begin{aligned}
& \quad p=a \vee(b \wedge c) \\
p_{a} & =p_{a=\text { true }} \oplus p_{a=\text { false }} \\
& =(\text { true } \vee(b \wedge c)) \oplus(\text { false } \vee(b \wedge c)) \\
& =\operatorname{true} \oplus(b \wedge c) \\
& =!(b \wedge c) \\
& =!b \vee!c
\end{aligned}
$$

- "! $b \vee!c$ " means a determines $p$ when either $b$ or $c$ is false


## Exercise 1: using the definitional way

- Let $p=a \vee b$. What values of $b$ will cause $a$ to determine $p$ ?

$$
\begin{aligned}
p_{a} & =p_{a=t r u e} \oplus p_{a=\text { false }} \\
& =(\text { true } \vee b) \oplus(\text { false } \vee b) \\
& =\text { true } \oplus b \\
& =!b
\end{aligned}
$$

|  | $\mathbf{a}$ | $\mathbf{b}$ | $\mathbf{a} \vee \mathrm{b}$ |
| :---: | :---: | :---: | :---: |
| 1 | T | T | T |
| 2 | T | F | T |
| 3 | F | T | T |
| 4 | F | F | F |

- "! $b$ " means a determines $p$ when $b$ is false
- We obtained the same result from reasoning about the truth table


## Exercise 2: using the definitional way

- Let $\mathrm{p}=\mathrm{a} \leftrightarrow \mathrm{b}$. What values of b will cause $a$ to determine $p$ ?

|  | $\quad p=a \leftrightarrow b$ |
| ---: | :--- |
| $p_{a}$ | $=p_{a=t r u e} \oplus p_{a=\text { false }}$ |
|  | $=($ true $\leftrightarrow b) \oplus($ false $\leftrightarrow b)$ |
|  | $=b \oplus!b$ |
|  | $=$ true |


|  | $\mathbf{a}$ | $\mathbf{b}$ | $\mathbf{a} \leftrightarrow \mathbf{b}$ |
| :---: | :---: | :---: | :---: |
| 1 | T | T | T |
| 2 | T | F | F |
| 3 | F | T | F |
| $\mathbf{4}$ | F | F | T |

- "true" means that a always determines $p$
- We obtained the same result from reasoning about the truth table


## Is there a problem with Active Clause Coverage?

- Step 1: For each $p$ in $P$ and each major clause $c_{i}$ in $C p$, choose minor clauses $c_{j}, j!=i$, so that $c_{i}$ determines $p$.

Active Clause Coverage (ACC) : TR has two requirements for each $c_{i}: c_{i}$ evaluates to true and $c_{i}$ evaluates to false.

- Ambiguity : Must minor clauses have the same values when the major clause is true and when the major clause is false?


## Illustrating the ambiguity in ACC

- Recall: a determines $p$ when "! $b \vee!c$ ", i.e., when either $b$ or $c$ is false

```
p=a\vee(b\wedgec)
Major clause : a
a = true, b = false, c = true
a = false, b = false, c = false

\section*{Three options for resolving ACC ambiguity}
- Minor clauses do not need to be the same
- Minor clauses must be the same
- Minor clauses allow the predicate to become both true and false

\section*{Option 1: minor clauses don't need to be the same}
- Step 1: For each \(p\) in \(P\) and each major clause \(c_{i}\) in \(C_{p}\), choose minor clauses \(c_{j}, j!=i\), so that \(c_{i}\) determines \(p\).
- Step 2 (ACC): TR has two requirements for each \(c_{i}\) : \(c_{i}\) evaluates to true and \(c_{i}\) evaluates to false.

General Active Clause Coverage (GACC) : The values chosen for the minor clauses \(c_{j}\) do not need to be the same when \(c_{i}\) is true as when \(c_{i}\) is false, that is, \(c_{j}\left(c_{i}=\right.\) true \()=c_{j}\left(c_{i}=\right.\) false \()\) for all \(c_{j}\) OR \(c_{j}\left(c_{i}=\right.\) true \()!=c_{j}\left(c_{i}=\right.\) false) for all \(c_{j}\).

\section*{Problem: GACC doesn’t subsume Predicate Coverage}

Major clause : a
\begin{tabular}{c|c|c|c|}
\hline & \(\mathbf{a}\) & \(\mathbf{b}\) & \(\mathbf{a} \leftrightarrow \mathbf{b}\) \\
\hline 1 & T & T & T \\
2 & T & F & F \\
3 & F & T & F \\
\(\mathbf{4}\) & F & F & T \\
\hline
\end{tabular}

\section*{Option 2: minor clauses do need to be the same}
- Step 1: For each \(p\) in \(P\) and each major clause \(c_{i}\) in \(C_{p}\), choose minor clauses \(c_{j}, j!=i\), so that \(c_{i}\) determines \(p\).
- Step 2 (ACC): TR has two requirements for each \(c_{i}: c_{i}\) evaluates to true and \(c_{i}\) evaluates to false.

Restricted Active Clause Coverage (RACC) : The values chosen for the minor clauses \(c_{j}\) must be the same when \(c_{i}\) is true as when \(c_{i}\) is false, that is, it is required that \(c_{j}\left(c_{i}=\right.\) true \()=c_{j}\left(c_{i}=\right.\) false) for all \(c_{j}\).

\section*{Exercise 3: using the definitional way}
- Let \(\mathrm{p}=\mathrm{a} \wedge(\mathrm{b} \vee \mathrm{c})\). What values of b and c will cause a to determine p ?
\[
\begin{aligned}
& \quad p=a \wedge(b \vee c) \\
p_{a} & =p_{a=\text { true }} \oplus p_{a=\text { false }} \\
& =(\text { true } \wedge(b \vee c)) \oplus(\text { false } \wedge(b \vee c)) \\
& =(b \vee c) \oplus \text { false } \\
& =(b \vee c) \\
& =b \vee c
\end{aligned}
\]
- " \(b \vee c\) " means a determines \(p\) when either \(b\) or \(c\) is true

\section*{Example on Restricted Active Clause Coverage}

Major clause : \(\mathrm{a}, \mathrm{P}_{\mathrm{a}}=\mathbf{b} \vee \mathbf{c}\)
\begin{tabular}{|c|c|c|c|c|}
\hline & a & b & c & a \(\wedge(b \vee c)\) \\
\hline \(\mathbf{1}\) & T & T & T & T \\
2 & T & T & F & T \\
3 & T & F & T & T \\
\(\mathbf{4}\) & T & F & F & F \\
\hline \(\mathbf{5}\) & F & T & T & F \\
\(\mathbf{6}\) & F & T & F & F \\
\(\mathbf{7}\) & F & F & T & F \\
\(\mathbf{8}\) & F & F & F & F \\
\hline
\end{tabular}

> RACC \(\left(c_{i}=a\right)\) can only be satisfied by row pairs \((1,5),(2,6)\), or \((3,7)\)
> Only three pairs can be used

\section*{Notes on RACC}
- Does RACC subsume predicate and clause coverage?
- RACC was a common interpretation by developers for FAA
- Problem: RACC often leads to infeasible test requirements

\section*{Option 3: minor clauses allow predicate to be true and false}
- Step 1: For each \(p\) in \(P\) and each major clause \(c_{i}\) in \(C p\), choose minor clauses \(c_{j}, j!=i\), so that \(c_{i}\) determines \(p\).
- Step 2 (ACC): TR has two requirements for each \(c_{i}: c_{i}\) evaluates to true and \(c_{i}\) evaluates to false.

Correlated Active Clause Coverage (CACC) : The values chosen for the minor clauses \(c_{j}\) must cause \(p\) to be true for one value of the major clause \(c_{i}\) and false for the other, that is, it is required that \(p\left(c_{i}=t r u e\right)!=\) \(p\left(c_{i}=\right.\) false).

\section*{Example on CACC}

\[
\text { a determines } P \text { when ( } b=\text { true or } c=\text { true) }
\]

CACC ( \(\mathrm{c}_{\mathrm{i}}=\mathrm{a}\) ) can be satisfied by choosing any of rows 1, 2, 3 AND any of rows 5, 6, 7-a total of nine pairs

\section*{Notes on CACC}
- CACC implicitly allows minor clauses to have different values
- CACC explicitly subsumes predicate coverage
- Does CACC subsume clause coverage?

\section*{Does CACC subsume clause coverage?}

\[
\text { a determines } P \text { when ( } b=\text { true or } c=\text { true) }
\]

CACC ( \(\mathrm{c}_{\mathrm{i}}=\mathrm{a}\) ) can be satisfied by choosing any of rows 1, 2, 3 AND any of rows 5, 6, 7-a total of nine pairs

\section*{Infeasibility}
- Consider the predicate: \((a>b \wedge b>c) \vee c>a\)
- Infeasible: \((a>b)=\) true, \((b>c)=\operatorname{true},(c>a)=\) true is infeasible
- As with other criteria, infeasible test requirements must be recognized and dealt with
- Recognizing infeasible test requirements is hard, and in general, undecidable

\section*{Subsumption among Logic coverage criteria}


\section*{An end-to-end example with RACC}
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline & & & & \multicolumn{5}{|l|}{\begin{tabular}{lll}
\(b \&\) & Al & Fin \\
For & Likewise, for clause \(c\), only one pair, \\
diffe & ce cal \\
determme TIE & TFT and TFF, cause \(c\) to determine the
\end{tabular}
value of \(p\)} \\
\hline & a & b & C & \(a \wedge(b \vee c)\) & \(P_{a}\) & \(P_{b}\) & \(P_{c}\) & \\
\hline I & T & T & T & T & 4 & & & In sum, three \\
\hline 2 & T & T & F & T & & 3 & & separate pairs of \\
\hline 3 & T & F & T & T & \(\cdots\) & & 3 & rows can cause a to determine the \\
\hline 4 & T & F & F & F & & 3 &  & value of \(p\), and \\
\hline 5 & F & T & T & F &  & & & only one pair each for \(b\) and \(c\) \\
\hline 6 & F & T & F & F &  & & & \\
\hline 7 & F & F & T & F & 3 & & & \\
\hline 8 & F & F & F & F & & & & \\
\hline
\end{tabular}

How many tests does RACC yield, compared to Combinatorial
Clause Coverage?

\section*{A more subtle exercise on determination}
\[
p=(a \wedge b) \vee(a \wedge!b)
\]
\[
\begin{aligned}
p_{\mathrm{a}} & =p_{\mathrm{a}=\text { true }} \oplus p_{a=\text { false }} \\
& =((\text { true } \wedge b) \vee(\text { true } \wedge!b)) \oplus((\text { false } \wedge b) \vee(\text { false } \wedge!b)) \\
& =(b \vee!b) \oplus \text { false } \\
& =\text { true } \oplus \text { false } \\
& =\text { true }
\end{aligned}
\]
\[
p=(a \wedge b) \vee(a \wedge \neg b)
\]
\[
\begin{aligned}
p_{b} & =p_{b=t r u e} \oplus p_{b=\text { false }} \\
& =((a \wedge \text { true }) \vee(a \wedge \neg \text { true })) \oplus((a \wedge \text { false }) \vee(a \wedge \neg \text { false })) \\
& =(a \vee \text { false }) \oplus(\text { false } \vee a) \\
& =a \oplus a \\
& =\text { false }
\end{aligned}
\]

\title{
A more subtle exercise on determination (2)
}

\section*{\(p=(a \wedge b) \vee(a \wedge!b)\)}
- \(a\) always determines the value of this predicate
- \(b\) never determines the value \(-b\) is irrelevant !
- So, why would anyone write a predicate like this?

\section*{Logic Coverage Summary}
- Predicates are often very simple-in practice, most have <3 clauses
- In fact, most predicates only have one clause!
- Only clause? PC is enough
- 2 or 3 clauses? CoC is practical
- Advantages of ACC criteria can be significant for large (no. of) predicates
- CoC is impractical for predicates with many clauses

Next
- Applying Logic Coverage to source code```

